CHAPTER : 4

The prediction of mass of $Z'$ boson from $B^0_q - \overline{B}^0_q$ mixing
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4.1 Introduction

One of the most promising ways to detect the effects of new physics (NP) on $B$ decays is to look for deviations of flavour-changing neutral current (FCNC) processes from their Standard Model (SM) predictions [1]. FCNC process occurs at loop-level in the SM. Its rate is suppressed by small electroweak gauge coupling, CKM matrix elements and loop factors [2]. On one hand these processes are very sensitive probe of NP beyond the SM because some of these suppression factors can be enhanced in NP models. On the other hand FCNC processes of $K$, $B_d$ and $B_s$ mesons [3] are still large enough to be studied experimentally as well as theoretically. $B^0_q - \bar{B}^0_q$ mixing ($q = d, s$), meson-antimeson mixing [4-6], plays an outstanding role in this direction. These meson-antimeson oscillations occur at time scales of the order of the meson lifetimes. Furthermore, the SM contribution to these mixing is loop-suppressed and comes with two or more small elements of the CKM matrix [7,8]. The presence of non-SM particles in the loop process would significantly change the rate of these oscillations.

In $B^0_q - \bar{B}^0_q$ mixing, an initially present $B^0_q$ state evolves into a time-dependent linear combination of $B^0_q$ and $\bar{B}^0_q$ flavour states. The oscillation frequency of this phenomenon is characterized by the mass difference of the ‘heavy’ and ‘light’ mass eigenstates:

$$\Delta M_{B_q} = M_H(B_q) - M_L(B_q) = 2|M_{12}(B_q)|.$$  \hspace{1cm} (4.1)

The determination of $B^0_q - \bar{B}^0_q$ mass difference $\Delta M_{B_q}$ has been a major objective of particle physics. The phenomenon of $B^0_q$ oscillations is well established [9], with a precisely measured mass difference $\Delta M_{B_s}$. In the SM, this parameter is proportional to the combination $(V_{td}^* V_{tb})^2$ of CKM matrix elements. Since the matrix element $V_{ts}$ is larger than $V_{td}$, the expected mass difference $\Delta M_{B_s}$ is higher. Hence, the mass
differences $\Delta M_{B_s}$ and $\Delta M_{B_t}$ can be used to determine CKM matrix elements $V_{td}$ and $V_{ts}$ respectively. In the SM [10], $B_s^0 - \overline{B_s^0}$ and $B_d^0 - \overline{B_d^0}$ mass differences are found to be:

$$\left( \Delta M_{B_s} \right)_{SM} = (0.543 \pm 0.091) \text{ ps}^{-1},$$  \hspace{1cm} (4.2)

$$\left( \Delta M_{B_t} \right)_{SM} = (17.30 \pm 2.6) \text{ ps}^{-1},$$  \hspace{1cm} (4.3)

From the recent experiments, $B_s^0 - \overline{B_s^0}$ and $B_d^0 - \overline{B_d^0}$ mass differences are found to be:

$$\Delta M_{B_s} = 0.507 \pm 0.004 \text{ ps}^{-1} \text{ (ALEPH, CDF, DO, DELPHI, L3, OPAL, BABAR, BELLE, ARGUS, CLEO)} [10,11]$$  \hspace{1cm} (4.4)

$$\Delta M_{B_t} = 17.73 \pm 0.05 \text{ ps}^{-1} \text{ (CDF, DO, LHCb)} [12].$$  \hspace{1cm} (4.5)

and $17 < \Delta M_{B_t} < 21 \text{ ps}^{-1}$ (90 % CL) (D0) [13].  \hspace{1cm} (4.6)

Although these experimental values are a little bit different from their SM values, for large hadronic uncertainties we can not strongly argue that it is a NP signal. However, these measurements may give constraints on the NP models, which predict $b \rightarrow s (d)$ FCNC transitions. This is why the $B_s^0 - \overline{B_s^0}$ mixing is one of the most important and interesting portals for detection of NP models [14,15].

