CHAPTER : 1

Introduction
Introduction

1.1 Motivation

The aim of the high energy physics is to understand the fundamental interactions among the elementary particles in nature. The mathematical tool that is used to describe these interactions is quantum field theory. With the work of Feynman, Schwinger, Tomonoga and others, quantum field theory has remained as one of the most important tool in understanding the microscopic world. With the ideas of gauge symmetry as a guiding principle, it is possible to get a better understanding of the particle interactions. In particular non-abelian gauge theories are of crucial important for describing all known interactions.

In general, neutral gauge bosons with spin–one exchange forces in gauge theories [1,2]. One well-known neutral gauge boson is the photon. The gauge symmetry of the electromagnetic interaction is U(1)$_{\text{em}}$. The photon is connected with the U(1)$_{\text{em}}$ gauge symmetry of electrodynamics. The photon being massless reflects that U(1)$_{\text{em}}$ is a good symmetry of the vacuum. Glashow [3] put forward an idea that the electromagnetic and weak interactions may be unified in a gauge theory based on the group $SU(2)_L \times U(1)_{\gamma}$, introducing an additional electrically neutral intermediate vector boson. According to the suggestion of Glashow, the neutral vector boson of the group $SU(2)_L$ and that of $U(1)_{\gamma}$ mix so as to give one linear combination, which we identify with photon, whereas the other orthogonal combination is the Z boson, which couples to the neutral weak currents. Thus, the predictions included the existence of four physical vector boson eigenstates $W^+, W^-, Z$, and the photon, obtained from rotations of the weak eigenstates. The electroweak mixing angle $\theta_w$ is generally expressed in terms of $\sin^2 \theta_w$. The W boson could mediate weak interactions that violate parity completely, whereas the Z boson, because of the electroweak mixing, has also a component that describes parity respecting part. If the value of $\sin^2 \theta_w$ is 0.25, it means that Z will interact electromagnetic like in about 25%. Another consequence of the mixing angle is that the mass of Z is not same as the $W^{\pm}$. The low energy phenomenology of weak interaction suggests that the weak interaction quanta
are massive vector particles. The problem of generating masses, consistent with gauge invariance, was solved by Weinberg and by Salam [3], using the idea of spontaneous symmetry breaking through the mechanism due to Higgs. The electromagnetic and weak interactions thus, have been unified in the Glashow-Weinberg-Salam model. This model is based on two independent local gauge symmetries: an Abelian gauge symmetry $U(1)_Y$ and a non-Abelian gauge symmetry $SU(2)_L$. The model has the total symmetry $SU(2)_L \times U(1)_Y$. At low energies, we can only observe the $U(1)_{em}$ symmetry. Therefore, the $SU(2)_L \times U(1)_Y$ gauge symmetry must be broken at some energy scale $E_{weak}$. At energies $E \ll E_{weak}$, only the $U(1)_{em}$ symmetry of electrodynamics is observed. At energies $E > E_{weak}$, the interaction has the full $SU(2)_L \times U(1)_Y$ gauge symmetry. From experiments, it is observed that $E_{weak} \approx O(100)$ GeV. This theory predicted the existence of massive force carrying particles along with the massless photon, and these (the $W^\pm$ and $Z$ particles) were discovered at CERN in 1983. Carlo Rubbia and Simon Vander Meer received the Nobel Prize in Physics in 1984 for this discovery.

Besides the electroweak interactions, there exist two other fundamental interactions, the strong and gravitational interaction. The Standard Model (SM) unifies the strong, electromagnetic and weak forces. It does not take into account the gravitational forces. That is why physicists are not satisfied with the unification at this stage. The ultimate goal is to construct a unified theory that would reveal how all observed particles and forces are just different manifestations of a single underlying system, which can be expressed within a common mathematical framework. Many Physicists don’t like the complicated gauge group of the SM. They suppose that strong and electroweak interactions can be described by one simple gauge group $G$ at high energies $E > E_{GUT}$. Such theories are called Grand Unified Theories (GUTs) [1,2,4]. For energies $E \ll E_{GUT}$, the gauge group $G$ must be broken to retain the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. One can consider this symmetry breaking similar to the breaking of $SU(2)_L \times U(1)_Y$ symmetry to $U(1)_{em}$ in the SM.

The simplest example of a GUT, which can contain the SM, is the SU(5) gauge theory proposed by Georgi and Glashow [4] in 1974. In this theory, fermions of each generation are placed in one 5-dimensional and one 10-dimensional representations of SU(5). The transformations of SU(5) are parameterised by
5^2 - 1 = 24 real parameters. The number of neutral gauge bosons \( n \) of a GUT is given by the rank of the theory i.e. \( n = \text{rank}[G] \), for example, rank of \([SU(5)] = 4\). Therefore, there is no room for additional neutral gauge bosons in the SU(5) GUT. All GUTs with gauge groups larger than SU(5) predict at least one extra neutral gauge boson \( Z' \). In 1975, Fritzsch and Minkowski [5] showed that the next interesting gauge group larger than SU(5) is SO(10). The SO(10) theory predicts one extra gauge boson because rank \([SO(10)] = 5\). GUTs with gauge groups larger than SO(10) predict more than one extra neutral gauge bosons and many new fermions. These new (exotic) fermions can mix with the SM fermions. Such mixing induces flavor-changing neutral currents (FCNCs) [6]. Mixing between ordinary (doublet) and exotic singlet left-handed quarks induces FCNC, mediated by the SM Z boson [7]. The mixing of the right-handed ordinary and exotic quarks, all SU(2)_L singlets, induces FCNC, mediated by \( Z' \) boson.

