CHAPTER 3
MATHEMATICAL FORMULATION OF THE MIXER IN
NATURAL CO-ORDINATE SYSTEM

3.1 THEORETICAL CONSIDERATIONS

A jet in crossflow, or a transverse jet, in a flow field where a jet of fluid enters and interacts with a cross flowing fluid. The JICF is a very pleasant flow configuration with regard to mixing. It is one of the most effective ways to mix two fluids in a limited space, which is superior to other flow constellations like the mixing layer or the jet in cross flow (Broadwell and Breidenthal 1984).

In order to predict the mixing of jet in cross flow of venturi-jet mixer, one must first predict the characteristics of this flow phenomenon. There are many research works on jets in cross flow, which are concerned with mixing problems.

Among them an interesting one was written by Hirst (1971), who studied the characteristics of round, turbulent and buoyant jets discharging to flowing stratified ambient fluid in a “natural” coordinate system.

As an extreme case of the work, the present research work predicts the trajectory of round jet in cross flow using this mathematical model in a natural coordinate system.
3.2 FORMULATION OF THE PROBLEM IN NATURAL COORDINATE SYSTEM

The natural coordinate system used to develop mathematical model for incompressible transverse jet discharge into cross flow of venturi-jet mixer due to suction effect at large Reynolds number is shown in Figure 3.1.

**Figure 3.1 Coordinate system and relevant variables in venturi-jet mixer**

In the present work, \( \nu \) is the horizontal crossflow speed and \( \mu \) is the average (top-hat) jet velocity. The jet stream diffuses along its trajectory in the stream wise direction. As the jet stream section gradually expands, the velocity increases due to the effect of the turbulent mixing and entrainment.

Most theoretical attempts to explain the jet motion involve integral methods, and, of necessity, many simplifying assumptions, particularly with regard to entrainment (Broadwell and Breidenthal 1984).

The jet density is assumed to be the same as that of the cross flow. It is also assumed that the flow is steady and fully turbulent and the fluid properties are constant.

The motion of the jet is determined by the initial conditions at the nozzle exit and the cross flow conditions, such as exit velocity, outlet orientation, nozzle diameter, as well as the crossflow velocity. The basic
equations of this case are (1) conservation of mass, (2) conservation of momentum and (3) conservation of tracer concentration.

The relative importance of JICF is a complex issue that has been discussed by numerous researches interested in the smoke issuing from chimneys, pollutant dispersal and many other engineering applications (Li et al. 2006).

The following analysis builds upon these engineering results and adopts their findings to the venturi-jet mixer. The theoretical model is based on an integral approach that considers the entrained mass and momentum from the crossflow by tracking the development of jet in a jet-centered coordinate system. The equations expressing the co-ordinate transformation between \((x,z)\) and \((s,\theta)\) are given by:

\[
x = \int \cos \theta ds \tag{3.1a}
\]
\[
z = \int \sin \theta ds \tag{3.1b}
\]

Numerous workers have investigated the use of different entrainment-velocity relationships (Wright 1984). The local entrainment velocity has been obtained (Hewett et al. 1971) as:

\[
u_e = \alpha |u - v\cos \theta| + \beta |v\sin \theta| \tag{3.2}
\]

where \(\alpha |u - v\cos \theta|\) is entrainment by radial inflow minus the amount swept tangentially along the jet margin by the cross flow, \(\beta |v\sin \theta|\) is entrainment from cross flow; \(\alpha\) is tangential entrainment parameter, and \(\beta\) is the normal entrainment parameter.
Values of the constant entrainment parameters $\alpha = 0.11$ and $\beta = 0.6$ are used for analysis as obtained by Hoult and Weil (1972). The equations of motion have been re-derived for venturi-jet mixer case (Hoult and Weil 1972). For the configuration in Figure 3.1, mass conservation (continuity) equation is shown in Equation (3.3).

$$\frac{d}{ds} \left( \pi b^2 \rho u \right) = 2\pi b \rho u_e$$

(3.3)

where $b$ is characteristic jet radius and $\rho$ is density of mixing fluids. The conservation of tangential momentum can be written as:

$$\frac{d}{ds} \left( \pi b^2 \rho u^2 \right) = -b^2 \Delta \sin \theta + v \cos \theta \frac{d}{ds} \left( \pi b^2 \rho u \right)$$

(3.4)

The first term on the right hand side of Equation (3.4) denotes change in momentum caused by the component of pressure force in the cross flow direction, and the second term represents entrainment of momentum from cross flow. The overall pressure drop is denoted by $\Delta p$. The conservation of radial momentum is given by,

$$\left( \pi b^2 \rho u^2 \right) \frac{d\theta}{ds} = -\pi b^2 \Delta \cos \theta - v \sin \theta \frac{d}{ds} \left( \pi b^2 \rho u \right)$$

(3.5)

In Equation (3.5), the left-hand side represents the change in $\theta$ caused by the entrainment of momentum at an angle to the jet axis by both pressure force and cross flow.

