CHAPTER – V

MATHEMATICAL MODELING OF DRYING BEHAVIOR OF CASHEW IN A HYBRID DRYER

5.1 Introduction

Modeling of the drying kinetics is one of the most important aspects of drying technology and it is considered as tool for process control. The study of drying behavior of the different material has received renewed interest by various investigators. Drying influences the physical, chemical and quality characteristics of products.

The purpose of modeling is to find out the size of drying equipment and drying chamber based on the operating conditions of the dryer. The prediction of drying kinetics of agricultural products under various conditions is very useful in the design and optimization of the dryers. Evaluation of drying kinetics as a function of drying condition could help us in drying simulation for predicting the suitable drying condition.
Thin layer drying modeling is always used in order to understand and estimate the drying characteristics of agricultural products. For proper understanding of process drying and production of quality dried products it is essential to know the thin layer drying characteristics.

Mathematical modeling of drying behaving of different agricultural products often requires statistical methods of regression and correlation analysis. Linear and non linear regression models are important tools to find the relationship between different variables. Many models have been proposed by several authors for the estimation of drying rate of biological materials that have finally led to different expressions for the prediction .Yet none of the empirical and semi empirical models can be used over a wide range of foods and drying conditions. Each of the models may account for varying degrees of effect of drying conditions on drying process which must be considered during the drying process.

Mortazaaghbashlo et.al.[104] investigated the modeling of thin layer drying behavior of potato slices in a semi industrial continuous band dryer. In order to describe the drying behavior of potato slices, three drying models were fitted to the drying data. The page model was selected as the best according to R^2, χ^2 and RMSE. The effective diffusivity varied between $3.17 \times 10^{-7}$and 15.45×10^{-7} m²/s and the energy of activation was found in the range of 39.49-42.34 KJ/mol.

Kavak Akpinar et al.[105] investigated the mathematical modeling of thin layer drying process of long green pepper in solar drying and open sun drying. The drying data were fitted to thirteen different mathematical models. Among the models, logarithmic model was found to be the most suitable model for describing the drying curve of the thin layer forced solar drying process of long green peppers with R^2 of 0.98815, χ^2 of 0.001742354 and RMSE of 0.040998285. However, the midilli and kucuk model has the best drying curve of long green peppers with R^2 of 0.99656, χ^2 of 0.000598087 and RMSE of 0.02404129 for the open sun drying mode.

Jompob Waewsak et al,[106] carried out a mathematical modeling study of hot air drying for some agricultural products. In his work the biomass longan dryer is used to dry agricultural products such as red chili peppers, lemon grass and leech lime leakers. The results have shown that among the thirteen different models, Midilli model was found to be the best for describing the drying behavior of red chilli peppers and leech lime leaves, whereas the wangh and singh model was the most suitable for lemon grass.

C. L. Hii et al.[107] has done the modeling using a new thin layer drying model and product quality of cocoa. In this paper drying kinetics and modeling of the artificial drying process of cocoa beans were investigated. The dryer was tested with a temperature of 60°C, 70°C and 80°C.The result showed that new model was best fit for the drying behavior of cocoa beans.

Tuncay Gunhan et al.[108] has reported the work on the mathematical modeling of drying of bay leaves. The investigation was conducted with constant air velocity of 1.5m/sec, relative humidity of 5%, 15% and 25% and temperatures of 40^oC, 50^oC and 60^oC. The drying data were fitted with fifteen different Mathematical drying models on the basis of correlation co-efficient (R^2), root mean square error (RMSE), mean bias error (MBE), Chi-square χ^2 and t- statistics method. The result showed that page model was most suitable for drying of bay leaves.

Zomorodianet al.[109] developed a mathematical model of forced convection solar drying of cuminum cyminum using mixed and indirect drying method Eleven different mathematical models was studied and the pertinent coefficients was determined for each model by non-linear regression analysis technique. The best results were found for diffusion model with R^2 =0.995, χ^2 =0.0023 and

RMSE=0.0199 in mixed mode and the Midilli model with R^2 =0.995 , $\chi 2$ =0.023 and RMSE=0.0225 in indirect mode. The best model was selected based on high pertinent coefficient.