\( Z' \) gauge bosons are predicted by a wide variety of extensions of the SM [8,9]. In particular, they often occur in GUTs, left-right symmetric models, Little Higgs models, superstring theories and theories with large extra dimensions. The mass of \( Z' \) boson is not constrained by theory. It can be anywhere between \( E_{\text{weak}} \) and \( E_{\text{GUT}} \). A broad class of supersymmetric extensions of the Standard Model predict a \( Z' \) boson whose mass is naturally in the range \( 250 \text{ GeV} < M_{Z'} < 2 \text{ TeV} \) [10]. The current experimental searches of the \( Z' \) boson from Drell-Yan cross sections at Tevatron have put lower limits on the mass range \( 0.6 - 1.0 \text{ TeV} \) at 95 % CL depending on the specific models [9]. From the electroweak precision data analysis, the improved lower limits on the \( Z' \) mass are given in the range \( 1.1 - 1.4 \text{ TeV} \) at 95 % CL [11]. These limits on \( Z' \) boson mass favours higher energy (\( \geq 1 \text{ TeV} \)) collisions for direct observation of the signal. It is also possible that the \( Z' \) bosons can be much heavy or weak enough to escape beyond the discovery reach expected at the LHC. In this case, only the indirect signatures of \( Z' \) exchanges may occur at the high energy colliders [12]. For an experimentalist a \( Z' \) is a resonance ‘bump’ more massive than the \( Z \) of the SM which can be observed in Drell-Yan production followed by its decay into lepton-antilepton pairs [13]. For a phenomenologist a \( Z' \) boson is a new massive electrically neutral, colourless gauge boson (equal to its own antiparticle) which couples to SM matter. For a theorist it is useful to classify the \( Z' \) according to its spin, even though actually measuring its spin will require high statistics.
The $B$ meson decays [14–18] provide information about the flavor structure of the SM, the origin of CP violation, the dynamics of hadronic decays, and to search for any signals of new physics beyond the SM. The $B$ factories, such as Belle (KEK) [19,20] and BaBar (SLAC) [21,22] have provided us huge data in this direction. The main objective of these $B$ factories is to critically test the SM predictions and to look for possible signatures of new physics (NP). The $B$ meson decays [23] induced by the flavor-changing neutral current transitions are very important to probe the quark-flavor sector of the SM. In the SM they arise from one-loop diagrams and are generally suppressed in comparison to the tree diagrams. Nevertheless, one-loop FCNC processes can be enhanced by orders of magnitude in some cases due to the presence of new physics. New physics comes into play in $B$ meson decays in two different ways: (a) through a new contribution to the Wilson coefficients, and (b) through a new structure in the effective Hamiltonian, which are absent in the SM. The effects of $Z'$ boson in B sector have been investigated in a number of papers [24–26]. Although the presence of NP in the $B$ decays is not yet conclusively established, there exist several signals which will be verified in the LHCb experiment and the forthcoming SuperB factories. Therefore it is interesting to explore as many $B$ meson decays as possible to find an indication of NP.

This introductory chapter is organized as follows. In Section 1.2, we briefly discuss some defects of the Standard Model of elementary particle physics. In Section 1.3, we discuss the outlines of the extensions to the Standard Model that predict the existence of $Z'$ boson. In Section 1.4, we briefly discuss extra $Z'$ gauge boson in different gauge theories. $Z$-$Z'$ mixing is discussed in Section 1.5. $Z'$ boson at the LHC and at the Future Linear Colliders are briefly discussed in Sections 1.6 and 1.7 respectively. In Section 1.8, we present a brief introduction to $B$ meson decays and the effect of $Z'$ boson on them.

1.2 Drawbacks of the Standard Model

The Standard Model of particle physics is the renormalizable quantum field theory of subatomic particles. It describes the strong and electroweak interactions of fermions (spin–$\frac{1}{2}$), gauge bosons (spin–1) and a final vital ingredient – the spin–0 Higgs boson. It is combining quantum mechanics with special relativity. The final particle of the SM, the Higgs boson, has not been discovered yet. It is very important because it
is responsible for the mechanism (the so-called Higgs mechanism [27]) by which all other particles acquire mass. The SM suggests that just after the big bang all particles were massless. As time passed on, the universe cooled and temperature fell below a critical value, an invisible field called the ‘Higgs field’ filled all space. The particle associated with the Higgs field is called the Higgs boson. Although the Higgs field is not directly measurable, accelerators can excite this field and can detect the Higgs boson. So far, experiments using the world’s most powerful accelerators have not observed any Higgs bosons, but indirect experimental evidence suggests that it is possible in future. Since the Higgs field is a scalar field, the Higgs boson has no spin, and hence no intrinsic angular momentum. The Higgs boson is also its own antiparticle and is CP-even. One of the important properties of this field is that the Higgs field is exactly the same everywhere whereas the magnetic or gravitational fields vary from place to place. When particles are moving in a uniform Higgs field, they change their velocities i.e. they accelerate. The Higgs field exerts a certain amount of resistance or drag, this is the origin of the inertial mass.

The SM has been extremely successful in the description of nature. To date, almost all experimental tests of the three forces described by the SM have agreed with its predictions. The SM, although successful, has some problems. Firstly, the theoretical structure of the SM does not include gravity. Since it incorporates three of four fundamental interactions, if a fundamental theory of all interactions can be called a “Theory of Everything”, then the Standard Model will be called as a theory of “Three-Fourth of Everything”. Some theorists think that the string theory will provide a “Theory of Everything”. Yet there is no direct experimental evidence that string theory is the fundamental theory of nature. Secondly, the SM is arbitrary in gauge sector. The gauge group \( G_{321} \) i.e. \( SU(3) \times SU(2) \times U(1) \) is complicated. The three coupling constants in different gauge sectors are independent and that is why the model cannot explain their relative magnitudes. Thirdly, at present the SM is not able to answer why the number of quarks and leptons is what it is. However, the SM predicts that the number of lepton and quark pairs observed in nature should be same. Fourthly, in the SM, only left-handed particles have a doublet structure. The right-handed quarks and right-handed charged leptons are all singlets and there are no right-handed neutrinos. Hence, the SM is asymmetric in handedness of the theory. Fifthly, the SM does not explain the existence of dark matter and dark energy in the universe.
Sixthly, it does not explain the existence of massive neutrino, which is recently observed from the neutrino oscillation experiments. Moreover, fermion masses and the CKM mixing matrix elements are free parameters in the theory, which is aesthetically not naïve. Looking into the different fermion masses, one observes that the relative ratio of light to heavy fermions is much less than $10^{-8}$. The origin of the mixing angle has no explanation and the complex phase factor of the CKM matrix accounts CP violation in weak sector but one fails to understand why CP is suppressed in strong interaction. Due to the above defects, the SM is incomplete and it needs extension to understand physical phenomena beyond the SM.