Finally, the conservation of tracer concentration $c$ is given by,

$$\frac{d}{ds} \left( \pi b^2 cu \right) = 0$$

(3.6)
A jet expands in the cross flow with turbulent entrainment. An important parameter is the ratio of the jet velocity to the cross flow velocity. Four length scales can be defined to determine the scale of jet trajectory within the venturi tube (Forney and Lee 1982). Each length gives a clear physical interpretation and this allows useful flow analogies to be made.

The length scales are: jet orifice diameter \( d = 2b_o \); the throat diameter \( D \); the length \( l_m = dR \sin \theta \) (measure of distance over which the jet travels before it bends over in the cross flow); the length \( l_q = (\pi / 4)^{0.5} d \) (measure of distance over which the initial volume flux is important); where \( R = u_o / v \) here refers to dimensionless jet to cross flow velocity. The conservation Equations (3.3) to (3.6) are rewritten as,

\[
\frac{du}{ds} = -\Delta psin\theta + \frac{2(u - v\cos\theta)u_e}{bu} \tag{3.7}
\]

\[
\frac{db}{ds} = \frac{\Delta psin\theta}{bu^2 \rho} + \frac{(2u - v\cos\theta)u_e}{u^2} \tag{3.8}
\]

\[
\frac{d\theta}{ds} = -\Delta p\cos\theta - \frac{2v\sin\theta u_e}{\rho u^2} \tag{3.9}
\]

\[
\frac{dc}{ds} = -\Delta psin\theta - \frac{2cu_e}{2\rho u^2} \tag{3.10}
\]

\[
\frac{dz}{ds} = \cos\theta \tag{3.11}
\]

\[
\frac{dx}{ds} = \sin\theta \tag{3.12}
\]

The boundary conditions at the jet orifice can be specified as: \( s = 0, \theta = \theta_o, u = u_o, b = b_o, c = c_o \).
3.3 RESULTS AND DISCUSSIONS

The set of coupled first order ordinary differential Equations (3.7) through (3.12) are solved by using fourth order Range-Kutta method. For the purpose of simulation, an inventory of data related to initial value of jet velocity, jet concentration and pressure drop must be kept. The dimension details of venturi-jet mixer used is shown in Figure 2.1.

Twenty five runs were organised, comprising 5 runs for jet with $\theta=45^\circ$, 5 runs for $\theta=60^\circ$, 5 runs for $90^\circ$, 5 runs for $120^\circ$ and 5 runs for $135^\circ$. For each test run condition 50 samples of mixed liquid at each location and other related data were obtained from the mixer.

Table 3.1 Experimental conditions and computational parameters

<table>
<thead>
<tr>
<th>Liquids used: (i) cross flow – water &amp; (ii) jet - water +tracer</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_o$ (deg)</td>
<td>45,60,90,120,135</td>
</tr>
<tr>
<td>$\rho_l$ &amp; $\rho_{cf}$ (kg/m$^3$)</td>
<td>1000</td>
</tr>
<tr>
<td>d (mm)</td>
<td>1</td>
</tr>
<tr>
<td>D (mm)</td>
<td>11</td>
</tr>
<tr>
<td>v (m/s)</td>
<td>4.384 - 7.892</td>
</tr>
<tr>
<td>$u_o$ (m/s)</td>
<td>2.995 - 7.317</td>
</tr>
<tr>
<td>Re$_{cf}$</td>
<td>58002 - 104403</td>
</tr>
<tr>
<td>Re$_j$</td>
<td>3603 - 7008</td>
</tr>
<tr>
<td>Sh$_o$</td>
<td>265 - 460</td>
</tr>
<tr>
<td>Re</td>
<td>31917 - 57451</td>
</tr>
<tr>
<td>R</td>
<td>0.614 - 0.812</td>
</tr>
</tbody>
</table>

The computational parameters obtained through the present experiment are shown in Table 3.1. The cross flow velocity is varied to test the effect on profiles of concentration and jet trajectory in which the cross
flow is constant with downstream distance in throat. Jets are simplified to have a Gaussian in vertical cross-section.