The $Z'$ is a hypothetical massive, electrically-neutral spin 1 gauge boson [4]. These bosons are predicted by a wide variety of extensions of the SM [16–22]. Theoretically it is predicted that they exist in Grand Unified Theories (GUTs), left-right symmetric models, Little Higgs models, superstring theories and theories with large extra dimensions. But experimentally $Z'$ boson is not conclusively discovered so far. Hence, the exact mass of $Z'$ boson is not known. The experimental searches of the $Z'$ boson from Drell-Yan cross sections at Tevatron have put lower limits on the mass range 0.6 – 1.0 TeV at 95 % CL depending on the specific models [23]. However, the lower mass limit can be as low as [24] 130 GeV if the coupling is weak. For an experimentalist a $Z'$ is a resonance ‘bump’ more massive than the $Z$ of the SM which can be observed in Drell-Yan production followed by its decay into lepton-antilepton pairs [25]. For a phenomenologist a $Z'$ boson is a new massive electrically neutral, colourless boson (equal to its own antiparticle) which couples to SM matter.
For a theorist it is useful to classify the $Z'$ according to its spin, even though actually measuring its spin will require high statistics.

There are many models beyond the SM predict more than one extra neutral gauge bosons and many new fermions. These new (exotic) fermions can mix with the SM fermions. Such mixing induces FCNCs \cite{26,27}. Mixing between ordinary (doublet) and exotic singlet left-handed quarks induces FCNC, mediated by the SM Z boson. In these models \cite{28-30}, one introduces an additional vector-singlet charge $-1/3$ quark $h$, and allows it to mix with the ordinary down-type quarks $d$, $s$ and $b$. Since the weak isospin of the exotic quark is different from that of the ordinary quarks, FCNCs involving $Z$ are induced. The $Z$-mediated FCNC couplings $U^Z_{dh}$, $U^Z_{db}$ and $U^Z_{sb}$ which are in general complex, are constrained by a variety of processes. $U^Z_{dh}$ is bounded by the measurements of $\Delta M_K$ ($K^0 - \bar{K}^0$ mixing), $|\epsilon|$ (the CP-violating parameters in the kaon system) and $K_L \rightarrow \mu^+ \mu^-$ \cite{28-30}, while the constraints on $U^Z_{db}$ and $U^Z_{sb}$ come principally from the experimental limit on $B \left( B \rightarrow \ell^+ \ell^- X \right)$ \cite{31-34}. The constraints on $U^Z_{db}$ and $U^Z_{sb}$ allow significant contributions to $B_q - \overline{B}_q$ mixing ($q = d$, $s$). Models of NP, which contain exotic fermions also predict the existence of additional neutral $Z'$ gauge bosons. The mixing among particles which have different $Z'$ quantum numbers will induce FCNCs due to $Z'$ exchange \cite{35,36}. With FCNCs, the $Z'$ boson contributes at tree level, and its contribution will interfere with the SM contributions.

In this chapter, we have considered $B_q - \overline{B}_q$ and $B_d - \overline{B}_d$ mass differences taking the effect of both $Z$- and $Z'$-mediated FCNCs in the $B_q^0 - \overline{B}_q^0$ mixing. The $B_q^0 - \overline{B}_q^0$ mixing in $Z'$ model is also studied by several authors \cite{2,38}. But we are different from them in the way that we have tried to estimate the mass of $Z'$ boson from $B_q^0 - \overline{B}_q^0$ mass differences.

This chapter is organized as follows: In Section 4.2, we discuss the phenomenology of $B_q^0 - \overline{B}_q^0$ mixing ($q = d$, $s$) in the Standard Model. In Section 4.3, we discuss about our model and evaluate the mass matrix elements considering contributions from both the $Z$ boson and $Z'$ boson. In Section 4.4, we evaluate the
$B_s^0 - B_s^0$ and $B_d^0 - B_d^0$ mass differences. We summarize our numerical results in Section 4.5.

4.2 $B^0_q - ar{B}^0_q$ mixing in the Standard Model

In the Standard Model, the $B^0_q - ar{B}^0_q$ mixing is due to the weak interaction. At the lowest order, this mixing is described by box diagrams involving two W bosons and two up-type quarks (Fig. 1) [4,39]. In this case, the long range interactions arising from intermediate virtual states are negligible because the large $B$ mass is off the region of hadronic resonances. In the SM, $M_{12}$ and $\Gamma_{12}$ are computed from the box diagram and read as [4,40,41]:

$$M_{12}^{SM} (B_q) = \frac{G_F^2 M_W^2 M_{B_q} \eta_{B_q} f_{B_q}^2 f_{B_{q'}} S_0(x_i) (V_{tq}^* V_{tb})^2}{12 \pi^2}, \quad (4.7)$$

$$\Gamma_{12} = \frac{G_F^2 m_q^2 \eta_{B_q} M_{B_q} f_{B_q}^2 f_{B_{q'}}}{8 \pi} \times \left[ (V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{c_b} O\left(\frac{m_c^2}{m_b^2}\right) + (V_{c_q}^* V_{c_b})^2 O\left(\frac{m_c^4}{m_b^4}\right) \right], \quad (4.8)$$