### 1.3 Extensions of the Standard Model

To overcome the above defects, different extensions of the Standard Model have been proposed. Many of these extensions [28,29] introduce additional symmetries beyond the $SU(3) \times SU(2) \times U(1)$ gauge symmetry of the SM. The group $G_{321}$ is a group of rank 4 and it has four neutral gauge bosons, i.e., gauge bosons, whose interactions with fermions, change none of the fermions’ quantum numbers.

The group $G$ which contains $G_{321}$ must have a rank greater than or equal to four. If the rank of $G$ is greater than four, the gauge theory based on it will have additional neutral gauge bosons, which are generically known as $Z'$ bosons. Thus, $Z'$ bosons are a generic feature of any theory that includes a gauge group of rank greater than four. They appear naturally in many different extensions of the SM [8]. Since the SM is incomplete, it is thus very plausible that $Z'$ bosons exist. This does not, of course, mean that they are observable. Since the $Z'$ has not yet been discovered, its exact mass is unknown. However, the $Z'$ mass is constrained by the Fermilab, weak neutral current data and precision studies at LEP and the SLC [30-33], giving a model-dependent lower bound around 500 GeV, if the interaction is comparable to the other couplings of the SM. But, the lower mass limit can be as low as 130 GeV [34], if the coupling is weak. In a study of $B$ meson decays with $Z'$-mediated flavor-changing neutral currents [35], they study the $Z'$ boson in the mass range of a few hundred GeV to 1 TeV. The LHC has the potential of discovering the $Z'$ up to $M_{Z'} = 4.5$ TeV with 100 fb$^{-1}$ data at centre of mass energy $\sqrt{s} = 14$ TeV [36]. These limits on $Z'$ boson mass favours higher energy ($\geq 1$ TeV) collisions for direct observation.
of the signal. It is also possible that the $Z'$ bosons can be much heavy or weak enough to escape beyond the discovery reach expected at the LHC. In that case, only the indirect signatures of $Z'$ exchanges may occur at the high energy colliders [12]. Recently in model-independent searches for the abelian $Z'$ boson at modern hadron colliders the authors of [37] expect the $Z'$ boson with mass between 400 GeV and 1.2 TeV. There is also an extensive study of $Z'$ boson in Ref 38 for fixing its mass limit.

1.4 Extra $Z'$ gauge boson in different gauge theories

Here, we briefly give some examples of the extended gauge theories. The properties of these theories are representative of models with extra gauge bosons and motivate us for further study.

1.4.1 $E_6$ Based Models

Extra U(1) gauge symmetries appear in the decomposition of the SO(10) or $E_6$ GUT groups. The group $E_6$ has rank 6. So the $E_6$ model predicts the existence of $Z'$ boson. Additional $Z'$ bosons, originating from $E_6$ grand unified theories can be expressed in terms of the decay chain [1,2]:

$$E_6 \rightarrow SO(10) \times U(1) \psi$$

$$\rightarrow SU(5) \times U(1)_x \times U(1)_\psi \rightarrow SM \times U(1)_{\theta_{11}}, \tag{1.1}$$

where $U(1)_{\theta_{11}}$ remains unbroken at low energies.

Now consider the models in which the linear combination [11],

$$U(1)_x' = \cos \beta \ U(1)_{\chi} + \sin \beta \ U(1)_\psi, \tag{1.2}$$

survives down to the EW scale, using a convention in which the mixing angle in equation (2) satisfies $-90^0 < \beta < 90^0$.

(a) $Z_x': \beta = 0^0 \Rightarrow Z' = Z_x$. This $Z_x'$ boson is also defined in $SO(10) \rightarrow SU(5) \times U(1)_{\chi}$. This is the unique solution to the conditions of (i) family universality, (ii) no extra matter other than the right-handed neutrino, (iii) absence of gauge and mixed gauge/gravitational anomalies, and (iv) orthogonality to the hypercharge generator.
(b) $Z_\varphi : \beta = 90^\circ \Rightarrow Z' = Z_\varphi$. This $Z_\varphi$ boson is also defined by $E_6 \rightarrow SO(10) \times U(1)_\varphi$. It possesses only axial-vector couplings to the ordinary fermions.

(c) $Z_\eta : \beta = -\arctan \sqrt{5/3} \approx -52.2^\circ \Rightarrow Z' = \sqrt{3/8} Z_x - \sqrt{5/8} Z_\varphi \equiv Z_\eta$. It occurs in Calabi-Yau compactification scheme [39] of the heterotic string [40] if $E_6$ breaks directly to a rank-5 subgroup via the Hosotani mechanism [41].

(d) $Z_I : \beta = \arctan \sqrt{3/5} \approx 37.8^\circ \Rightarrow Z' = \sqrt{5/8} Z_x + \sqrt{3/8} Z_\varphi \equiv -Z_I$. This boson is orthogonal to $Z_\eta$. This boson [42] has the defining property of vanishing couplings to up-type quarks. Its production is thus suppressed at hadron colliders, especially at the Tevatron since in high energy $p\bar{p}$ collisions $Z'$ production through down quarks is suppressed by a factor of 25 relative to up quarks [43].