A numerical solution to the crossflow and jet equations provided the jet trajectory and mass concentration decay profile in the parametric form \( x=x(s), z=z(s) \) and \( c=c(s) \). To track the jet trajectory in numerical simulations and experiments, different definitions have been used by the researchers.

In the current work, the jet trajectory is defined as locus of velocity and concentration in the central plane (Yuan and Street 1998). The quality of the cross flow prior to jet exit has been quantified in terms of flow uniformity, turbulence intensity and boundary layer (Maruyama et al. 1982). To obtain the fully developed pipe flow condition, the jet tube length of 100d is used in the experiment.

The flow uniformity at \( v=4 \text{m/s} \) have been evaluated by the spatial standard deviation using a number of samples of the order of 50. The standard deviation is 0.0019 m/s. The turbulence intensity at the core of a fully-developed flow is estimated between 3% - 5%, since the range of Reynolds numbers is 31917-57451.

### 3.3.1 Jet Trajectories and its Correlation

Jet trajectory, one of the important characteristics of a jet in a crossflow, is defined as the locus of the local maximum velocity and also the locus of local maximum scalar concentration. From the analysis of the trajectory, we can obtain a qualitative representation of the flow field and jet deflection (Liscinsky et al. 1993).

The main flow and mixing parameters are, the Reynolds number \( \text{Re}=\text{Ud}_c/\gamma \), and Sherwood number \( \text{Sh}=h_m d_c/D_{AB} \) where \( U \) is bulk mean velocity; \( d_c \) is characteristic dimension; \( \gamma \) is kinematic viscosity; \( h_m \) is mass
transfer coefficient; $D_{AB}$ is diffusion coefficient of jet in cross flow. The non-dimensional gradients of species and flow at the mixture interface are $Sh$ (function of Schmidt number and Reynolds number) and $Re$.

In the present work, the local velocity maxima and local scalar concentration maxima are used for describing the jet trajectory (Yuan and Street 1998, Kamotani and Greber 1992). For convenience the $x$ and $z$ are normalised as done by Li et al. (1998) with respect to $d$. The flow field of a vertical jet in cross flow is observed to be influenced by the square root of fluid momentum ratio shown in Equation (3.13),

$$ R = \left( \frac{\rho_f u^2}{\rho_{cf} \nu^2} \right)^{1/2} $$  \hspace{1cm} (3.13)

For incompressible flow, Equation (3.13) can be simplified as effective velocity ratio $R = u/v$. One of the important characteristics of jet in cross flow is the jet penetration and trajectory which directly affects the mixer dimensions and design.

There are several theoretical and experimental research works available in the literature, which address the jet breakup and trajectory. Wu et al. (1997) experimentally and theoretically analysed liquid jet injected into a cross flow air stream ($R \sim 4-185$; $d \sim 0.5, 1, 2$ mm; $u \sim 9-38$ m/s; $v \sim 70-141$ m/s). Their correlation for jet trajectory is as follows:

$$ \frac{z}{d} = 1.37 \sqrt{R} \frac{x}{d} $$  \hspace{1cm} (3.14)

This is in general a valid conclusion that the spray trajectory is mostly controlled by the momentum ratio: however, as shown in this work in
many cases this ratio is not adequate in predicting the jet trajectory, particularly in elevated conditions.

Tambe et al. (2005) studied the jet characteristics in crossflow for three liquids (R ~ 1-10; d ~ 0.38, 0.76 mm; u ~ 3-26 m/s; v ~ 89-215 m/s). They correlated the jet trajectory with R in a logarithmic form, similar to Wu et al. (1997):

$$\frac{z}{d} = 1.55R^{0.53} \ln \left( 1 + 1.66 \frac{x}{d} \right)$$

(3.15)

Forney et al. (1999) first employed an arbitrary injection angle analysis in turbulent tube flow to conclude the jet trajectory. Their empirical downstream correlation of numerical results for diameter ratios d/D ≤ 0.25 and injection angles 30° ≤ θ_o ≤ 150° is given as:

$$\frac{z}{Rd} = \left( 0.57 - 0.00426 (90° - \theta_o) \right) \left( \frac{d}{D} \right)^{-0.39} \left( \frac{x}{Rd} \right)^{0.26}$$

(3.16)

Amighi et al. (2009) performed a preliminary analysis to investigate the approximate correlation between the trajectory and principal variables such as jet diameter, crossflow velocity and jet velocity. They revealed that the jet velocity has a greater effect on trajectory than the crossflow velocity.