Agnieszka Kaleta et al.[110] formulated three new types of drying model for drying apple.The three developed models were compared with the sixteen models available from the literature for accuracy. Their accuracies were measured on the basis of correlation co-efficient (R^2),root mean square error (RMSE) and reduced chi-square (X^2). Drying behavior of apple was investigated in fluidized bed dryer. At the end of this study the page model and one of the empirical models formulated by the author was considered as the most suitable with R^2>0.9977, RMSE =0.0094-0.0167, $\chi 2$ =0.0001-0.0002.

Navneetkumar et al.[111] designed mathematical modeling of thin layer hot air drying carrot pomace. The experiments were carried out at 60°C, 65°C,70°C and 75°C and at an air velocity 0.7m/sec. The average value of effective diffusivity ranged from 2.74×10^{-9} to 4.64×10^{-9} m^2/sec and the activation energy value was 23.05kJ/mole for drying of carrot pomace. With increase of temperature, the drying time of the carrot pomace decreases where as drying temperature, increased the effective diffusivity.

Basunia et al.[112] investigated the best fitted thin-layer re-wetting model for medium grain rough rice. Five models namely diffusion model, page model, exponential approximate model, diffusion model and polynomial model were compared with experimental data on the basis of standard error of estimate (SEE). The comparison showed that both the diffusion and page models fitted closely with the experimental data

5.2 Materials and Methods

5.2.1 Modeling of thin layer drying curves

The moisture ratio (MR) was defined as $MR = (X-X_e)/(X_0-X_e)$. The values of X_e are relatively small compared to X or X_e for the drying time, thus the MR can be simplified to $MR= X/X_0$. For investigating the drying characteristics of cashew kernel it is important to model the drying behavior effectively. In this study, the experimental thin layer drying data for cashew kernel were fitted with 15 commonly used mathematical drying models [113] listed in Table 5.1.

Table 5.1. Thin layer drying mathematical models.

S. No	Model	Model equation	References
1	Lewis	$MR = \exp(-kt)$	Lewis (1921)
2	Page	$MR = \exp(-kt^n)$	Page (1949)
3	Modified page	$MR = \exp[(-kt^n)]$	Overhults et al (1973)
4	Henderson and Pabis	$MR = a \exp(-kt)$	Westerman (1973)
5	Logarithmic	$MR = a \exp(-kt)+c$	Togrul and Pehlivan (2003)
6	Two-term	$MR = a \exp(-k_0t)+b \exp(k_1t)$	Henderson (1974)
7	Two-term exponential	$MR = a \exp(-kt)+(1-a)\exp(-kat)$	Sharaf-Elden et al (1980)
8	Wangh and Singh	$MR = 1+at+bt^2$	Wangh and Singh (1978)
9	Diffusion approach	$MR = a \exp(-kt)+(1-a)\exp(-kbt)$	Demir et al (2007)
10	Verma et al.	$MR = a \exp(-kx)(1-a)\exp(-gx)$	Verma et al (1985)
11	Modified Henderson and Pabis	$MR = a \exp(-kt)+b \exp(-gt)+c \exp(-ht)$	Karathanos (1999)
12	Simplified Fick's diffusion equation	$MR = a \exp(-k(t/L^2))$	Diamente and Munro (1991)
13	Midilli et al.	$MR = a \exp(-kt^n)+bt$	Midlli et al (2002)
14	Demir et al.	$MR = a \exp(-kt^n)+b$	Demir et al (2007)
15	Weibull	$MR = \exp(-(t/a)^b)$	Corzo et al (2008)

To validate the goodness of the fit, three statistical criteria, namely root mean square error (RMSE), reduced chi square (χ^2) and coefficient of correlation (R^2) were calculated. Higher R^2, lower χ^2 and RMSE value leads to better goodness of fit.