(e) $Z_S : \beta = \arctan \sqrt{5/27} \approx 23.3^\circ \Rightarrow Z' = \sqrt{27/32} Z_x + \sqrt{5/32} Z_\varphi \equiv Z_S$, numerically close to the $Z_I$. A supersymmetric model with a secluded $U(1)'$ breaking sector and a large supersymmetry breaking A-term was introduced (i) to provide an approximately flat potential allowing the generation of a $Z-Z'$ mass hierarchy [44] and (ii) to produce a strong first order EW phase transition for EW baryogenesis [45].

(f) $Z_N : \beta = \arctan \sqrt{15} \approx 75.5^\circ \Rightarrow Z' = (Z_x + \sqrt{15} Z_\varphi)/4 \equiv Z_N$. The $Z_N$ boson appears in the ESSM [46] or the $E_6$SSM [47].

(g) $Z_R$ : All models discussed so far neglected the kinetic mixing between the gauge kinetic term for the $U(1)'$ and $U(1)_R$ gauge boson. In the $E_6$ context, one can write the $Z'$ as the general combination [48]:

$$Z' = \cos \alpha \cos \beta Z_x + \sin \alpha \cos \beta Z_\varphi + \sin \beta Z_\varphi.$$  \hspace{1cm} (1.3)

$$\alpha = \arctan \sqrt{3/2} \approx 50.8^\circ \Rightarrow Z' = \sqrt{2/5} Z_x + \sqrt{3/5} Z_\varphi \equiv Z_R$$ \hspace{1cm} (1.4)

(h) $Z_{LR}$ : The group SO(10) has rank 5, so the SO(10) GUT does predict the existence of a $Z'$. Since SO(10) has a larger rank than that of $G_{321}$, there are
several ways in which it can be broken down to $G_{321} = SU(3)_C \times SU(2)_L \times U(1)_Y$. For example:

$$SO(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$\rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L},$$

(1.5)

(1.6)

Where $B$ is baryon number and $L$ is lepton number. The first chain gives the additional boson $Z'_{x}$, while second chain yields the left-right symmetric model [49]. The SM gauge group is extended to $SU(2)_L \times SU(2)_R \times U(1)$ resulting in a right-handed charged boson and an additional $Z'$. 

$$Z_{LR} = \sqrt{3/5} \left( \alpha Z_R - Z_{B-L}/2\alpha \right)$$

and

$$Z' = \sqrt{3/5} Z_x - \sqrt{2/5} Z_y$$

where

$$\alpha = -\arctan \frac{2\sqrt{3}}{3} \approx -39.2^\circ.$$ 

The parameter

$$\alpha = \frac{g_R^2 / g_L^2 \cot^2 \theta_W - 1}{\theta_W}$$

is the weak mixing angle. Manifest left-right symmetry requires $g_L = g_R$, while very strong coupling limit ($\alpha, g_R / g_L \rightarrow \infty$) implies $Z_{LR} \rightarrow Z_R$.

(i) $Z':$ A leptophobic $Z'$ has vanishing $U(1)'$ charges to charged leptons and left-handed neutrinos. The choice [50] $(\alpha, \beta) = (\arctan \sqrt{8/27}, -\arctan \sqrt{9/7}) \approx (28.6^\circ, -48.6^\circ)$ implies

$$Z' = \frac{\sqrt{27/80}}{3/4} Z_x + \frac{1}{\sqrt{10}} Z_y - 3/4 Z'_\nu = Z_4.$$ 

The effects of a leptophobic $Z'$ are very difficult to observe but it can be searched for in the dijet [51] and $t\bar{t}$ [52] channels at hadron colliders.

1.4.2 Sequential $Z'$

The $Z_{SM}$ boson has the same couplings to fermions as the SM $Z$ boson. Such a boson is not observed in the gauge theories unless it has different couplings to exotic fermions than the ordinary $Z$. However, it serves as a useful reference case when comparing constraints from various sources.

1.4.3 Superstring $Z'$

The $Z_{str}$ boson appears in a specified model [53] based on the free fermionic string construction with real fermions. While this model itself is not realistic the predicted
\(Z_{\text{strong}}\) is itself not ruled out. Its coupling strength is predicted and so are its fermionic couplings. Such a \(Z_{\text{strong}}\) can be naturally at the electroweak scale [34,54]. The existing mass limits of \(Z'\) boson in different models are given [11,55] in Table 1.

**Table 1: 95\% C. L. limits on \(M_{z'}\) for different models [11,55].**

<table>
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<th>(Z')</th>
<th>(M_{z'}) [GeV]</th>
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<td>(Z_{\text{string}})</td>
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### 1.5 Mixing

The \(Z - Z'\) mixing is essential for the mass eigenstates \(Z_1\) and \(Z_2\) whose masses are identified with \(M_Z\) and \(M_{Z'},\) the diagonal elements of the mass matrix. Since, there are no quantum numbers, which forbid a mixing of neutral gauge bosons, the \(Z - Z'\) mixing arises naturally in many models. We discuss the mixing as described in [1,2].

#### 1.5.1 Kinetic mixing

The effective gauge group at low energies can be written as [1,2]

\[
SU(3)_C \times SU(2)_L \times U(1)_{Y} \times U'(1). \tag{1.7}
\]

Let us consider the case where all neutral gauge bosons are massless and assume diagonal kinetic terms of the gauge fields. The neutral current Lagrangian for the group equation (1.7) is [1,2,56]

\[
-\mathcal{L}_{NC} = \bar{f} \gamma^{\beta} \left( g_1 W_{\gamma,\beta} + g_1 W_{\gamma,\beta} + g_1^{ij} Y^{i f} Z_{\beta}^{2f} + g_2^{ij} Q^{i f} B_{\beta} + g_2^{ij} Q^{i f} Z_{\beta}^{2f} \right) f. \tag{1.8}
\]
where $T_3'$ is the third component of the SM isospin, $Y'$ is the hypercharge and $Q'^f$ is the charge due to the new $U'(1)$ and $f$ stands for fermions. The particles associated with the $U(1)_Y$ and $U'(1)$ gauge groups are denoted as $B'$ and $Z'_2$ respectively. $W_{3\beta}$ is the same field as in the SM.