where $M_{12}$ and $\Gamma_{12}$ are the off-diagonal elements of the mass and decay matrices, $G_F$ is the Fermi constant, $M_W$ is the W boson mass, $m_i$ is the mass of quark $i$, $x_i = m_i^2 / M_W^2$; $M_{B_q}$, $f_{B_q}$ and $B_{B_q}$ are the $B^0_q$ mass, weak decay constant and bag parameter respectively. The Inami – Lim function $S_0(x_i)$ is approximated as 0.784 $x_i^{0.76}$ [42], $V_{ij}$ are the elements of the CKM matrix [7,8]; $\eta_B$ and $\eta_{B_q}$ are QCD corrections.

Fig. 1: Box diagrams for $B^0_q - ar{B}^0_q$ mixing ($q = d, s$).
The phases of $M_{12}$ and $\Gamma_{12}$ satisfy $\phi_M - \phi_v = \pi + O\left(\frac{m_c^2}{m_b^2}\right)$, \hfill (4.9)

implying that the mass eigenstates have mass and width differences of opposite signs. The heavy state is expected to have smaller decay width than that of the light state. Hence, $\Delta \Gamma = \Gamma_L - \Gamma_H$ is expected to be positive in the SM. Recently, the LHCb collaboration has found that $\Delta \Gamma_s$ is positive.

The quantity $\left|\frac{\Gamma_{12}}{M_{12}}\right| \sim O\left(\frac{m_b^2}{m_c^2}\right)$ is very small. In the absence of CP violation in the mixing, the ratio $\frac{\Delta \Gamma}{\Delta M_q}$ is equal to the small quantity $\left|\frac{\Gamma_{12}}{M_{12}}\right|$ which is independent of CKM matrix elements. Hence, it is same for $B^0_s - \bar{B}^0_s$ and $B^0_d - \bar{B}^0_d$ systems. From the current knowledge on the mixing parameter $x_q = \Delta M_q / \Gamma_q$ \hfill [43], we have

\begin{align*}
  x_d &= 0.774 \pm 0.008 \quad (B^0_d - \bar{B}^0_d \text{ system}), \\
  x_s &= 26.2 \pm 0.5 \quad (B^0_s - \bar{B}^0_s \text{ system}). \hfill (4.10)
\end{align*}

Furthermore, the Standard Model predicts that $\Delta \Gamma_d / \Gamma_d$ is very small (below 1%), but $\Delta \Gamma_s / \Gamma_s$ is considerably larger (\sim 10%) \hfill [4]. These width differences are caused by the existence of final states to which both the $B^0$ and $\bar{B}^0$ mesons decay. The $M^q_{12}$ is very sensitive to NP both for $B^0_d$ and $B^0_s$. $\Gamma_{12}$ stems from Cabbibo-favoured tree-level decays and possible NP effects are expected to be smaller than the hadronic uncertainties but in the case of $\Gamma^d_{12}$, the contributing decays are Cabbibo-suppressed.

New physics in $M^q_{12}$ will not only affect the neutral-meson mixing parameters, but also the time-dependent analyses of decays corresponding to interference between mixing and decay. The $\Delta M_{\eta_c}$ and $\Delta M_{\eta_s}$ mass differences in the SM are given in equations (4.2) and (4.3).
In extended quark sector model [28–30,44], besides the three standard generations of the quarks, there is an $SU(2)_L$ singlet of charge $-1/3$. This model allows for $Z$-mediated FCNCs. This model has already discussed in chapter 2, Section 2.3 (Equations 2.22–2.27). Now consider the $B_q^0 - \bar{B_q}^0$ mixing ($q = d, s$) in the presence of $Z$-mediated FCNC [28–30,44] at tree level (Fig. 2) [45,46]. The $Z$-mediated FCNC couplings $U_{qb}^Z$ and $U_{zh}^Z$, which affect the $B_q^0 - \bar{B_q}^0$ mixing, are constrained from the experimental limit on $B(B \to \ell^+ \ell^- X)$ [31–34]. The $Z$-mediated flavour-changing couplings $U_{qb}^Z$ can contribute to $B_q^0 - \bar{B_q}^0$ mixing [44]:

$$M_{12}^Z(B_q) = \frac{\sqrt{2}G_F M_{B_q} \eta_{B_q} M_q^2}{12} f_{B_q}^2 B_{B_q} (U_{qb}^Z)^2.$$  

(4.11)

