Chapter 4

WAVE PROPAGATION IN THERMOELASTIC PLATES WITH VOIDS

4.1 INTRODUCTION

The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. It concerns with elastic materials consisting of a distribution of small pores (voids) and in which the void volume is included in kinematics variables. This theory has practical utility for investigating the various types of geological and biological materials to which the elastic theory is inadequate.

In this chapter, the propagation of thermoelastic waves in homogeneous isotropic plates, both rectangular and cylindrical, with voids subjected to stress free, insulated / isothermal and rigidly fixed, insulated / isothermal boundary conditions has been investigated. The Lord-Shulman nonclassical theory of thermoelasticity has been employed to study the problem. Secular equations for the plate in the closed form and isolated mathematical conditions for symmetric and skew symmetric wave mode propagation in completely separate terms are derived. It is observed that the motion for SH modes get decoupled from rest of the motion and remains unaffected due to thermo-mechanical coupling, thermal relaxation and voids effects. The results for coupled and uncoupled theories of thermoelasticity have been obtained as particular cases from the derived secular equations. At short wavelength limits, the secular equations for symmetric and skew symmetric waves in stress free insulated and isothermal plate reduce to Rayleigh surface frequency equations. It is also noticed that the Rayleigh-Lamb type equation also govern circular crested thermoelastic waves in a plate with
voids as was in case of elastokinetics. Although the frequency wave number relationship holds whether the waves are plane or radial, the displacement and stresses vary according to Bessel functions rather than trigonometric functions as far as radial coordinate is concerned. Finally, the analytical results obtained have been computed numerically and the corresponding dispersion curves, attenuation coefficient, thermomechanical coupling and specific loss profiles, for symmetric and skew-symmetric wave modes are presented graphically in order to illustrate and compare the theoretical result.

4.2 FORMULATION OF THE PROBLEM

We consider a homogeneous isotropic, thermally conducting elastic plate with voids of thickness 2d in the undeformed state and at uniform temperature $T_0$ initially. We take origin of the rectangular cartesian coordinate system in the middle plane of the plate such that $-\infty < x < \infty$, $-\infty < y < \infty$ and $-d < z < d$. The x-y plane is chosen to coincide with the middle plane of the plate and z-axis normal to it along the thickness as illustrated in the Figure 4.1. The field quantities are assumed to be independent of y-coordinate.

![Figure 4.1: Geometry of the problem.](image-url)
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The surfaces \( z = \pm d \) are assumed to be (i) stress free, thermally insulated / isothermal and (ii) rigidly fixed thermally insulated / isothermal boundaries. The basic governing equations and constitutive relations (1.5.2)-(1.5.5) for thermoelastic solid with voids, in the absence of body forces, equilibrated forces and heat sources, in this case are given by:

\[
\mu \nabla^2 \ddot{u} + (\lambda + \mu) \nabla \cdot \ddot{u} + b \nabla \phi - \beta \nabla T = \rho \ddot{u} \quad (4.2.1)
\]

\[
\alpha \nabla^2 \phi - b \nabla \cdot \ddot{u} - \xi_1 \phi - \xi_2 \dot{\phi} + mT = \rho \chi \ddot{\phi} \quad (4.2.2)
\]

\[
K \nabla^2 T - \rho C_r \left( \dot{T} + t_0 T \right) - mT \left( \ddot{\phi} + t_0 \phi \right) = \beta t_0 \nabla \left( \ddot{u} + t_0 \dot{u} \right) \quad (4.2.3)
\]

\[
\sigma_{ij} = \lambda e_{ik} \delta_{ij} + 2\mu e_{ij} + (b \phi - \beta T) \delta_{ij}, \quad i, j = x, y, z \quad (4.2.4)
\]

where \( \ddot{u}(x, z, t) = (u, 0, w) \) is the displacement vector, \( T(x, z, t) \) is the temperature change, \( \lambda, \mu \) are Lame' parameters; \( K \) is thermal conductivity; \( \rho \) and \( C_r \) are respectively the density and specific heat at constant strain; \( \phi \) is change in volume fraction, \( \beta = (3\lambda + 2\mu) \alpha \), \( \alpha \) is linear thermal expansion; \( \sigma_{ij} \) and \( e_{ij} \) are respectively stress and strain tensor; \( \alpha, b, \xi_1, \xi_2, m \) and \( \chi \) are material constants due to presence of voids and \( \delta_{ij} \) is Kronecker's delta.

Upon using relevant quantities from equation (2.2.4), the governing equations and constitutive relations (4.2.1)-(4.2.4) in non-dimensional form can be written as
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\[ \delta^2 \nabla^2 \ddot{u} + (1 - \delta^2) \nabla \cdot \ddot{u} + a_i \nabla \phi - \nabla T = \ddot{u} \] \hspace{1cm} (4.2.5)

\[ \nabla^2 \phi - a_2 (\nabla \cdot \ddot{u}) - a_3 (\phi + \xi \dot{\phi}) + a_4 T = \frac{\dot{\phi}}{\delta_i^2} \] \hspace{1cm} (4.2.6)

\[ \nabla^2 T - (1 + t_0 \ddot{u}) - \xi \nabla \cdot (\ddot{u} + t_0 \ddot{u}) - a_3 \left( \phi + t_0 \ddot{\phi} \right) = 0 \] \hspace{1cm} (4.2.7)

\[ \sigma_{ij} = (1 - 2\delta^2) e_{ij} \delta_{ij} + 2\delta^2 e_{ij} + (a_3 \phi - T) \delta_{ij} \] \hspace{1cm} (4.2.8)

where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \).

### 4.3 BOUNDARY CONDITIONS

The non-dimensional mechanical and thermal boundary conditions at \( z = \pm d \) are:

(i) Stress free boundary

\[ \sigma_{zz} = 0 , \quad \sigma_{xz} = 0 , \quad T_{zz} + hT = 0 , \quad \phi_{zz} = 0 \] \hspace{1cm} (4.3.1)

(ii) Rigidly fixed boundary

\[ u = w = 0 , \quad T_{zz} + hT = 0 , \quad \phi_{zz} = 0 \] \hspace{1cm} (4.3.2)

where \( h \) is the surface heat transfer coefficient with \( h \to 0 \) corresponding to thermally insulated boundary and \( h \to \infty \) referring to isothermal one.