Kinetic mixing (KM) essentially arises due to the existence of incomplete GUT representations at the low energy scale. While the transition from GUT energies to the weak scale occur, the non-zero contributions proportional to $g_{12}$ and $g_{21}$ arise, where $g_{ij}'$ is the $2 \times 2$ matrix which can be made triangular by a rotation of the two Abelian gauge bosons around the angle $\theta_K$,  

$$
\begin{pmatrix}
B'_{\beta} \\
Z'_{2\beta}
\end{pmatrix} =
\begin{pmatrix}
c_K & -s_K \\
s_K & c_K
\end{pmatrix}
\begin{pmatrix}
B_{\beta} \\
Z'_{\beta}
\end{pmatrix},
$$

(1.9)

where $\theta_K$ is the kinetic mixing angle and $c_K = \cos \theta_K, s_K = \sin \theta_K$. Now, the Lagrangian given in equation (1.8) can be written as

$$
L_{NC} = - \bar{f} Y' (g_{12}' W_3' + g_{11}' Y' B_{\beta} + g_{12}' Y' Z'_{\beta} + g_{22}' Q'^f Z'_{\beta} ) f,
$$

(1.10)

where $g_{11}' = g_{11} c_K + g_{12} s_K$, $g_{12}' = - g_{11} s_K + g_{12} c_K$, $g_{21}' = g_{21} c_K + g_{22} s_K$ and $g_{22}' = - g_{21} s_K + g_{22} c_K$.

The effects of kinetic mixing on the $Z'$ couplings can be sufficiently large to obtain leptophobic conditions [50] and can communicate SUSY-breaking to the visible sector [57]. However, this mixing between the gauge bosons $W_{3\beta}$ and $B_{\beta}$ is forbidden in the SM. The influence of gauge kinetic mixing leads to enrichment in the phenomenology of new gauge bosons like $Z'$ boson.

1.5.2 Mass mixing

At low energies, the gauge symmetry is broken to describe massive gauge bosons. In the SM, the mass matrix can be written [1,2]

$$
L_{M} = \frac{1}{2} (B, W_3) M_M^2 \begin{pmatrix}
B \\
W_3
\end{pmatrix},
$$

$$
M_M^2 = \begin{pmatrix}
M_B^2 & -M_W M_B \\
-M_W M_B & M_W^2
\end{pmatrix},
$$

(1.11)

and
\[ \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \langle \phi^0 \rangle = \frac{v}{\sqrt{2}} g, \quad M_B = \frac{v}{2} g_{11}, \quad M_W = \frac{v}{2} g, \quad (1.12) \]

where \(v\) is the vacuum expectation value of the Higgs field. \(M_{SM}^2\) is diagonalized by a rotation of the symmetry eigenstates around the Weinberg angle \(\theta_w\), \(c_w = \cos \theta_w\), \(s_w = \sin \theta_w\). The mass eigenstates of the photon and the Z boson can be written as

\[ \begin{pmatrix} \gamma \\ Z \end{pmatrix} = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} B \\ W_s \end{pmatrix}, \quad (1.13) \]

The masses of the mass eigenstates are

\[ M_Z^2 = M_W^2 + M_B^2, \quad M_\gamma^2 = 0. \quad (1.14) \]

The photon is connected with U(1)_{em} gauge symmetry. According to Noether’s theorem, this symmetry corresponds to a conserved quantity, the electric charge. The U(1)_{em} gauge symmetry is exact. Therefore, the mass of the photon is zero. This symmetry protects the photon from further mixing. The Weinberg angle \(\theta_w\), is related to the entries of the mass matrix equation (1.11) as:

\[ \tan 2 \theta_w = \frac{2 M_B M_W}{M_W^2 - M_B^2}. \quad (1.15) \]

The relations between the Weinberg angle and the mass values can be written as

\[ l_w^2 = s_w^2 = \frac{M_Z^2 - M_\gamma^2}{M_B^2}, \quad M_B M_W = s_w c_w (M_Z^2 - M_\gamma^2) = s_w c_w M_Z^2. \quad (1.16) \]

The mass matrix of the Z and Z’ takes non-diagonal entries \(\delta M^2\), which are related to the vacuum expectation values of the Higgs fields,

\[ L_M = \frac{1}{2} (Z, Z') M_{ZZ}^2 \begin{pmatrix} Z \\ Z' \end{pmatrix}, \quad M_{ZZ}^2 = \begin{pmatrix} M_Z^2 & \delta M^2 \\ \delta M^2 & M_{Z'}^2 \end{pmatrix}. \quad (1.17) \]

The vacuum expectation values of the Higgs fields are assumed to be real. The mass matrix equation (1.17) is diagonalized by a rotation of the fields Z and Z’ around the mixing angle \(\theta\), \(c_\theta = \cos \theta\), \(s_\theta = \sin \theta\) leading to the mass eigenstates \(Z_1\) and \(Z_2\),
\[
\begin{pmatrix}
Z_1 \\
Z_2
\end{pmatrix} =
\begin{pmatrix}
c_\theta & s_\theta \\
-s_\theta & c_\theta
\end{pmatrix}
\begin{pmatrix}
Z
\end{pmatrix} .
\]

The masses \( M_1 \) and \( M_2 \) of the mass eigenstates \( Z_1 \) and \( Z_2 \) are

\[
M_{1,2}^2 = \frac{1}{2} \left[ M_Z^2 + M_{Z'}^2 \pm \sqrt{(M_Z^2 - M_{Z'}^2)^2 + 4(\delta M^2)^2} \right],
\]

which gives

\[
M_1 < M_Z < M_2, \text{ and } \rho_{\text{mix}} = \frac{M_{Z'}^2}{M_1^2 c_w^2} > \frac{M_Z^2}{M_2^2 c_w^2} = \rho_0 = 1 .
\]

When \( \theta = 0 \), we have \( M_Z = M_1 \) and \( M_{Z'} = M_2 \).