Further, correlating the trajectory merely with the momentum ratio R that ascribes equal contributions to crossflow and jet velocities was inadequate. Therefore it was proposed to correlate the jet trajectories with initial injection angle θ_o, Reynolds number Re based on the hydraulic diameter of the channel. Re is important as it is a measure of the turbulence intensity in the crossflow that has direct effect on wave growth and liquid breakup.
Figure 3.2 Normalised jet trajectories with \( d \) for different injection angle

(a) 45°, (b) 60°, (c) 90°, (d) 120°, (e) 135°
As a part of comprehensive study on liquid jet in crossflow correlations are obtained to the trajectory of tracer injected into crossflow water in venturi-jet mixer for the present study. Among the employed types of correlations used in the literature, the power law is utilized owing to its simplicity and popularity (Amighi et al. 2009).

To obtain the leeward and windward trajectories, a nonlinear multiple regression analysis was done. The correlations arrived for jet centre-line (Equation (3.17)) and windward trajectories (Equation (3.18)) are:

\[
\frac{Z}{d} = \left(0.648 - 0.00407(90^\circ - \theta_o)\right) \left(\frac{x}{d}\right)^{0.51} R^{0.33} \text{Re}^{0.0219} \tag{3.17}
\]

\[
\frac{Z}{d} = \left(0.693 - 0.00482(90^\circ - \theta_o)\right) \left(\frac{x}{d}\right)^{0.53} R^{0.35} \text{Re}^{0.0247} \tag{3.18}
\]

The standard error for the centre-line correlation is 0.27 and for the windward correlation is 0.315. The coefficient of determination of the centre-line correlation is 0.94 and that of windward correlation is 0.93.

Based on Equations (3.7) to (3.12) the trajectories (axes) of all jets are normalised with \(d\), (for \(45^\circ < \theta_o < 135^\circ\) and \(0 < x/d < 30\)) and plotted as shown in Figure 3.2. Predictions of weak and moderate turbulent plane jet in a strong crossflow have not been found in the literature. The lack of enough experimental data may be one of the reasons for this. To validate the mathematical model of the present work, the available experimental observations of a traditional JICF along a flat plate presented by Yuan and Street (1998) on jet trajectory are compared with predicted numerical results. The jet issued into a cross flow over a flat plate boundary layer is a reasonable substitute for the flow in venturi-jet mixer because both flows are
complex and have the formation of counter rotating vortex pair. The plots show that predicted profiles agree well with experimental observations.

As shown in Figure 3.2, the numerical solutions for the jet trajectories are judged to be excellent match with experimental results of Yuan and Street (1998) for injection angle $\theta_o \geq 90^\circ$, both in downstream and perpendicular to direction of the cross flow but fair for angles $\theta_o \leq 90^\circ$. Figure 3.2 shows that the higher the value of $Re$ and lower the value of $R$, the higher the jet deflection. Due to the mixing of the two streams the cross stream is completely blocked by the jet stream in the near field and then the jet stream becomes deflected under the influence of the cross stream.

The amount of penetration of the jet into the crossflow and deflection of the jet also depends upon the initial injection angle $\theta_o$. From the present predictions (Figure 3.2) we can observe that the higher the value of $\theta_o$, the more is the penetration and lesser is deflection.

### 3.3.2 Comparison of Jet Trajectory Correlation with Numerical Simulation

The predicted trajectories for the cases of injection angles greater than $90^\circ$ show that in the vicinity of the jet origin (-0.2<x/d<0.5), jet traverse against the crossflow for a distance of x/d=0.2 and beyond that distance the jet aligns with cross flow. A similar observation is also reported by Maruyama et al. (1982). However, no experimental data are available in that region for comparison with the predictions for venturi-jet mixer.