### 4.4 SOLUTION OF THE PROBLEM

Introducing the potential functions \( G \) and \( \psi = (0, -\psi, 0) \), through the relations

\[ \ddot{u} = \nabla G + \nabla \times \psi , \quad \nabla \cdot \psi = 0 \] \hspace{1cm} (4.4.1)

in equations (4.2.5)-(4.2.7), we obtain

\[ \nabla^2 G - \ddot{G} + a_1 \phi - T = 0 \] \hspace{1cm} (4.4.2)

\[ - a_2 \nabla^2 G + \left( \nabla^2 - a_2 (1 + \xi \frac{\partial}{\partial t} - \frac{1}{\delta_i^2} \frac{\partial^2}{\partial t^2}) \phi + a_4 T = 0 \right. \] \hspace{1cm} (4.4.3)
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\[(\nabla^2 - \frac{\partial}{\partial t} - t_0 \frac{\partial^2}{\partial t^2})T - \varepsilon_T \nabla^2 \left( \ddot{G} + t_0 \dddot{G} \right) - a_3 \left( \ddot{\phi} + t_0 \dddot{\phi} \right) = 0 \] (4.4.4)

\[\nabla^2 \psi - \frac{1}{\delta^2} \psi = 0 \] (4.4.5)

We take the solution of the form

\[(G, \psi, \phi, T) = (f(z), g(z), h(z), \bar{T}(z)) e^{ik(x-ct)}\] (4.4.6)

where \(c = \omega / k\) is the phase velocity, \(\omega\) being circular frequency and \(k\) is the wave number. Upon using solutions (4.4.6) in equations (4.4.2)-(4.4.5) and solving the resulting system of equations, the expression for \(G, \phi, T\) and \(\psi\) are obtained as

\[G = \sum_{i=1}^{3} \left( A_i \sin m_i z + B_i \cos m_i z \right) \exp\left\{ ik(x-ct) \right\}\]

\[\phi = \sum_{i=1}^{3} W_i \left( A_i \sin m_i z + B_i \cos m_i z \right) \exp\left\{ ik(x-ct) \right\}\]

\[T = \sum_{i=1}^{3} S_i \left( A_i \sin m_i z + B_i \cos m_i z \right) \exp\left\{ ik(x-ct) \right\}\]

\[\psi = (A_4 \sin m_4 z + B_4 \cos m_4 z) \exp\{ik(x-ct)\}\] (4.4.7)

where

\[m_i^2 = k^2(\lambda_i^2 c^2 - 1), i = 1, 2, 3, \quad m_4^2 = k^2 \left( \frac{c^2}{\delta_2^2} - 1 \right)\] (4.4.8)

\[\sum \lambda_i^2 = 1 + \tau_0 (1 + \varepsilon_T) + \frac{1}{\delta_1^2} - \frac{a_3}{\omega^2} \left( \frac{\varepsilon_b}{\varepsilon_T} - \frac{\varepsilon_\phi}{\varepsilon_T} \right)\] (4.4.9)

\[\sum \lambda_1^2 \lambda_2^2 = \tau_0 + \frac{1 + \tau_0 (1 + \varepsilon_T)}{\delta_1^2}\]

\[+ \frac{a_3}{\omega^2} \left[ \frac{\varepsilon_b}{\varepsilon_T} (1 + \tau_0 (1 + \varepsilon_T)) + \frac{\tau_0 \varepsilon_\phi}{\varepsilon_T} (\varepsilon_T - \varepsilon_b (2 \varepsilon_T + \varepsilon_\phi)) \right]\] (4.4.10)

\[\lambda_1^2 \lambda_2^2 \lambda_3^2 = \tau_0 \left[ \frac{1}{\delta_1^2} - \frac{a_3}{\omega^2} (\varepsilon_\phi + \varepsilon_b) \right]\] (4.4.11)
In the absence of voids (i.e. for generalized thermoelasticity) \( m = 0 = b \) so that \( \varepsilon = 0 = \varepsilon_a \) and we have

\[ \lambda_2^2 = \frac{1}{\delta_1^2} - \frac{a_3 \bar{\varepsilon}_0}{\omega^2}, \]

\[ \lambda_1^2 + \lambda_2^2 = 1 + \tau_0 (1 + \varepsilon_r), \]

\[ \lambda_1^2 \lambda_2^2 = \tau_0 \]

\[ W_i = \begin{cases} 0, & \text{for } i = 1, 3 \\ 1, & \text{for } i = 2 \end{cases}, \]

\[ S_i = - \begin{cases} m_i^2 - k^2 (c^2 - 1), & \text{for } i = 1, 3 \\ 0, & \text{for } i = 2 \end{cases} \]

In case of elasticity with voids \( m = 0 \neq b \Rightarrow \varepsilon_r = 0 = \varepsilon, \) the characteristic roots are given by

\[ \lambda_1^2 + \lambda_2^2 = 1 + \frac{1}{\delta_1^2} \left( a_1 \bar{\varepsilon}_0 - a_3 \bar{\varepsilon}_a \right), \]

\[ \lambda_1^2 \lambda_2 = \frac{1}{\delta_1^2} - \frac{a_3 \bar{\varepsilon}_0}{\omega^2}, \]

\[ \lambda_3^2 = \tau_0 \]

\[ W_i = \begin{cases} \left[ m_i^2 - k^2 (c^2 - 1) / a_i^2 \right], & i = 1, 2 \\ 0, & i = 3 \end{cases}, \]

\[ S_i = \begin{cases} 0, & i = 1, 2 \\ 0, & i = 3 \end{cases} \]
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For elasticity without voids \((\varepsilon_h = 0, \varepsilon_r = 0 = \varepsilon_\phi)\), we have

\[
\lambda_1^2 = 1, \quad \lambda_2^2 = \frac{1}{\sigma_1^2} - \frac{a_1 m_0^2}{\omega^2}, \quad \lambda_3^2 = \tau_0
\]

\[
W_i = \begin{cases} 0, & \text{for } i = 1, 3 \\ 1, & \text{for } i = 2 \end{cases}
\]

\[
S_i = \begin{cases} 0, & i = 1, 2 \\ 1, & i = 3 \end{cases}
\]

(4.4.17)

The displacement component \(u\) and \(w\) are obtained as

\[
u = ik \sum_{i=1}^{3} (A_i \sin m_i z + B_i \cos m_i z) + m_i (A_i \cos m_i z - B_i \sin m_i z) \exp(i \omega t - ikz) \quad \text{(4.4.18)}
\]

\[
w = \sum_{i=1}^{3} (m_i (A_i \cos m_i z - B_i \sin m_i z) - ik (A_i \sin m_i z + B_i \cos m_i z)) \exp(i \omega t - ikz) \quad \text{(4.4.19)}
\]

The stresses, temperature gradient and gradient of volume fraction can be computed on the same lines.