Like the Standard Model, the mixing angle \( \theta \) is related to the entries of the mass matrix equation (1.17) as :

\[
\tan 2\theta = \frac{2\delta M^2}{M_Z^2 - M_{Z'}^2}.
\]

The relations between \( \theta \) and the mass values can be written as

\[
t_\theta^2 = \frac{s_\theta^2}{c_\theta^2} = \frac{M_Z^2 - M_1^2}{M_2^2 - M_Z^2},
\]

and

\[
\delta M^2 = s_\theta c_\theta (M_1^2 - M_Z^2).
\]

From equation (1.23), it is clear that \( \theta \) and \( M_2 \) are related independently of the Higgs sector, for the fixed value of \( M_1 \) and \( M_Z \). This constraint on \( \theta \) is known as the mass constraint [2,58]. It predicts \( \theta \sim 1/M_2 \) for large \( M_2 \). Again from equation (1.24), for a fixed Higgs sector, \( \delta M^2 \) is also fixed leading to the constraint on \( \theta \). It is called the Higgs constraint. For large \( M_2 \), it is stronger than the mass constraint predicting the asymptotic behaviour \( \theta \sim 1/M_2^2 \). Thus the mixing is well-constrained.

There are stringent limits on the mass of an extra \( Z' \) from the non-observation of direct production followed by decays into \( e^+e^- \) or \( \mu^+\mu^- \) by CDF [30], while indirect constraints from precision data also limit the \( Z' \) mass (weak neutral current...
processes and LEP II) and severely constrain the $Z - Z'$ mixing angle $\theta$ [59,60]. These limits are model-dependent, but are typically in the range $M_{Z'} > O(500)$ GeV and $\theta < \text{ few } \times 10^{-3}$ for standard GUT models. Anoka, Babu and Gogoladze [61] analyse a special class of supersymmetric $Z'$ models wherein the $Z'$ properties get essentially fixed from constraints of SUSY breaking. They found $M_{Z'} = 2 - 4$ TeV and the $Z - Z'$ mixing angle $\theta = 0.001$. Constraints from the electroweak precision observable are satisfied, with the $Z'$ model giving a slightly better fit compared to the Standard Model.

1.6 $Z'$ Boson at the LHC

Both at the Tevatron and at the LHC, we concentrate on the processes $p \bar{p} \to (p p) \to (Z' \to \ell^+ \ell^-) + X, \ (\ell = e, \mu)$: these two very clean channels to look for, and after simple generous cuts, the irreducible background is dominated by the well-understood SM Drell-Yan (DY) processes [62]. In the Tevatron experimental limits, they use the most recent available results from CDF (on $Z' \to e^+e^-$ [52] and $Z' \to \mu^+\mu^-$ [63]) and D0 (on $Z' \to e^+e^-$ [64]). They directly provide the 95% CL bounds on product $\sigma(p \bar{p} \to Z'X) \times BR(Z' \to \ell^+\ell^-)$ and give the mass of the $Z'$ boson.

The $Z'$ boson has also been studied at the LHC [65,66]. The main discovery mode for a $Z'$ boson at a hadron collider is Drell-Yan production of a dilepton resonance $pp \to Z' \to \ell^+\ell^-$ where $\ell = e$ or $\mu$ [11,67-70]. In this production channel at LHC, one would be able to measure the mass $M_{Z'}$, the width $\Gamma_{Z'}$, and the leptonic cross section $\sigma_{Z'} = \sigma_{Z'} B_t$. The cross section $\sigma_{Z'}$ is a useful indirect probe for the existence of the exotics or super partners. Again $\sigma_{Z'} = \sigma_{Z'} \Gamma_t$, probes the absolute magnitude of the quark and lepton couplings. If the $Z'$ bosons couple to quarks and leptons not too weakly and if their mass is not too large, they are expected to be discovered in the LHC program [71,72]. The $Z'$ boson can also be studied by studying the cross sections $\sigma(pp \to Z' \to b\bar{b})$ and $\sigma(pp \to Z' \to \tau \bar{\tau})$ [72], as described by the Drell-Yan cross section with the addition of a $Z'$ [73].

The $Z'$ may be discovered by detecting excess signals from backgrounds near its resonance in the dilepton invariant mass distribution [36]. With 100 fb$^{-1}$ of data,
the LHC can discover a $Z'$ with a mass up to 4.5 TeV at $\sqrt{s} = 14$ TeV. $Z'$ physics with early LHC data has been discussed in [13]. They show that the LHC at 7 TeV with integrated luminosity of 500 pb$^{-1}$ will greatly improve on current Tevatron mass limits on $Z'$ boson. Their results are based on the narrow width approximation in which the leptonic Drell-Yan $Z'$ boson cross-section depends on the $Z'$ boson mass. In the B–L model, the $Z'$ boson predominantly couples to leptons. Considering both the $Z'_{B-L} \rightarrow e^+e^-$ and $Z'_{B-L} \rightarrow \mu^+\mu^-$ decay channels, $Z'_{B-L}$ discovery potential at the LHC for 7 TeV has been discussed in [74]. The LHC is being able to discover the $Z'_{B-L}$ boson up to masses of 1.2 TeV for 1 fb$^{-1}$, while at the Tevatron a 5 $\sigma$ discovery will be possible up to the mass 0.9 TeV. From the recent CMS collaboration analyses [75] the mass limits for the sequential standard model $Z'$ and the superstring inspired $Z'_{\nu}$ are about 2590 GeV and 2260 GeV respectively at 95% CL.

If a $Z'$ boson is discovered at the LHC, it will be important to compare its properties as measured at the LHC with the constraints of the electroweak (EW) fits [76]. The accurate measurements of its properties and its invisible decay rates are of great interest for possible observation of extra dimensions. When new physics has been discovered and studied at the LHC, we want to consider how it affects the EW fits. But it is not entirely certain that the LHC will find the $Z'$ boson.