$$RMSE = \sqrt{\left[\frac{1}{N} \sum_{i=1}^{N} \left(MR_{\exp,\,\mathrm{i}} - MR_{pre,\,i} \right)^2 \right]} \tag{5.1}$$

$$x^2 = \frac{\sum_{i=1}^{N} \left(MR\exp,i - MRpre,i \right)^2}{N-p} \tag{5.2}$$

$$R^2 = \frac{\left[\sum_{i=1}^{N} \left(MR_{\exp,i} - \overline{MR_{\exp}} \right) \left(MR_{pre,i} - \overline{MR_{pre}} \right) \right]^2}{\sum_{i=1}^{N} \left(MR_{\exp,i} - \overline{MR_{pre}} \right)^2 \sum_{i=1}^{N} \left(MR_{pre,i} - \overline{MR_{pre}} \right)^2} \tag{5.3}$$

Expression for parameters of selected model equations

The empirical expression for the selected model (page, logarithmic, two terms and midilli model) and their constant and coefficient are as follows.

The two parameters (n' and 'k') in the page model are expressed as a linear function of drying air temperature and relative humidity. The expressions for these parameters are given below.

$n = 0.068293+0.01094T+0.17036RH$ (5.4)

$k = 0.00561-0.000073T-0.0469RH$ (5.5)

The parameter in the logarithmic model is a, k and c and their expressions are [114]

$a=0.001T+0.945$ (5.6)

$k= 1.49E\text{-}06T^2- 0.0001T+0.005$ (5.7)

$c=8E\text{-}05T^2 – 0.01T + 0.256$ (5.8)

The expression for two term model constant and co efficient are

$MR= a \exp(-k_0t)+b \exp(-k_1t)$ (5.9)

$a = 3.28882-0.0982883(T)+0.000980965(T^2)$ (5.10)

$b = -2.18886+0.0943443(T)-0.000942365(T^2)$ (5.11)

$k0= -0.017314+0.000795566(T)-0.00000729165(T^2)$ (5.12)

$k1= -0.0142761+0.00057601(T)-0.0000058283(T^2)$ (5.13)

MIDILLI-KUCUK Model. The most commonly used empirical equation to describe the drying characteristic are as follows [114]

$MR = a\exp(-ktn)+ bt$ (5.14)

$a = 0.98190+0.00071(T)-0.000008(T^2)$ (5.15)

$k =0.01103(T0.31093)$ (5.16)

$n =0.67687-0.00552(T)+0.00007(T^2)$ (5.17)

$b =-0.00003-0.000002(T) +0.00000002(T^2)$ (5.18)

The parameters used in Lewis models are k, n and their expressions are;

$K = 0.03801T+0.5292RH-3597.49RH-0.7864$ (5.19)

$n = -0.00081T^2-0.02198RH^2-807840RH^2+0.09872T-0.00094RH+321.92RH-1.961$ (5.20)