### 4.5 DERIVATION OF THE SECULAR EQUATIONS

Invoking the boundary conditions (4.3.1) and (4.3.2) at free surface \(z = \pm d\) of the plate and using equations (4.4.7) and (4.4.8), we obtain a system of eight simultaneous linear equations which have a non trivial solution if and only if the determinant of coefficients of amplitudes \((A_1, B_1, A_2, B_2, A_3, B_3, A_4, B_4)^T\) vanishes. This after lengthy algebraic reductions and manipulations leads to secular equations for stress free and rigidly fixed thermally insulated and isothermal boundaries of the plate.

**Stress free boundary**

\[
\begin{bmatrix}
T_1 \\
T_4
\end{bmatrix}^{11} + \begin{bmatrix}
m_1 (W_3 S_1 - W_1 S_3) \\
m_2 (W_2 S_3 - W_3 S_2)
\end{bmatrix}^{11} T_2 \begin{bmatrix}
T_3 \\
T_4
\end{bmatrix}^{11} + \begin{bmatrix}
m_1 (W_1 S_2 - W_3 S_1) \\
m_3 (W_2 S_3 - W_3 S_2)
\end{bmatrix}^{11} T_3 \begin{bmatrix}
T_3 \\
T_4
\end{bmatrix}^{11}
\]

\[
= \frac{-4k^2 m_2 m_3}{(k^2 - m_1^2)^2} \left[ 1 + \frac{W_3 S_1 - W_1 S_3}{W_2 S_3 - W_3 S_2} + \frac{W_1 S_2 - W_3 S_1}{W_2 S_3 - W_3 S_2} \right]
\]

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for insulated boundary \((h \to 0)\) and

\[
\begin{bmatrix}
T_1 \\
T_4
\end{bmatrix}
+ \frac{m_1 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)}}{m_1 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)}}
\begin{bmatrix}
T_2 \\
T_4
\end{bmatrix}
+ \frac{m_1 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)}}{m_1 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)}}
\begin{bmatrix}
T_3 \\
T_4
\end{bmatrix}
\]

\[
(4.5.2)
\]

\[
-4k^2 m_4 \left(1 + \frac{m_2}{m_1} \frac{(m_3 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)})}{m_1 (m_2 S_3 S_3^{(1)} - m_1 W_1 S_1 T_1^{(1)})} + \frac{m_1}{m_2} \frac{(m_3 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)})}{m_1 (m_2 S_3 S_3^{(1)} - m_1 W_1 S_1 T_1^{(1)})}ight)
\]

for isothermal boundary \((h \to \infty)\).

**Rigidly fixed boundary**

\[
\begin{bmatrix}
T_1 \\
T_4
\end{bmatrix}
+ \frac{m_1 (W_2 S_2 - W_2 S_2)}{m_2 (W_2 S_2 - W_2 S_2)}
\begin{bmatrix}
T_2 \\
T_4
\end{bmatrix}
+ \frac{m_1 (W_2 S_2 - W_2 S_2)}{m_2 (W_2 S_2 - W_2 S_2)}
\begin{bmatrix}
T_3 \\
T_4
\end{bmatrix}
\]

\[
(4.5.3)
\]

\[
-\frac{m_4 m_2}{k^2} \left(1 + \frac{W_2 S_2 - W_2 S_2}{W_2 S_2 - W_2 S_2} + \frac{W_2 S_2 - W_2 S_2}{W_2 S_2 - W_2 S_2}ight)
\]

for insulated boundary of plate and

\[
\begin{bmatrix}
T_1 \\
T_4
\end{bmatrix}
+ \frac{m_1 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)}}{m_1 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)}}
\begin{bmatrix}
T_2 \\
T_4
\end{bmatrix}
+ \frac{m_1 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)}}{m_1 W_3 S_3 T_3^{(1)} - m_1 W_1 S_1 T_1^{(1)}}
\begin{bmatrix}
T_3 \\
T_4
\end{bmatrix}
\]

\[
(4.5.4)
\]

\[
-\frac{m_4 m_2}{k^2} \left(1 + \frac{m_2 W_3 S_3 T_3^{(1)} - m_2 W_3 S_3 T_3^{(1)}}{m_2 W_3 S_3 T_3^{(1)} - m_2 W_3 S_3 T_3^{(1)}} + \frac{m_1 W_3 S_3 T_3^{(1)} - m_1 W_3 S_3 T_3^{(1)}}{m_2 W_3 S_3 T_3^{(1)} - m_2 W_3 S_3 T_3^{(1)}}ight)
\]

for isothermal boundary.

Here superscript \((+1)\) corresponds to skew symmetric mode and \((-1)\) refers to symmetric modes.. The equations \((4.5.1)-(4.5.4)\) are the secular equations for propagation of guided poro-thermoelastic waves in plates. We refer to such waves as thermoelastic plate waves, rather than Lamb waves whose properties were derived by Lamb (1917) for isotropic elastic solids. These secular equations govern the symmetric
and skew symmetric motion of the plate with stress free and rigidly fixed, thermally
insulated / isothermal boundaries.