The same idea can be applied to a $Z'$-boson i.e., mixing among particles which have different $Z'$ quantum numbers will induce FCNCs due to $Z'$ exchange [27,47–52]. Since the $U_{pq}^{Z'}$ are generated by mixing that breaks weak isospin, they are expected to be at most $O(M_1/M_2)$, where $M_1(M_2)$ is typical light (heavy) fermion mass. On the other hand, the $Z'$-mediated coupling $U_{pq}^{Z'}$ can be generated via mixing of particles with same weak isospin and, so, suffer no suppression. Even though $Z'$-mediated interactions are suppressed relative to $Z$, these are compensated by the factor $U_{pq}^{Z'} / U_{pq}^Z \sim (M_2/M_1)$. Thus, the new contributions from $Z'$ boson are exactly in the similar manner as in the $Z$ boson (Fig. 2) [45,46]. Therefore, the contribution of $Z'$-mediated FCNCs to $B_q^0 - \bar{B_q}^0$ mixing [27] is,

$$M_{12}^{Z'}(B_q) = \frac{\sqrt{2}G_F M_{B_q} \eta_{B_q} M_q^2}{12} M_{Z'}^2 f_{B_q}^2 B_{B_q} (U_{qb}^{Z'})^2.$$  

(4.12)

Now considering the contributions from $Z$- and $Z'$-mediated FCNC, we can write the mass matrix element for $B_q^0 - \bar{B_q}^0$ mixing as:

$$M_{12}(B_q) = M_{12}^{SM}(B_q) + M_{12}^Z(B_q) + M_{12}^{Z'}(B_q).$$  

(4.13)
Fig. 2: Feynman diagrams for $B_q^0 - \overline{B_q^0}$ ($q = s, d$) mixing in the extended quark model, where the blob represents the tree level flavour changing vertex.

4.4 Evaluation of $B^0_q - \overline{B^0_q}$ mixing mass differences

The $B^0_q - \overline{B^0_q}$ ($q = s, d$) mixing mass differences can be evaluated by substituting equations (4.7), (4.11), (4.12) and (4.13) in equation (4.1). Thus, considering the contributions from Z- and Z'-mediated FCNC, we can write the $B^0_{b} - \overline{B^0_{b}}$ mass difference as:

$$\Delta M_{b} = 2 \left[ \frac{G_f^2 M_w^2 M_B \eta_{B_s}}{12 \pi^2} f_B \beta_1 S_0 \left( x_s \right) \left( V_{ub}^* V_{ub} \right)^2 + \frac{\sqrt{2} G_f M_B \eta_{B_s} f_B} {12} B_s (U_{ub}^x)^2 \right]$$

$$+ \frac{\sqrt{2} G_f M_B \eta_{B_s} M_Z^2}{12} f_B \beta_1 (U_{ub}^z)^2$$

(4.14)
Similarly, the $B^0_d - \bar{B}^0_d$ mass difference can be written as:

\[
\Delta M_{B_d} = \frac{G^2 F M^2_W M_{B_d} \eta_{B_d}}{12 \pi^2} f^2_{B_d} B_{B_d} S_0(x_t) (\gamma_1^* \gamma_2)^2 + \frac{\sqrt{2} G_F M_{B_d} \eta_{B_d}}{12} f^2_{B_d} B_{B_d} (U^{Z*}_{a b})^2
\]

(4.15)

The equations (4.14) and (4.15) are used in the next section for our calculations.

4.5 Results and Discussions

We estimate the mass of $Z'$ boson using the experimental values of mass differences i.e. $\Delta M_{B_d} = 17.73 \pm 0.05$ ps$^{-1}$ [12] in equation (4.14) and $\Delta M_{B_s} = 0.507 \pm 0.004$ ps$^{-1}$ [10,11] in equation (4.15). We have taken the recent data from [4]: $G_F = (1.16637 \pm 0.00001) \times 10^{-5}$ GeV$^{-2}$, $M_{W} = (80.399 \pm 0.23)$ GeV, $M_{B_s} = (5279.5 \pm 0.5)$ MeV, $M_{Z} = (91.1876 \pm 0.0021)$ GeV. Using the lattice QCD calculations [53], $f_{B_d} \sqrt{B_{B_d}} = (216 \pm 9 \pm 13)$ MeV, $f_{B_s} \sqrt{B_{B_s}} = (275 \pm 7 \pm 13)$ MeV and assuming $|V_{ub}| = 1$, one finds $|V_{td}| = (8.4 \pm 0.6) \times 10^{-3}$, and $|V_{ub}| = (38.7 \pm 2.1) \times 10^{-3}$. The Inami-Lim function [3] $S_0 = 2.35$, and $\eta_{B_s} = \eta_{B_d} = 0.552$ [1]. The value of $|U^{Z*}_{a b}| \approx 10^{-3}$ [54] and $|U^{Z*}_{b b}| \approx 10^{-3}$ [28–30]. From the study of $B^0_s - \bar{B}^0_s$ mixing in leptophobic $Z'$ model, they [2] obtained $|U^{Z*}_{s b}| \leq 0.036$ for $M_{Z'} = 700$ GeV and $|U^{Z*}_{b b}| \leq 0.051$ for $M_{Z'} = 1$ TeV. We take $|U^{Z*}_{s b}| \approx 0.04$ and $|U^{Z*}_{b b}| \approx 7.8 \times 10^{-3}$ for our calculations. With these values, we observe that the value of $\Delta M_{B_s}$ is consistent with the mass of $Z'$ boson in the range $989$ GeV $-$ $1665$ GeV.