5.3 **Result and Discussion**

5.3.1 Variation of MR experimental vs predicted for forced solar dryer

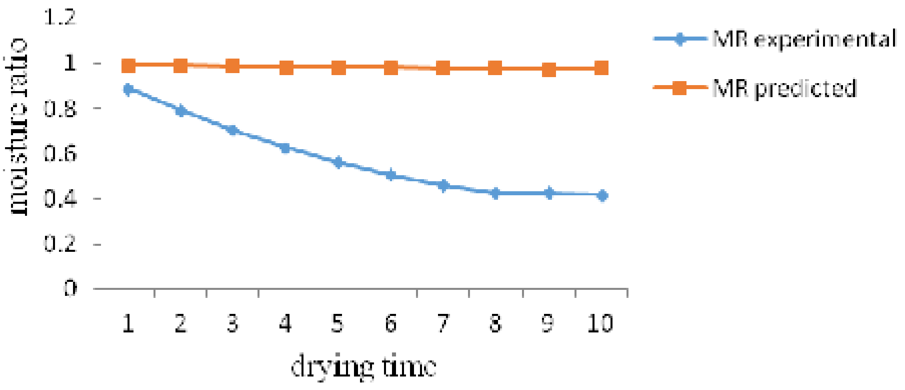
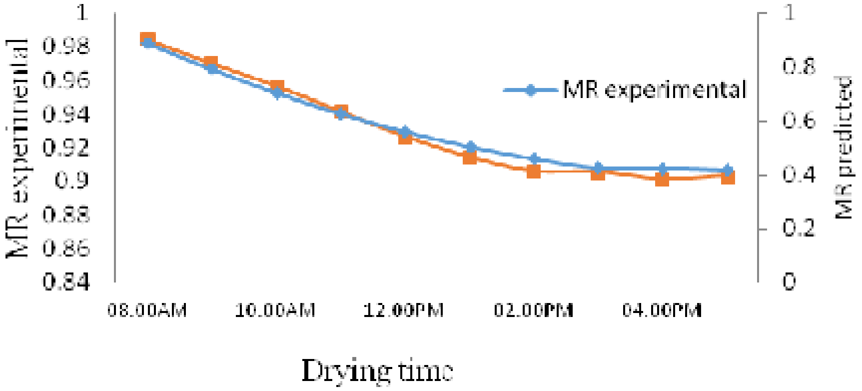


Fig. 5.1. Variation of moisture ratio -experimental and predicted with drying time for forced solar dryer

The variation of moisture ratio of the cashew kernel with time for forced solar drying using two term model is shown in Fig 5.1. The moisture ratio calculated based on experimental value decreases from 0.9 to 0.4 during the stipulated drying period. The predicted moisture ratio value reduces from 1 to 0.98.

5.3.2 Variation of MR experimental vs predicted for hybrid dryer

Fig. 5.2.Variation of moisture ratio - experimental and predicted with drying time for hybrid



dryer

The page model is suitable for representing drying behavior of cashew kernel in forced hybrid drying. The predicted and experimental values of MR are plotted and shown in Fig 5.2. The result showed that if the drying time increases moisture ratio decreases. The experimental moisture ratio decreases from 0.98 to 0.9 and the predicted moisture ratio decreases from 0.98 to 0.92.

5.3.3 Variation of MR experimental vs predicted for biomass dryer

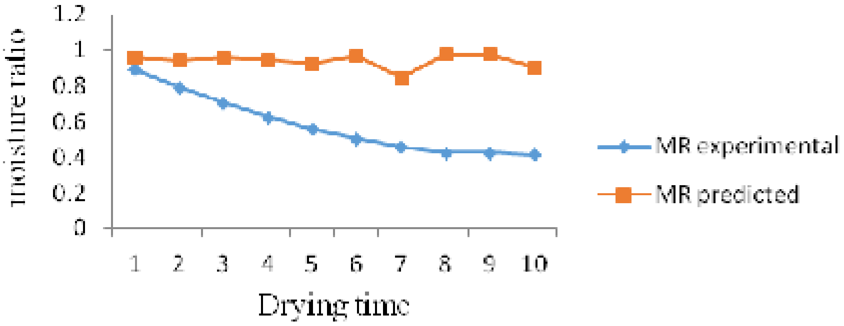


Fig. 5.3 Variation of moisture ratio - experimental and predicted with drying time for biomass dryer

The moisture ratio predicted and experimental values are plotted with drying time for biomass dryer (Fig 5.3). The result showed that if the drying time increases moisture ratio decreases. The experimental moisture ratio decreases from 0.98 to 0.9 and the predicted moisture ratio decreases from 0.9 to 0.5.

5.3.4 Average value of R^2, χ^2 and RMSE for solar biomass hybrid dryer

The average value of R^2, χ^2 and RMSE are shown in Fig 5.4. The calculated value has maximum R^2 and minimum χ2 and RMSE value in all the three modes of operation of the dryer.