4.6 PARTICULAR CASES OF THE SECULAR EQUATION

4.6.1 Thermoelastic Plate

In case of coupled thermoelasticity (CT) the thermal relaxation time vanishes ($\tau_0 = 0$),
the secular equations are again given by (4.5.1) to (4.5.4) with reduced values of
characteristic roots $m_i$, $i = 1, 2, 3$. For generalized thermoelasticity, the effect of
voids is absent, so that $m_a = 0 = b$, i.e. $\epsilon_a = 0 = \epsilon_b$, and consequently,

$$W_i = 0, \text{ for } i = 1, 3; \ W_2 \rightarrow 1, \ S_2 = 0, \ S_i = -m^2 + k^2(c^2 - 1), \ i = 1, 3.$$  

Thus the secular equations (4.5.1)-(4.5.2) for stress free, thermally insulated and stress
free, isothermal boundaries of the plate respectively, reduce to

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}^{21} - \begin{bmatrix} m_1(\alpha^2 - m_i^2) \\ m_3(\alpha^2 - m_3^2) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}^{21} = -\frac{4k^2m_1m_3(m_2^2 - m_3^2)}{(k^2 - m_2^2)(\alpha^2 - m_3^2)} \quad (4.6.1)$$

$$\begin{bmatrix} T_1 \\ T_4 \end{bmatrix}^{21} - \begin{bmatrix} m_1(\alpha^2 - m_i^2) \\ m_3(\alpha^2 - m_3^2) \end{bmatrix} \begin{bmatrix} T_1 \\ T_4 \end{bmatrix}^{21} = -\frac{(k^2 - m_2^2)(m_2^2 - m_3^2)}{4k^2m_1m_3(\alpha^2 - m_3^2)} \quad (4.6.2)$$

The secular equations (4.5.3) and (4.5.4) in case of rigidly fixed, thermally insulated and
rigidly fixed, isothermal boundaries of the plate respectively, become

$$\begin{bmatrix} T_1 \\ T_4 \end{bmatrix}^{21} - \begin{bmatrix} m_1(\alpha^2 - m_i^2) \\ m_3(\alpha^2 - m_3^2) \end{bmatrix} \begin{bmatrix} T_1 \\ T_4 \end{bmatrix}^{21} = -\frac{m_1m_3(m_2^2 - m_3^2)}{k^2(\alpha^2 - m_3^2)} \quad (4.6.3)$$

$$\begin{bmatrix} T_1 \\ T_4 \end{bmatrix}^{21} - \begin{bmatrix} m_3(\alpha^2 - m_i^2) \\ m_1(\alpha^2 - m_3^2) \end{bmatrix} \begin{bmatrix} T_1 \\ T_4 \end{bmatrix}^{21} = -\frac{k^2(m_2^2 - m_3^2)}{m_1m_3(\alpha^2 - m_3^2)} \quad (4.6.4)$$

where $\alpha^2 = k^2(c^2 - 1)$. 
4.6.2 Elastic Plate

For a elastic plate with voids, the thermal effect are ignored and secular equations can be written from above equations (4.6.1) to (4.6.4) by replacing the subscript 3 with 2. Further in case of elastic plate, the voids effects are ignored along with thermal one. This leads to the reduction of the above secular equations to

\[
\frac{T_1}{T_3} = \left[ \frac{4k^2m_4m_4}{(k^2 - m_4^2)} \right]^{\frac{1}{2}} \quad (4.6.5)
\]

\[
\frac{T_1}{T_3} = \left[ \frac{m_4m_4}{k^2} \right]^{\frac{1}{2}} \quad (4.6.6)
\]

The secular equations (4.6.1) to (4.6.6) are the same as obtained and discussed by Sharma et al. (2000), Sharma (2001b), Graff (1991) and Achenbach (1973) for homogeneous isotropic thermoelastic and elastic plates apart from notations, if any, subjected to corresponding boundary conditions.

4.7 DISCUSSIONS OF THE SECULAR EQUATIONS

4.7.1 Regions of the Secular Equations

Depending upon whether \( m_i (i = 1, 2, 3, 4) \) being purely imaginary or complex, the frequency equations (4.5.1) to (4.5.4) will get altered. Here we take secular equation (4.5.1) for illustration purpose.

Region I:

When the characteristics roots given by (4.4.8) are of type \( m_i^2 = -m_i''^2, i = 1, 2, 3, 4 \); then \( m_i = im_i'' (i = 1, 2, 3, 4) \) are purely imaginary. It ensures that the superposition of partial waves has the property of exponential decay. In this case the secular equations can be obtained from (4.5.1) by replacing circular tangent functions of \( m_i, k = 1, 2, 3, 4 \) with hyperbolic tangent functions of \( m_i'' \). We obtain
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\[
\begin{align*}
\left[ \frac{Th_1}{Th_4} \right]^{21} &+ \left[ \frac{m'_i(W_i'S'_i' - W_i'S_i')}{m'_i(W_i'S'_i - W_i'S_i)} \right] \left[ \frac{Th_2}{Th_4} \right]^{11} + \left[ \frac{m'_i(W_i'S'_i' - W_i'S_i')}{m'_i(W_i'S'_i - W_i'S_i)} \right] \left[ \frac{Th_3}{Th_4} \right]^{11} \\
= & \frac{4k^2m'_i m_4}{(k^2 + m_4^2)} \left[ 1 + \frac{W_i'S_i' - W_i'S_i'}{W_i'S_i - W_i'S_i} + \frac{W_i'S_i' - W_i'S_i'}{W_i'S_i - W_i'S_i} \right]
\end{align*}
\]

(4.7.1)

where \( Th_k = \tanh(m_\kappa d) \); \( k = 1,2,3,4 \) and \( W'_i \) and \( S'_i \) can be obtained from \( W_i \) and \( S_i \) by replacing \( m_\kappa \) with \( im'_i \) (\( k = 1,2,3,4 \)).