Present predictions of the jet windward trajectories for three different injection angles and $R<1.0$ are compared with the trajectories reported by Yuan and Street (1998).
Figure 3.3  Jet windward trajectories for different injection angles
(a) 45°, (b) 90°, (c) 120°: Numerical simulation, this work
correlation, Yuan and Street (1998) (R=2 and 3.3 and Re=2100)
Figure 3.3 shows that the agreement between the predicted values by simulation and present correlation and the experimental results by Yuan and Street (1998) for the jet windward trajectories in the x-direction is reasonably good. The comparison of profiles of the trajectories (Figure 3.3) shows that the predicted values are higher than the values of Yuan and Street for x/d>10 and $\theta_o \geq 90^\circ$.

### 3.3.3 Jet Trajectories Normalised with d, Rd, and R^2d

The jet trajectories for all 5 injection angles are plotted in double logarithmic scale with three different normalisations: d, Rd and R^2d as proposed by Li et al. (2006). Figure 3.4(a and b) shows simulated jet trajectory normalised with d in double logarithmic scale for jet injection angle of 45° and 135°, and compares the trajectory of Yuan and Street (1998) (d=13.44mm, R=2 and $Re_{cf}=2100$).

It appears obvious from Figure 3.4 that the jet injection angle, $\theta_o$ and cross flow Reynolds number, $Re_{cf}$ had a greater effect on jet penetration and deflection. Physically, an increase in jet injection angle, results in an increase in jet penetration. While the cross flow is stronger, the centre-line trajectory is more deflected and the jet penetration is reduced.

Figure 3.4 displays a larger jet penetration for the lower cross flow Reynolds number ($Re_{cf} = 58002$) and follow on the top branch, while the lower set of points relate to $Re_{cf} = 104403$. The numerical solutions slightly underestimate the trajectories at injection angles less than 90° as compared with the trajectory of Yuan and Street (1998). But at lower injection angle and cross flow Reynolds number, the jet trajectory is somewhat more in agreement with the trajectory of Yuan and Street (1998).
Figure 3.5(a-f) shows typical jet trajectories normalised with $R_d$ and $R^2d$ for injection angles $45^\circ \leq \theta_0 \leq 135^\circ$, in double logarithmic scales for the all values of $Re_{cf}$. As it is drawn on double scale, the power-law relationship appears as straight line.

The jet trajectory is directly dependent upon the parameters such as $\theta_0$, $R$, $Re_{cf}$, and $Re_j$ (Leong et al. 2000, Amighi et al. 2009). Often this functional dependence is best characterised by multivariate power equations.
The correlation of jet trajectory for jet in cross flow in venturi-jet mixer based on velocity ratio, crossflow Reynolds number and jet Reynolds number is also obtained by multivariate-linear regression analysis (Steven et al. 2000) as given in Equation (3.19):

\[
\frac{Z}{R_d} = \left(0.614 - 0.0047 (90^\circ - \theta_0)\right)\left(\frac{X}{R_d}\right)^{0.502} R^{0.333} Re_{cf}^{0.0187} Re_j^{0.0176} \tag{3.19}
\]

The coefficients are generated with 95% of confidence intervals. The coefficient of determination is 0.938 and the standard error is 0.37 for the jet trajectory correlation. The difference between the jet trajectory of correlation and numerical simulation is within ±10% at any location.

Figure 3.5 Jet trajectories normalised with \(R_d\), (a) \(Re_{cf} = 58002\), (b) \(Re_{cf} = 81202\), (c) \(Re_{cf} = 104403\) Jet trajectories normalised with \(R^2d\), (d) \(Re_{cf} = 69602\), (e) \(Re_{cf} = 81202\), (f) \(Re_{cf} = 92803\)
3.3.4 Jet Width Growth

The expansion of jet issuing from the venturi-jet mixer and pipeline mixer are depicted in Figure 3.6 for initial injection angle of 60°. The boundaries of the jet are identified as the radius of the jet measured perpendicular to the jet trajectory along the downstream direction from injection.

The jet lower boundary of pipeline mixer aligns with pipe wall from the injection without much deviation. In contrast, the jet lower boundary of venturi-jet mixer moves away from the wall for a distance of 15x/d and further downstream aligns with the wall.

The jet width growth is high for venturi-jet mixer for all the injection angles (not shown in Figure 3.6) as compared to pipeline mixer with uniform cross section from Forney et al. (1999).

A high jet width growth in venturi-jet mixer may be ascribed to the expansion of cross flow prior to impact. This is induced by the effect of the expansion in the downstream side and the volumetric flow rate of jet entering the nozzle on the throat. This in turn reduce the downward momentum component of the cross flow.