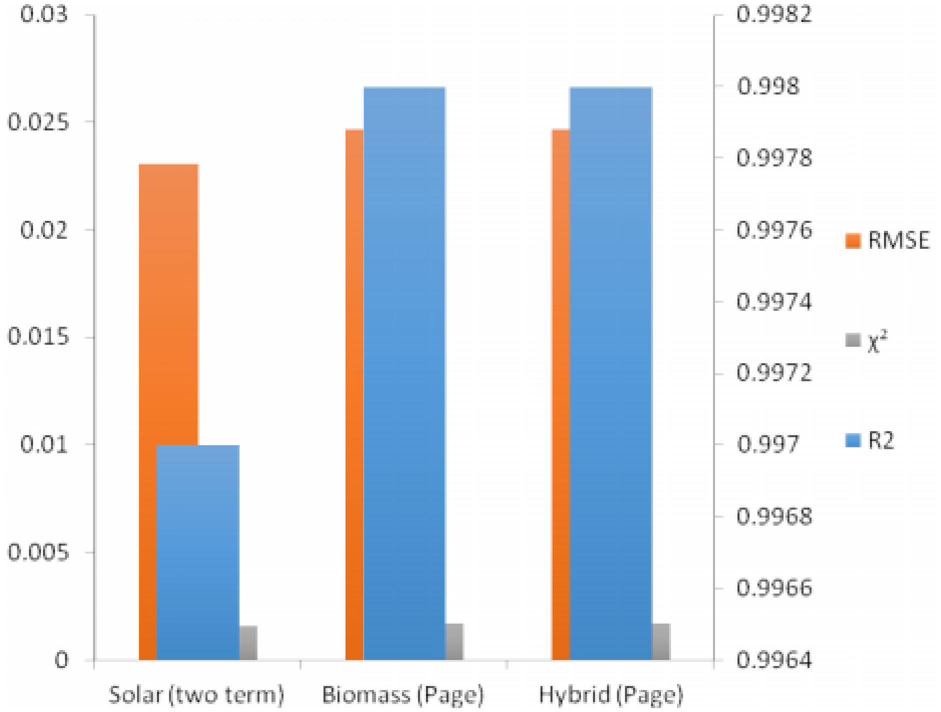


Fig. 5.4. Average value of R^2, χ^2 and RMSE

5.3.5 Statistical results for forced hybrid dryer

By carrying out the multiple regression analysis by using moisture ratio, temperature and relative humidity the outcome of the accepted model constants and coefficients are listed in Table 5.2.

From these results in page model the R^2, χ^2, RMSE values are 0.998, 0.0017 and 0.0247 respectively followed by logarithmic model values of 0.972, 0.0082 and 0.0638. Modified page, two term, exponential two term and Midilli models gave the closest results but the selected page model gave the best fit and simpler than all the other models.

Table 5.2. Statistical results from various thin layer drying models for forced hybrid drying

Model name	Model Coefficients	R^2	RMSE	χ^2
Lewis	K=0.030214	0.916	0.0773	0.0313
Page	K=0.28496, n=0.455996	0.998	0.0247	0.0017
Modified page	K=0.0812,n=0.3205	0.943	0.0712	0.0032
Henderson and Pabis	a=0.06149, K=0.023	0.911	0.0921	0.0046
Logarithmic	a=0.761761, K=0.0921	0.972	0.0638	0.0082
Two term	a=0.00312,b=0.2813,k_1=0.143,k_0=1.273	0.933	0.0376	0.0041
Exponential two term	a=0.0179,k=0.111	0.920	0.0375	0.1060
Wang and Singh	a=1.00032,b=0.7932	0.873	0.0316	0.0238
Thompson	a=0.000944,b=0.1932	1.011	0.1955	0.0412
Diffusion Approximation	a=0.45651,K=0.0310,b=0.04732	1.271	0.0332	0.0132
Midilli et al	a=1.1143,K=0.1791,n=1.3215,b=0.00321	0.923	0.872	0.0712

5.3.6 Statistical results for forced convection solar dryer

The results obtained by regression analysis for forced solar dryer is listed in Table 5.3. The results shows that, in two term, Midilli, exponential two term, modified page and Henderson the values of R^2, has the maximum and lowest value of χ^2 and RMSE were obtained. Among the above 5 models two term and Midilli models are very close to the result.