**Region II:** In case three roots \( m_\kappa (k = 1,2,3) \) are of the type \( m_\kappa^2 = -m_i^2 \), \( k = 1,2,3 \) then secular equation for this case can be obtained from (4.5.1) by replacing circular tangent function of \( m_\kappa \), \( k = 1,2,3 \) with hyperbolic tangent functions. We obtain

\[
\begin{align*}
\left[ \frac{Th_1}{T_4} \right]^{21} &+ \left[ \frac{m'_i(W_i'S'_i' - W_i'S_i')}{m'_i(W_i'S_i' - W_i'S'_i)} \right] \left[ \frac{Th_2}{T_4} \right]^{11} + \left[ \frac{m'_i(W_i'S'_i' - W_i'S_i')}{m'_i(W_i'S_i' - W_i'S'_i)} \right] \left[ \frac{Th_3}{T_4} \right]^{11} \\
= & \frac{-4k^2m'_i m_4}{(k^2 - m_4^2)} \left[ 1 + \frac{W_i'S'_1' - W_i'S'_3}{W_i'S_1 - W_i'S_3} + \frac{W_i'S'_2' - W_i'S'_1}{W_i'S_2 - W_i'S_1} \right]
\end{align*}
\]

(4.7.2)

where \( Th_k = \tanh(m_\kappa d) \); \( k = 1,2,3 \) and \( W'_i \) and \( S'_i \) can be obtained from \( W_i \) and \( S_i \) by replacing \( m_\kappa \) with \( im'_i \) (\( k = 1,2,3 \)).

**Region III:** When roots of characteristics equation (4.4.8) are of type \( m_i^2 \), \( i = 1, 2, 3, 4 \), the secular equation is given by (4.5.1).

**4.7.2 Thin Plate Results**

The thin plate limits are specified by \( kd \ll 1 \), when the transverse wavelength with respect to thickness is quite large, In this case the characteristics roots lies in region I and II. In region I, the secular equation can be written from (4.5.1) by replacing circular tangent function of \( m_\kappa \) (\( k = 1, 2, 3, 4 \)) with hyperbolic tangent functions of \( m'_i \).
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Expanding tangent hyperbolic functions in series and retaining the first two terms in secular equation (4.7.1), we obtain

\[
\left( k^2 - m_i^4 \right)^2 + \frac{4}{3} d^2 k^2 m_i^4 - \frac{1}{3} d^2 \frac{G}{F} (k^2 + m_i^4)^2 = 0 \quad (4.7.3)
\]

where

\[
F = W_1 (S_2 - S_1) + W_2 (S_3 - S_1) + W_3 (S_1 - S_2)
\]

\[
G = W_1 (m_i^2 S_1 - m_i^2 S_i) + W_2 (m_i^2 S_2 - m_i^2 S_2) + W_3 (m_i^2 S_1 - m_i^2 S_2)
\]

and dashes have been dropped for convenience here.

In case of generalized thermoelasticity without voids, the equation (4.7.3) reduces to

\[
\left( k^2 - m_i^4 \right)^2 + \frac{4}{3} d^2 k^2 m_i^4 - \frac{1}{3} d^2 \alpha'^2 (k^2 + m_i^4)^2 = 0 \quad (4.7.4)
\]

where \( \alpha'^2 = k^2 (1 - c_i^2) \).

This equation agrees with Sharma and Pathania (2005b) and has been discussed there, in the absence of liquid. In region II, neglecting the terms of second and higher order in the expansions of circular and hyperbolic functions, equation (4.7.2) reduces to

\[
m_i^2 W_1 (m_i^2 S_2 - m_i^2 S_1) + m_i^2 W_2 (m_i^2 S_3 - m_i^2 S_i) + m_i^2 W_3 (m_i^2 S_1 - m_i^2 S_2)
\]

\[
= \frac{4 k^2 m_i^2 m_i^2 m_i^2 F}{(k^2 - m_i^2)^2} \quad (4.7.5)
\]

Here primes have been suppressed for convenience.

In the absence of voids we have \( m_i = 0 = b \), i.e. \( \epsilon_i = 0 = \epsilon_b \), and hence \( W_i = 0 \), for \( i = 1, 3 \); \( W_2 \to 1 \), \( S_2 = 0 \) and \( S_i = -m_i^2 + k^2 (c_i^2 - 1) \), \( i = 1, 3 \). Therefore for generalized thermoelasticity without voids, the equation (4.7.5) reduces to

\[
m_i^2 + m_i^2 - \alpha'^2 = \frac{4 k^2 m_i^2 m_i^2}{(k^2 - m_i^2)^2} \quad (4.7.6)
\]
This equation also agrees with Sharma and Pathania (2005b) and discussed there in detail, in the absence of liquid.

### 4.7.3 Waves of Short Wavelength

Some information on the asymptotic behavior is obtained by letting \( k \to \infty \). In this case the characteristic roots lie in region I and we have

\[
\frac{\tanh m_i d}{\tanh m_1 d} \to 1, \ i = 1, \ 2, \ 3,
\]

so that the secular equation (4.5.1) reduces to

\[
(\beta'^2 + 1)^2 \left[ W_2 \alpha_i \alpha_1 (S_3 \alpha_1 - S_1 \alpha_i) + W_1 \alpha_i (S_3 \alpha_1 - S_2 \alpha_3) + W_1 \alpha_i (S_2 \alpha_2 - S_1 \alpha_3) \right]
\]

\[
= 4 \beta' \alpha_i \alpha_3 \alpha_1 \left[ W_2 (S_3 - S_1) + W_1 (S_1 - S_2) + W_1 (S_2 - S_1) \right]
\]

where

\[
\alpha_i^2 = 1 - \lambda_i^2 c^2, \ \beta'^2 = 1 - \frac{c^2}{\delta^2}
\]

and primes have been omitted.