Clearly, the venturi-jet mixer case shows a 50% increase in jet (centre line) penetration for 60° initial injection angle as compared to pipeline mixer. It reveals that the jet expands, penetrates and mixes with cross flow in the downstream direction well for venturi-jet mixer with expansion in the downstream as compared to the pipeline mixer (simple static mixer).
Figure 3.6  Jet width growths for 60° injection angle: (a) pipeline mixer, (b) venturi-jet mixer

The jet upper and lower boundary trajectories are simulated using Equations (3.7) to (3.12) with the initial conditions for cross flow speed, jet radius, axial jet velocity and the concentration of tracer.

Figure 3.7 displays the jet radius growth for injection angles $45° \leq \theta \leq 135°$ and cross flow Reynolds number 58202 and 104403.

The jets observed here show a faster growth in the radius along the stream wise direction for higher injection angle and consistently a slower radius growth with increase in cross flow Reynolds number. It reveals that the
jet expands, penetrates and mixes with cross flow in the downstream direction. Also it is observed that much faster deflection of jet from their initial direction for higher cross flow Reynolds number $Re_{cf}$ for a particular injection angle because of relatively small momentum ratio $R$ and predominance of conservation of momentum in the cross flow direction.

Figure 3.7  Jet radius normalised with $d$ for jet injection angle $\theta_o$: 45°, 90°, 135° and $Re_{cf}$: 58202 and 104403

It is evident from the Figure 3.7 that the jet penetrates deeper with increase in jet injection angle for the same cross flow Reynolds number. At
sufficiently high cross flow speeds or low mass entrainment rates, a jet cannot only ingest significant quantities of motive fluid, but also the centreline of the jet can become distorted or bent-over in the cross flow field.

The jet bends over because of the addition of horizontal momentum by the cross flow. A typical bent-over jet does not have a constant radius of curvature along its entire path. The radius of curvature increases with height in a uniform field because the fractional increase in the horizontal component of momentum flux decreases as more of the cross flow’s momentum is taken into the jet (Bursik 2001). The velocity ratio, \( R \) of the jet to the cross flow strongly affects the penetration depth of the jet and the mixing of two flow streams.

### 3.3.5 Concentration Decay

The centreline decay of concentration is one of the parameters that could be used to evaluate large scale mixing (Li et al. 2006). Figure 3.8 shows the comparison of concentration decay from numerical and experimental results for all jet injection angles and cross flow Reynolds number.

The concentration of the mixture is normalised by the initial concentration of the tracer at the jet exit. The observation shows that larger injection angle \( \theta_o \) have higher decay at the same distance downstream. It is also observed that the numerical data is much closer to the experimental data.

The concentration decay is rapid up to \( x=15d \) for \( \theta_o \leq 90^\circ \) and \( x=20d \) for \( \theta_o > 90^\circ \). The reason for this is the jet expands quickly in the mixer due to turbulent entrainment and thus creates efficient macro mixing (Forney et al. 1999).

Also Figure 3.8 shows the behaviour of centreline dilution. At cases when the initial jet angle is \( 45^\circ \), results of numerical simulations of centreline
dilution are consistent with experimental results in both the near field and far field. However, model underestimates the data in the near and intermediate field for $\theta_o > 45^\circ$.

![Comparison of observed and predicted centreline dilution](image)

**Figure 3.8** Comparison of observed and predicted centreline dilution

(a) $\theta_o = 45^\circ$, $Re_{cf} = 58002$, (b) $\theta_o = 60^\circ$, $Re_{cf} = 69602$, (c) $\theta_o = 90^\circ$, $Re_{cf} = 81202$, (d) $\theta_o = 120^\circ$, $Re_{cf} = 92803$, (e) $\theta_o = 135^\circ$, $Re_{cf} = 104403$: (o experimental results, —— numerical results)