1.7 $Z'$ Boson at the Future Linear Colliders

The Linear Collider (LC) environment is one of the most suitable for $Z'$ physics due to two reasons [77]: (i) if a $Z'$ is found at the LHC, it is hard to identify at the hadronic machine; whereas the clean experimental environment of a LC is the ideal framework to establish the $Z'$ line shape (i.e. its mass and width) and to measure its couplings [78]. (ii) For a LC operating at TeV energies, there exists scope to discover a $Z'$ boson over regions of the B – L parameter space which can not be probed at the LHC, either directly through a resonance (when $\sqrt{s_{e^+e^-}} \geq M_{Z'}$) or indirectly through interference effects (when $\sqrt{s_{e^+e^-}} < M_{Z'}$). In both the cases, a LC proves to be more powerful than the LHC in accessing the region of small $Z'$ couplings.

A comparison between the $Z'_{B-L}$ discovery power at the LHC and at a future LC has been done in [77]. They have probed the $Z'$ sector of the minimal B – L
model at future LC in the $e^+e^- \rightarrow \mu^+\mu^-$ process. They have found that for $M_{Z'} = 1$ TeV, the LHC can discover a $Z'$ if the coupling $g' = 0.007$ while a LC can achieve this for $g' = 0.005$. Again, a LC can discover a $Z'$ with a 2 TeV mass for a $g'$ coupling which is a factor 8 smaller than the one for which the same mass $Z'$ can be discovered at the LHC. Altogether then, both an ILC ($\sqrt{s_{e^+e^-}} \leq 1$ TeV) [79] and a Compact Linear Collider (CLIC, $\sqrt{s_{e^+e^-}} \leq 3$ TeV) [80] may be able to outperform the LHC. For $g' = 0.1$ a CLIC type LC would be sensitive to a $Z'$ mass up to 10 TeV while the LHC can observe a $Z'$ with mass below 4 TeV (for the same coupling). Similarly, for $g' = 0.2$, an ILC would be sensitive to a $Z'$ with mass up to 7.5 TeV while the LHC would be able to observe a $Z'$ only below 4.7 TeV (for the same coupling). There are also further possibilities to explore the LC potential to study $Z'$ physics by exploiting beam polarisation and/or asymmetries in the cross section. The new gauge boson $Z'$ is the carrier of the new gauge force with smallest gauge group beyond the SM which plays the crucial roles in cosmology, GUT, SUSY and various strongly coupled new physics theories [8,81].

1.8 $B$ Meson Decays

The era of $B$ physics began about 35 years ago, after the discovery of bottom quark in 1977 [82]. The area of $B$ physics forms a part of the more general field of flavor physics, which deals with the six flavors of quarks: the origin of their masses, their electroweak interactions, mixing between them, and phenomena like charge-parity (CP) violation that are observed through their decays [83]. Flavor physics has now entered the era of precision measurements, and $B$ meson decays in particular are going to be instrumental in indirect search of new physics beyond the Standard Model.

The main reason to study $B$ meson decays is their sensitivity to the flavor structure of nature. Since $B$ mesons (those containing a $b$ quark) are much heavier than the $K$ (with an $s$ quark) and $D$ (with a $c$ quark), they can decay through more number of channels [83,84]. Again the large mass of $b$ quark makes the quantity $\Lambda_{QCD}/m_b$ small, so that a systematic expansion in this quantity can be carried out by Heavy Quark Effective Theory (HQET) [85] with acceptable accuracy. In this way, the problem always created due to the strong interaction effects can be overcome.
Generally $B$ physics describes weak decays of $B$ mesons. Here, we are coming across three different energy scales: (i) they are weak decays implies that they involve the scale of weak interactions, given by the mass of W boson, $M_w$, (ii) since the energy of the process is that of the decaying meson, there is another scale of energy i.e. the mass of the $B$ meson $m_B$, and (iii) As we are dealing with mesons, the physics of strong interactions of bound states is also important i.e. the hadronic scale $\Lambda_{QCD}$. Moreover, since we are looking for new physics, another energy scale $\Lambda_{NP}$, at which the SM breaks down as an effective theory, is required. These energy scales are given as:

$$\Lambda_{QCD} \ll m_B \ll M_w \ll \Lambda_{NP}$$

(1.25)

where, $\Lambda_{QCD} \sim 0.2 - 1.0$ GeV, $m_B \sim 5.2$ GeV, $M_w \sim 80.3$ GeV and $\Lambda_{NP} >$ few TeV.

The term ‘flavor changing’ refers to the processes where the initial and final flavor numbers (that is, the number of particles of a certain flavor minus the number of antiparticles of the same flavor) are different [86]. In ‘flavor-changing charged current’ processes, both up-type and down-type flavors, and/or both charged lepton and neutrino flavors are involved. Within the Standard Model, these processes are mediated by the W bosons and occur at tree level. In ‘flavor-changing neutral current’ (FCNC) processes either up-type or down-type flavors but not both, and/or either charged lepton or neutrino flavors but not both, are involved. Within the Standard Model, these processes do not occur at tree level, and are often highly suppressed.

The origin of flavor and CP violation is one of the most important issues in particle physics. Measurements of CP violating $B$ meson decays have established that the Kobayashi-Maskawa mechanism is the dominant source of the observed CP violation [86]. And measurements of flavor changing $B$ meson decays have established that the Cabbibo-Kobayashi-Maskawa mechanism is a major player in flavor violation.