The equation (4.7.7) is the dispersion equation for Rayleigh wave propagation in semi infinite thermoelastic solid with voids. In case of thermoelasticity without voids, this equation reduces to

\[
\left( 2 - \frac{c^2}{\delta^2} \right)^2 \left[ \alpha_i^2 + \alpha_i \alpha_1 + \alpha_3^2 - 1 + c^2 \right] = 4 \beta' \alpha_i \alpha_3 \left[ \alpha_1 + \alpha_3 \right]
\]

This is the same as obtained and discussed by Sharma et al. (2000) in case of homogeneous isotropic thermoelastic plate with stress free insulated boundary conditions. The reduction of other secular equations can also be obtained on similar lines.
4.8 AMPLITUDES OF DISPLACEMENT, TEMPERATURE AND VOLUME FRACTION FIELDS

Upon using equations (4.4.7), (4.4.18) and (4.4.19), the amplitudes of \( x \) and \( z \) components (\( u_{xy}, w_{xy}, \phi_{xy}, T_{xy} \)) of displacement, volume fraction field and temperature change, after lengthy algebraic reductions and simplifications, are obtained as

\[
u_{xy} = [i k (\cos m_1 z + L_1 \cos m_2 z + M_1 \cos m_3 z) + m_1 N_1 \cos m_4 z] B_1 e^{ik(x-ct)}
\]

(4.8.1)

\[
w_{xy} = [-m_1 \sin m_1 z + L_1 m_2 \sin m_2 z + M_1 m_3 \sin m_3 z + ik N_1 \sin m_4 z] B_1 e^{ik(x-ct)}
\]

(4.8.2)

\[
\phi_{xy} = [W_1 \cos m_1 z + L_1 W_2 \cos m_2 z + M_1 W_3 \cos m_3 z] B_1 e^{ik(x-ct)}
\]

(4.8.3)

\[
T_{xy} = [S_1 \cos m_1 z + L_1 S_2 \cos m_2 z + M_1 S_3 \cos m_3 z] B_1 e^{ik(x-ct)}
\]

(4.8.4)

where

\[
L_1 = \frac{s_4 (W_1 s_1 c_1 - W_1 s_3 c_3) - q^* c_4 s_2 s_1 m_1 m_4 (W_1 - W_3)}{s_4 (W_2 s_1 c_1 - W_3 s_3 c_3) - q^* c_2 s_2 s_1 m_2 m_4 (W_3 - W_2)}
\]

(4.8.5)

\[
M_1 = \frac{s_4 (W_1 s_1 c_1 - W_1 s_3 c_3) - q^* c_4 s_2 s_1 m_1 m_4 (W_1 - W_3)}{s_4 (W_2 s_1 c_1 - W_3 s_3 c_3) - q^* c_2 s_2 s_1 m_2 m_4 (W_3 - W_2)}
\]

(4.8.6)

\[
N_1 = \frac{2ik (c_1 s_1 s_2 m_1 m_4 (W_1 - W_3) + c_1 s_2 s_1 m_1 m_2 (W_1 - W_3) + c_1 s_3 s_1 m_1 m_3 (W_3 - W_2))}{p^* (s_4 (W_2 s_1 c_1 - W_3 s_3 c_3) - q^* c_2 s_2 s_1 m_2 m_4 (W_3 - W_2))}
\]

(4.8.7)

Here \( q^* = \frac{4k^2}{p^*}, \quad p^* = k^2 \left( \frac{c^2}{\delta^2} - 2 \right) \).

The expressions for \( w_{xy}, u_{xy}, T_{xy} \) and \( \phi_{xy} \) are given by

\[
w_{xy} = [m_1 \cos m_1 z + L_1' m_2 \cos m_2 z + M_1' m_3 \cos m_3 z - ik N_1' \cos m_4 z] A_1 e^{ik(x-ct)}
\]

(4.8.8)

\[
u_{xy} = [ik (\sin m_1 z + L_1' \sin m_2 z + M_1' \sin m_3 z) - m_1 N_1' \sin m_4 z] A_1 e^{ik(x-ct)}
\]

(4.8.9)

\[
T_{xy} = [S_1 \sin m_1 z + L_1' S_2 \sin m_2 z + M_1' S_3 \sin m_3 z] A_1 e^{ik(x-ct)}
\]

(4.8.10)

\[
\phi_{xy} = [W_1 \sin m_1 z + L_1' W_2 \sin m_2 z + M_1' W_3 \sin m_3 z] A_1 e^{ik(x-ct)}
\]

(4.8.11)
where $L_i', M_i', N_i'$ can be written from $L_i, M_i, N_i$ respectively by interchanging $c_i$ with $s_i$ ($i = 1, 2, 3$) and multiplying the latter with -1.

4.9 CIRCULAR CRESTED WAVES

In this section we consider the propagation of circularly crested generalized thermoelastic waves in an infinite homogeneous, isotropic, cylindrical plate with voids and of thickness ‘2d’ initially at uniform temperature $T_0$. We take origin of the coordinate system $(r, \theta, z)$ on the middle surface of the plate. The $r-z$ plane is chosen to coincide with middle surface and $z$-axis normal to it along the thickness. The surfaces $z = \pm d$ are subjected to stress free and thermally insulated or isothermal boundary conditions. We take $r-z$ plane as the plane of incidence and we assume that the plate is axis-symmetric with respect to $z$-axis as the axis of symmetry so that the solutions are explicitly independent of $\theta$ coordinate. Upon introduction of potential functions $G$ and $\psi$ defined by

$$u = \frac{\partial G}{\partial r} + \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial G}{\partial z} - \frac{\partial \psi}{\partial r} - \frac{\psi}{r} \quad (4.9.1)$$

The non-dimensional form of basic governing equations (4.2.5)-(4.2.7) leads to

$$\nabla^2 G + a_1 \phi - T = \ddot{G} \quad (4.9.2)$$

$$\nabla^2 \phi - a_3 (\phi + \zeta \dot{\phi}) - a_2 \nabla^2 G + a_4 T = \frac{\dot{\phi}}{\delta^2} \quad (4.9.3)$$

$$\nabla^2 T - (\ddot{T} + t_0 \dddot{T}) - 2 \nabla^2 \left( \dot{G} + t_0 \ddot{G} \right) - a_5 (\dot{\phi} + \phi \dot{\phi}) = 0 \quad (4.9.4)$$

$$\nabla^2 \psi - \frac{1}{r^2} \psi - \frac{\psi}{\delta^2} = 0 \quad (4.9.5)$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$.

We assume the solution of the form
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\[ (G, \phi, T) = [\bar{G}(z), \bar{\phi}(z), \bar{T}(z)] \ J_0(kr) \exp(-i\omega t) \]

(4.9.6)

\[ \psi = \bar{\psi}(z) \ J_1(kr) \exp(-i\omega t) \]

where \( \omega \) is the angular frequency, \( k \) is the wave number, and \( J_0(kr) \) and \( J_1(kr) \) are respectively, the Bessel functions of order zero or one.