### 3.3.6 Correlation for Concentration Decay

Any characteristic length $l (x, z, x_m, z_m)$ and tracer concentration $c$ can be normalised with initial Sherwood number $Sh_o$, due to occurrence of mass transfer phenomenon during mixing process. The Sherwood number is a function of Schimidt number ($Sc$) and it is significant in mass transfer phenomenon. Where $x_m$ is the axial distance along the jet axis and $z_m$ is the radial direction coordinate along the jet axis. The numerical and experimental data in dimensionless form, such as $l'=(l/d_o)Sh_o^{-1}$ and $c'_m=c_{nm}Sh_o$ ($c'_m$ is the normalised tracer concentration in jet axis with respect to $c_o$ and $Sh_o$) are used further in the analysis of concentration decay.
Figure 3.9 Concentration decay (experimental) for injection angle (a) 45°, (b) 60°, (c) 90°, (d) 120°, (e) 135°
The normalised concentration with respect to initial concentration at any point \((x,z)\) is \(c_n = (c_{cf} - c)/(c_{cf} - c_o)\). Where, \(c\) = tracer concentration at any location, \(c_{cf}\) = concentration of tracer in cross flow, and \(c_o\) = initial concentration of tracer in jet. For cross flow with zero tracer concentration, the normalised concentration with respect to initial concentration at any point is written as, \(c_n = c/c_o\). Along the jet axis, the equation for normalised concentration can be rewritten as \(c_{nm} = c_m/c_o\), and, while the concentration on jet trajectory (axis, \(x_m, z_m\)) is \(c_m\).

The trajectory (axial points \((x_m', z_m')\)) are determined by normalised local \(c'\) maxima. Figure 3.9 (a-e), show experimental concentration decay (axes) in terms of \((x_m', c_m')\), for \(\theta_o = 45^\circ, 60^\circ, 90^\circ, 120^\circ\) and \(135^\circ\), in double logarithmic scales for all 5 values of \(Sh_o\). From these figures it is concluded that the concentration decay have the form,

\[
y_m' = A (x_m')^B
\]

(3.20)

where \(A\) and \(B\) are determined from the measurements and numerical calculations, for any angle \(\theta_o\). For dimensionless concentrations \(c_m'\), in the ranges \(45^\circ < \theta_o < 135^\circ\) and \(265 < Sh_o < 460\), equation of the form \(c_m' = A(x_m')^{-1}\), was determined from the measurements and numerical calculations, where the arithmetic coefficient \(A\) has particular value depending on \(\theta_o\) (in degrees).

It is observed that coefficient \(A\) was found to vary from 0.4255 to 0.7675. The arithmetic \(A\) values were put in an auxiliary diagram against 5 injection angles \(\theta_o\) is shown in Figure 3.10 and the simple expression \(A = 4.6083\theta^{-0.4764}\) was determined. Thus, for \(45^\circ < \theta_o < 135^\circ\), the final equation for concentration on jets’ axes is

\[
c_m' = 4.6083\theta^{-0.4764} (x_m')^{-1}
\]

(3.21)
The decay of concentration obtained by experimental results is shown in Figure 3.9 for different injection angles and Sherwood number. For the present study the Sherwood number may be used to study the advection-diffusion mass transfer characteristics for venturi-jet mixer.

Figure 3.10 A values for $45^\circ < \theta_o < 135^\circ$ and $265^\circ < Sh_o < 460$

The rate of concentration decay in the downstream direction increases with increase of injection angle. This is because with jet penetration increases and the jet spread with increasing downstream distance. The effect of the aligned injection against the crossflow is to decrease jet penetration and increase spreading.

Each downstream distribution is very similar to the small injection angle case indicating that the larger injection angle case is primarily responsible for the increase in penetration and increase in spreading (observed in the distributions for $135^\circ$ injection angle).

Penetration is increased compared to transverse slot but mixing appears similar after a slow start due to the extra length of this alignment. Figure 3.11 compares the observed concentration decay with the calculated concentration decay based on Equations (3.1) to (3.12) and present correlation.
for injection angles $45^\circ$, $90^\circ$ and $135^\circ$. A good agreement between them under the jet condition $R \leq 1$ is obtained.

![Graph showing concentration decay trend line comparison for injection angles](image)

**Figure 3.11 Comparison of concentration decay trend line: Injection angle (a) $45^\circ$, (b) $90^\circ$, (c) $135^\circ$ (--- : experimental values, ----- : numerical values and o: correlation results)**

Figure 3.11 may be used to investigate the prediction ability of present correlations with respect to numerical and experimental data. From these plots it is clear that all configurations have similar mixing pattern with downstream distance, which is expected since the mixture uniformity increases with increase in jet radius along the flow direction.