The contribution of $Z'$-mediated FCNCs to $b \rightarrow s \nu \bar{\nu}$ yields the constraint [27]:

\[
|U^{Z*}_{a b}| \frac{M^2_{B_d}}{M^2_{Z'}} \leq 7.1 \times 10^{-3}.
\]

(4.16)
We take $|U_{sb}^{Z'}| \approx 0.04$, and hence our estimation of the mass of $Z'$ boson satisfies the bound of equation (4.16). This demonstrates the importance of $B_s^0 - \bar{B_s^0}$ mixing in constraining NP in the flavour sector.

Similarly, we take $|U_{d}^{Z'}| \approx 7.8 \times 10^{-3}$ for our calculations, which is satisfied the constraints obtained for the FCNC coupling $|U_{d}^{Z'}| < 0.61$ for $B \rightarrow \pi \nu \overline{\nu}$ decay [55]. We observe that the value of $\Delta M_{B_s}$ is consistent with the mass of $Z'$ boson in the range $1352 - 1824$ GeV. If one tries with any other values of $Z'$ boson mass, there is a discrepancy in the values of $\Delta M_{B_s}$ and $\Delta M_{B_d}$.

Since the $Z'$ has not yet been discovered, its exact mass is unknown. A broad class of supersymmetric extensions of the SM predict a $Z'$ boson whose mass is naturally in the range $250$ GeV $< M_{Z'} < 2$ TeV [56]. In a study of $B$ meson decays with $Z'$-mediated flavour-changing neutral currents [49], they study the $Z'$ boson in the mass range of a few hundred GeV to 1 TeV. The experimental searches of the $Z'$ boson from Drell-Yan cross sections at Tevatron have put lower limits on the mass range 0.6 – 1.0 TeV at 95 % CL depending on the specific models [23]. From the electroweak precision data analysis, the improved lower limits on the $Z'$ mass are given in the range 1.1– 1.4 TeV at 95 % CL [57]. The LHC has the potential of discovering the $Z'$ up to $M_{Z'} = 4.5$ TeV with 100 fb$^{-1}$ data at centre of mass energy $\sqrt{s} = 14$ TeV [58]. These limits on $Z'$ boson mass favour higher energy ($\geq 1$ TeV) collisions for direct observation of the signal. It is also possible that the $Z'$ bosons can be much heavy or weak enough to escape beyond the discovery reach expected at the LHC. In this case, only the indirect signatures of $Z'$ exchanges may occur at the high energy colliders [59]. Recently [60], it has been shown that one can probe a TeV scale $Z'$ boson at the LHC in longitudinal weak gauge boson scattering. More interestingly, our estimation of mass of $Z'$ boson lies in the range of $1352 - 1665$ GeV.

In conclusion, the FCNC processes of $B_d^0 - \overline{B_d^0}$ and $B_s^0 - \overline{B_s^0}$ mixing offer interesting probes to search for signals of physics beyond the SM. In this chapter, we have tried to estimate the mass of $Z'$ boson from $B_s^0 - \overline{B_s^0}$ mass differences. Our
estimation of mass of $Z'$ boson is consistent with the experimental values of $\Delta M_B$, and $\Delta M_{B_s}$. Despite of the success of the B-factories and the Tevatron, there is still considerable room for new physics in $B_d^0 - \overline{B_d^0}$ as well as $B_s^0 - \overline{B_s^0}$ mixing. We hope that the current exciting experimental situation will stimulate novel activities in this direction.
References


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