Upon using solution (4.9.6) in equations (4.9.2)-(4.9.5) and solving the resulting differential equations, the expressions for \( G, \phi, \psi, T \) are obtained as

\[ G = \sum_{i=1}^{3} (A_i \sin m_i z + B_i \cos m_i z) J_0(kr) \exp(-i\omega t) \]

(4.9.7)

\[ \phi = \sum_{i=1}^{3} W_i (A_i \sin m_i z + B_i \cos m_i z) J_0(kr) \exp(-i\omega t) \]

\[ \psi = (A_4 \sin m_4 z + B_4 \cos m_4 z) J_1(kr) \exp(-i\omega t) \]

\[ T = \sum_{i=1}^{3} S_i (A_i \sin m_i z + B_i \cos m_i z) J_0(kr) \exp(-i\omega t) \]

where \( m_i^2, S_i, W_i \) etc. are again given by (4.4.8) to (4.4.13). The displacements \( u \) and \( w \) are again obtained from equations (4.9.1) as

\[ u = \left[ \sum_{i=1}^{3} -k(A_i \sin m_i z + B_i \cos m_i z) + m_4 (A_4 \cos m_4 z - B_4 \sin m_4 z) \right] J_1(kr) \exp(-i\omega t) \]

(4.9.8)

\[ w = \left[ \sum_{i=1}^{3} m_i (A_i \cos m_i z - B_i \sin m_i z) - k(A_4 \sin m_4 z + B_4 \cos m_4 z) \right] J_0(kr) \exp(-i\omega t) \]

The stresses can also be computed on the similar lines. Now upon invoking the non-dimensional boundaries conditions

\[ \sigma_{zz} = 0, \quad \sigma_{zz} = 0, \quad \frac{\partial \phi}{\partial z} = 0, \quad \frac{\partial T}{\partial z} + hT = 0 \]

(4.9.9)

at the surface \( z = \pm d \), we again obtain a system of simultaneous linear equations. This system of equations after applying the lengthy algebraic reduction and manipulation
leads to the secular equations for the plate with thermally insulated and isothermal boundaries. Here we again arrive at the secular equations (4.5.1)-(4.5.4) for symmetric and skew symmetric vibrations of thermally insulated and isothermal thermoelastic plate with voids. These equations can be recognized as Rayleigh-Lamb equations for symmetric and antisymmetric modes of wave propagation in an infinite Cartesian plate with voids. Thus, it is noticed that the Rayleigh-Lamb type equation also govern circular crested thermoelastic waves in plate with voids as was in case of elastokinetics. Although the frequency wave number relationship holds whether the waves are plane or radial, the displacement and stresses vary according to Bessel functions rather than trigonometric functions as far as radial coordinate is concerned.

For large values of $r$, we have

\[
J_0(kr) = \frac{\sin kr + \cos kr}{\sqrt{\pi kr}}, \\
J_1(kr) = \frac{\sin kr - \cos kr}{\sqrt{\pi kr}}. \tag{4.9.10}
\]

Thus, far from the origin the motion becomes periodic in $r$. Actually “far” occurs rather rapidly, within four to five zeros of the Bessel function. When $r$ becomes very large, the straight crested behaviour is the limit of circular crested waves. This agrees with Sharma and Singh (2002) and Graff (1991). The results in case of conventional coupled thermoelasticity can be obtained from the above analysis by setting $t_0 = 0$. In case of thermally insulated plate with voids, the amplitudes of displacement, volume fraction field and temperature change during the symmetric and skew symmetric modes of vibrations, are obtained as

\[
u_{sy} = -k(\cos m_1 z + L_1^* \cos m_2 z + M_1^* \cos m_3 z + m_4 N_1^* \cos m_4 z) \rho J_1(kr) \exp(-i\omega t) \tag{4.9.11}
\]
where \( L_1^* = L_1, M_1^* = M_1, N_1^* = iN_1 \) and \( L_1^* = L_1, M_1^* = M_1, N_1^* = iN_1 \).

It is mentioned here that the displacements, temperature change, and volume fraction fields are varying according to Bessel functions rather than the trigonometric functions along the radial coordinate.

4.10 NUMERICAL RESULTS AND DISCUSSION

In order to illustrate and verify the analytical results obtained in the previous sections we present some numerical simulation results. The physical data of the material chosen for this purpose is given in Table 2.1. The real values of phase velocity (V) and attenuation coefficient (Q) are computed by using relation (2.3.17).

The phase velocities of symmetric and skew symmetric modes of various values of wave number (R) form the dispersion relation (4.5.1) and (4.5.2) for stress free insulated and isothermal boundary conditions. The corresponding dispersion curves for Rayleigh-Lamb type modes are presented in Figs 4.2 and 4.3 and attenuation coefficient profiles in Figs. 4.4 and 4.5, in the context of generalized (LS) theory of thermoelasticity.
The dispersion curves for symmetric and skew symmetric modes of vibration in a stress free isothermal plate with voids are given in Fig. 4.2. The phase velocity of the lowest skew symmetric mode is observed to increase from zero value at vanishing wave number to become closer to thermoelastic Rayleigh wave velocity at higher values of wave number whereas in case of lowest symmetric mode it decreases from a value nearly equal to unity towards thermoelastic Rayleigh wave velocity with increasing wave number. The phase velocity of symmetric and skew symmetric optical modes of wave propagation attain quite large value at vanishing wave number which decrease sharply to become steady and asymptotic to the reduced classical shear wave velocity at higher values of the wave number. The magnitude of phase velocity of optical symmetric and skew symmetric modes is observed to develop at a rate which is approximately \( n \)-times the magnitude of velocity of the first mode \((n=1)\). The asymptotic behaviour at large wave numbers is attributed to the fact that a finite thickness plate appears to be half-space in such situations and vibration energy is mainly transmitted through the surface (interface) of the plate. At short wavelengths the free surfaces admit a Rayleigh type surface wave with complex wave number and hence phase velocity. Consequently, the surface wave propagates with attenuation due to the radiation of energy into the medium. This radiated energy will be reflected back to the center of the plate by the lower and upper surfaces. Therefore, the attenuated surface wave on the free surface is enhanced by this reflected energy to form a propagating wave. In fact, the multiple reflections between the upper and lower surfaces of the plate form caustics at one of the free surface and a strong stress concentration arises due to which wave field becomes unbounded in the limit \( d \to \infty \). The unbounded displacement field is characterized by the singularities of circular
tangent functions. It is also observed that as the thickness of plate increases, the phase velocity decreases. This can be explained by the fact that as thickness of the plate increases, the coupling effect of various interacting fields decreases resulting in lower phase velocity. It can also be observed that the thermoelastic Rayleigh wave velocity is reached at higher wave number as the thickness increases, because the transportation of energy mainly takes place in the neighbourhood of the free surfaces of the plate in this case.