One way of searching new physics beyond the SM is by studying the $B$ meson decays [23] induced by the FCNC transitions. In the SM, they arise from one-loop diagrams and are generally suppressed in comparison to the tree diagrams. Nevertheless, one-loop FCNC processes can be enhanced by orders of magnitude in some cases due to the presence of new physics. It is interesting to note that the non-universal $Z'$ couplings could lead to FCNC in the tree level as well as introduce new weak phases, which are essential in inducing the CP asymmetries. The effect of $Z'$-mediated FCNC in different
$B$ meson decays have been investigated in a number of papers [24–26]. Some authors have tried to get the solution to $B \rightarrow \pi K$ puzzle in the flavor-changing $Z'$ model. Some authors have studied the branching ratio and direct CP asymmetry of decay modes $B_d \rightarrow \phi K$, $B_d \rightarrow \eta'K$, $B_s \rightarrow \phi \pi^0$ etc. The enhancement of both branching ratio and CP asymmetry may provide a signal of the non-universal $Z'$ model, which can be used to constrain the mass of $Z'$ boson. The authors in [26] have studied different $B$ meson decays such as $B_{d,s} \rightarrow \ell^+\ell^-$, $B \rightarrow \pi K$, $\Lambda_s \rightarrow \Lambda \ell^+\ell^-$, and $B \rightarrow \pi\pi$. They find the branching ratios of the decays are enhanced from its Standard Model value due to the effect of both $Z$ and $Z'$ mediated FCNCs, and gives the possibility of new physics beyond the SM. The contribution of $Z'$ boson depends upon the precise value of $M_{Z'}$. The SM predictions for the branching ratios are $(3.23 \pm 0.27) \times 10^{-9}$ for $B_s \rightarrow \mu^+\mu^-$ and $(1.07 \pm 0.10) \times 10^{-10}$ for $B_d \rightarrow \mu^+\mu^-$ [87]. Authors [26] have predicted the branching ratio for $B \left( B_d \rightarrow \mu^+\mu^- \right) = (1.47 \pm 0.09) \times 10^{-8}$ in the $Z'$ model. Again authors [26] have predicted the branching ratio $B \left( B_s \rightarrow \mu^+\mu^- \right) = 7.04 \times 10^{-8}$ in the $Z'$ model. New physics models like MSSM and SUSY can predict the branching ratio up to 100 times the SM predictions. The CDF measurement predicts the upper limits $B \left( B_s \rightarrow \mu^+\mu^- \right) < 3.1 \times 10^{-8}$ at 95% CL and $B \left( B_d \rightarrow \mu^+\mu^- \right) < 4.6 \times 10^{-9}$ at 95% CL and the D0 Collaboration predicts $B \left( B_s \rightarrow \mu^+\mu^- \right) < 1.5 \times 10^{-8}$ at 95% CL [88]. These upper limits are about an order of magnitude above the SM predictions. In [89], we have predicted the mass of $Z'$ boson from $B^0_s - \bar{B}^0_s$ mixing in the range 1352 – 1665 GeV. The $B \rightarrow K^*\ell^+\ell^-$ decays [90] are studied considering the effect of $Z'$-mediated FCNC. They find that the forward-backward asymmetry can be enhanced by $Z'$ contribution.

Flavor physics made important transition from the work on confirmation in Standard Model of particle physics to the phase of search for effects of new physics beyond the SM. The LHC era is in full swing, also in flavor physics. $B$ physics in the LHC era and at the Tevatron has been studied in [88,91,92] with a motivation for searching new physics beyond the SM. Although the presence of NP in the $B$ decays is not yet conclusively established, there exist several signals which will be verified in the LHCb experiment and the forthcoming SuperB factories. With large expectations the
whole physics community is positive about the future interesting results and the importance of $B$ meson decays for discovering, and/or understanding new physics beyond the Standard Model.

From the various experiments like the Tevatron, the LHC, the FLC etc; the discovery of $Z'$ boson is not ruled out, it is therefore interesting to study its effect on $B$ meson decays to reveal new physics beyond the Standard Model, which we carry out in this thesis to some extend. In chapter 2, we discuss the effect of both $Z$ and $Z'$-mediated flavor-changing neutral currents (FCNCs) on the $\Lambda_k \rightarrow \Lambda \ell^+ \ell^-$ ($\ell = \mu, \tau$) decay. We find the branching ratio is reasonably enhanced from its standard model value and gives the possibility of new physics beyond the Standard Model. The contribution of $Z'$-boson depends upon the precise value of $M_{Z'}$. In chapter 3, we have studied the effect of $Z'$- mediated FCNC on $B \rightarrow \pi\pi$ decays. The branching ratios of these decays can be enhanced remarkably in the non-universal $Z'$ model. Our estimated branching ratios $B\left( B^0 \rightarrow \pi^0 \pi^0 \right)$ are enhanced significantly from their Standard Model values. For $g'/g = 1$, the branching ratios $B\left( B^0 \rightarrow \pi^0 \pi^0 \right)$ are very close to the recently observed experimental values and for higher values of $g'/g$ branching ratios are more. Our calculated branching ratios $B\left( B^0 \rightarrow \pi^+ \pi^- \right)$ and $B\left( B^+ \rightarrow \pi^+ \pi^0 \right)$ are also enhanced from the SM value as well as the recently observed experimental values. These enhancements of branching ratios from their SM value give the possibility of new physics. In chapter 4, we have predicted the mass of $Z'$ boson from $B_{q} - B_{q}'$ mixing ($q = d, s$) taking the effect of both $Z$- and $Z'$-mediated flavour-changing neutral currents. Our estimated mass of $Z'$ boson lies in the range of 1352 – 1665 GeV. In chapter 5, we have evaluated $B_{q} - B_{q}'$ and $B_{d} - B_{d}'$ decay width differences in $Z'$ model. Our estimated $B_{q} - B_{q}'$ decay width differences $\Delta \Gamma_q = (0.100 \pm 0.008) - (0.105 \pm 0.007) \text{ ps}^{-1}$, $\Delta \Gamma_d = (45.20 \pm 0.20) \times 10^{-4} - (46.35 \pm 0.50) \times 10^{-4} \text{ ps}^{-1}$ are enhanced relative to the SM prediction. Lower is mass of $Z'$ boson, more is the enhancement. Hence, the $B_{q} - B_{q}'$ mixing could provide signals for new physics beyond the SM. Finally, in chapter 6, we discuss the results as obtained in the thesis and the future in this line of research.
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