Table 5.3. Statistical result from various thin layer drying models for forced solar drying

Model name	Model Coefficients	R^2	RMSE	χ^2
Newton	K=0.13241	0.847	0.1028	0.0100
Page	K=0.2779154, n=0.048795	0.891	0.0266	0.0033
Modified page	K=0.68921,n=0.2115	0.942	0.2204	0.0271
Henderson and Pabis	a=0.924798, K=0.418	0.931	0.0249	0.0036
Logarithmic	a=0.083952, K=0.0032145	0.945	0.0638	0.0032
Two term	a=0.049,b=0.47008,k_1=0.085,k_0=0.465214	0.997	0.0231	0.0016
Exponential two term	a=0.411528,k=0.189254	0.960	0.0792	0.0194
Wang and Singh	a=-0.03257,b=0.008524	0.916	0.1329	0.0258
Thompson	a=0.00497,b=0.3915	1.005	0.0324	0.0177
Diffusion Approximation	a=0.56132,K=0.0121,b=0.007895	1.324	0.0299	0.0029
Midilli et al	a=1.37457,K=0.1254,n=0.5698,b=0.0258	0.977	0.0243	0.0071

5.3.7 Statistical results for forced convection biomass dryer

Experimental moisture ratios obtained from the biomass dryer under forced convection mode were fitted to the selected thin layer drying model. By using regression analysis the value of correlation coefficient (R^2), the reduced chi-square (χ^2) and Root Mean Square Errors (RMSE) and their constants and coefficients are determined and listed in Table 5.4. The highest R^2 value and lowest values of χ^2 and RMSE were obtained in Henderson and Pabis model. Based on the statistical analysis, Henderson and pabis model can be selected as a suitable model to predict the drying characteristics of cashew.

Table 5.4. Statistical result from various thin layer drying models for forced biomass drying

Model name	Model Coefficients	R^2	RMSE	χ^2
Newton	K=0.054023	0.923	0.0827	0.0413

Page	K=0.28496, n=0.455996	0.997	0.0234	0.0016
Modified page	K=0.0681,n=0.5203	0.962	0.0676	0.0028
Henderson and Pabis	a=0.941654, K=0.049	0.932	0.0921	0.0346
Logarithmic	a=0.839717, K=0.095284	0.978	0.0617	0.0074
Two term	a=0.049,b=0.470827,k_1=0.049,k_0=0.470827	0.945	0.0776	0.0024
Exponential two term	a=0.259568,k=0.149879	0.972	0.0675	0.1060
Wang and Singh	a=-0.03146,b=0.000253	0.863	0.0516	0.0283
Thompson	a=0.003778,b=0.392219	1.023	0.0955	0.0314
Diffusion	a=0.766617,K=0.11179,b=0.053014	1.287	0.0432	0.0245
Midilli et al	a=1.000377,K=0.174944,n=0.657723,b=0.00125	0.943	0.0772	0.1712

5.4 Conclusion

In this study, the drying behavior of cashew kernel was found out in solar mode, biomass mode and hybrid mode of operation. In order to describe the drying behavior of cashew kernel eleven thin layer drying mathematical model were fitted to the drying data obtained from solar biomass hybrid dryer. Among the eleven thin layers drying model and based on their correlation co-efficient, reduced chi-square value and root mean square error value, the best curve fitting model is predicted. Two term model has the best curve fitting model for the experimental moisture ratio value for the forced solar dryer and page model for forced biomass and forced hybrid type of dryer respectively. It may be concluded that the fitted drying model adequately explains the drying behavior of the cashew kernel.