In Fig. 4.3, the trend of phase velocity for acoustic symmetric and skew symmetric modes in case of insulated thermoelastic stress free plate with voids is more or less same as in case of isothermal thermoelastic stress free plate. For the first mode \((n=1)\) the phase velocity for symmetric mode at vanishing wave number has increased reasonably and decreases sharply in comparison to isothermal one. For the second mode \((n=2)\) for \(1 \leq R \leq 3\), the phase velocity of skew symmetric modes becomes less than the phase velocity of symmetric modes which gets reversed in case of isothermal one. A similar behaviour is noticed in case of third mode \((n=3)\) in the wave number range \(1 \leq R \leq 2\). In all wave number ranges, the behaviour and trend of phase velocity profiles of optical modes closely follow each other in both thermally insulated and isothermal plates. From the comparison of these dispersion curves with that of Sharma et al. (2000), it is evident that the phase velocity of the symmetric and asymmetric modes has decreased significantly due to the presence of voids almost at all wave number values. Moreover, the phase velocity of optical asymmetric modes in isothermal plates is observed more than that of symmetric one at most of the considered wavelengths whereas the behaviour gets reversed at some of the wavelength region in case of insulated thermoelastic plate with voids as can be seen from the comparison of
Figs. 4.2 and 4.3. This clearly depicts the effect of thermal boundary conditions prevailing at the surface of the solid plate with voids.

In Fig. 4.4, the variation of attenuation coefficient with wave number for symmetric and asymmetric acoustic modes is represented in case of insulated thermoelastic plate with voids. The attenuation coefficient for acoustic symmetric mode has almost negligible variation with increasing wave number whereas for asymmetric mode it increases monotonically in $0 \leq R \leq 1.5$ and decreases slowly for $R \geq 1.5$ and ultimately it vanishes. It is also noticed that the attenuation attains large values in case of asymmetric acoustic mode as compared to symmetric mode. Fig. 4.5 represents the variations of attenuation coefficient with wave number for symmetric and skew symmetric acoustic mode in case of isothermal thermoelastic plate with voids. For both symmetric and asymmetric modes, the attenuation increases sharply at $R = 0.5$ and attains maximum value at $R = 1$ for symmetric mode and at $R = 1.5$ for asymmetric mode and then decreases slowly before it vanishes. The Fig. 4.6 deals with the variations of thermo-mechanical coupling factor with wave number for symmetric and asymmetric acoustic modes. For asymmetric mode, thermomechanical coupling is interactive for $0.5 \leq R \leq 2.5$ and attains its maximum value at $R = 1$ whereas for symmetric mode it is interactive for $0 \leq R \leq 4$ and attains its maximum value at $R = 2$.

From Fig. 4.7, it is clear that for acoustic symmetric mode in insulated thermoelastic plate with voids, the specific loss is negligibly small in the considered range of wave number ($0 \leq R \leq 6$). However for asymmetric acoustic mode, there is steep rise in the magnitude of specific loss factor at $R = 0.5$ to attain maximum value at $R = 1$ and decreases slowly before it becomes asymptotic to zero at large wave numbers. In Fig. 4.8, for symmetric acoustic mode in isothermal plate with voids, there is steep rise in
specific loss at $R = 0.5$ and attains its maximum value at $R = 1$ and then decreases monotonically for $1 \leq R \leq 5$ but for asymmetric acoustic mode, specific loss shows Gaussian behaviour and attains its maximum value at $R = 3$. From the comparison of Figs. 4.7 and 4.8 it is revealed that the behaviour of asymmetric and symmetric modes in the respective figures is same except difference in their magnitudes there by showing mode conversion at certain wave lengths due to change of thermal boundary conditions prevailing at the surfaces of the thermoelastic plate with voids.

4.11 CONCLUSIONS

The SH modes get decoupled from rest of the motion and remains unaffected due to thermo-mechanical coupling, thermal relaxation and voids effects. At short wavelengths, the secular equations for symmetric and skew symmetric waves reduce to Rayleigh surface frequency equations. It is also noticed that the Rayleigh-Lamb type equation also govern circular crested thermoelastic waves in a plate with voids as in case of elastokinetics. Although the frequency wave number relationship holds whether the waves are plane or radial, the displacement and stresses vary according to Bessel functions rather than trigonometric functions as far as radial coordinate is concerned. It is observed that as thickness of the plate increases, the phase velocity decreases because with increasing thickness the coupling effect of various interacting fields also increases resulting in lower phase velocity. The behaviour of asymmetric and symmetric modes revealed that except difference in their magnitudes there is mode conversion at certain wave lengths due to change of thermal boundary conditions prevailing at the surfaces of the thermoelastic plate with voids.
Fig. 4.2: Dispersion curves in isothermal thermoelastic plate with voids.

Fig. 4.3: Dispersion curves in insulated thermoelastic plate with voids.
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Fig. 4.4: Attenuation coefficient of acoustic modes in a insulated thermoelastic plate with voids.

Fig. 4.5: Attenuation coefficient of acoustic modes in a isothermal thermoelastic plate with voids.
Fig. 4.6: Variations of thermomechanical coupling for acoustic modes in thermoelastic plate with voids.

Fig. 4.7: Specific loss factor of acoustic modes in an insulated thermoelastic plate with voids.
Fig. 4.8: Specific loss factor of acoustic modes in a isothermal thermoelastic plate with voids.