### 3.3.7 Predicted Maximum Jet-velocity Decay

In Figure 3.12 the non-dimensional jet-velocity profile plots for the 5 injection angles are shown as a function of non-dimensionalised downstream distance for all the cross flow Reynolds number. The plots show that the normalised jet centreline velocity increase at rapid rate from injection point ($s=0$) to $s=5d$ and uniform thereafter for all the cases of downstream distance ($s>5d$). Due to the mixing of the two streams with momentum ratio $R<1$, the local maximum jet mean velocity ($u$) approaches the cross flow velocity in the downstream. The lower cross flow Reynolds number, lower the velocity magnitude of jets in cross flow. There is little effect on the jet-velocity profile for injection angles $45^\circ$-$135^\circ$ with the variation of cross flow Reynolds number.
Figure 3.12  Predicted maximum jet-velocity decay along trajectory for jet injection angle $\theta_o: 45^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, $135^\circ$
3.3.8 Velocity Ratio and Entrainment Ratio

The velocity ratio, $R$ and the entrainment ratio, $m_j/m_{cf}$ for all the experimental cases of venturi-jet mixer for runs with different cross flow Reynolds number, $Re_{cf}$ are shown in Figures 3.13 - 3.15. Figure 3.13 indicate the lower the cross flow velocity $v$ and Reynolds number $Re_{cf}$, more the jet entrains into the mixer and higher the velocity ratio $R$ for $45^\circ \leq \theta \leq 135^\circ$.

At cross flow velocity $v=4.3844$ m/s, a higher vacuum pressure prevails in the throat which sucks more suction fluid (jet) into the mixer for all the cases investigated. As cross flow velocity $v$ increases, the vacuum pressure decreases at all the injection angles which results in increase of suction effect.

On the contrary, velocity ratio and jet entrainment rate are decreasing as the cross flow Reynolds number increases. The entrainment rate of jet is influenced by the cross-section of the inlets of venturi and jet, cross flow velocity, level of tracer liquid from centre of venturi. From Figure 3.15 it is concluded that the mass entrainment with velocity ratio can be described by a linear fit.

$$\frac{m_j}{m_{cf}} = 0.008265 \left(\frac{u}{v}\right)$$  \hspace{1cm} (3.22)

Equation (3.22) shows that the velocity ratio has an effect on mass entrainment ratio because the mass entrainment is proportional to the velocity ratio. The pre-factor and exponent in the correlation could be functions of other parameters, such as cross flow and jet properties, the dimensions of the venturi-jet mixer and the angle of initial injection of jet.
Figure 3.13 Velocity ratio ($R$) variation (Notations same as Figure 3.14)

Figure 3.14 Entrainment ratios for all cases of mixer system

Figure 3.15 Linear fit for velocity ratio, $u/v$ Vs entrainment ratio, $m_f/m_{cf}$
(Notations same as Figure 3.14)
3.3.9 Mixer Pressure Drop

Comparison of experimental results of pressure drop of all the cases is a vital part of determination of performance characteristics of the mixer examined. The pressure drop caused by a mixer flow configuration and jet injection can be expressed by a dimensionless ratio, Z-factor and also by pressure drop normalised with maximum pressure drop in the mixer during a set of runs for a particular injection angle. The Z factor is the ratio of pressure drop across the venturi-jet mixer ($\Delta p_{VM}$) to the pressure drop in the empty pipe ($\Delta p_{EP}$), which indicates increase in energy costs when a static mixer is used in a continuous flow process (Zalc et al. 2002). Therefore Z-factor can be written as given in Equation (3.23):

$$Z = \frac{\Delta p_{VM}}{\Delta p_{EP}}$$

(3.23)

The Z-factor is examined in Figure 3.16 as a function of the cross flow Reynolds number, indicating that it increases slowly for 45° injection angle. In contrast, for other injection angles up to a cross flow Reynolds of about 82000, the Z-factor drops sharply and beyond which it remains virtually constant at about 10.

Figure 3.16 Variation of Z-factor (Notations same as Figure 3.18)
Many researchers report that Z-factors for the static mixers can range from 5 to 40, and our experimental results are within the specified range (Paul and Muschelknautz 1982 and Alloca 1982). Figure 3.17 represents the experimental pressure drop, $\Delta p$ normalised with maximum pressure drop, $\Delta p_{\text{max}}$ in the same set of experiments in the mixer. The experimental results for the normalised pressure drop across the mixer for all the initial injection angles, $\theta_o$ are compared for the studied venturi-jet mixer systems and plotted against cross flow Reynolds number, $Re_{cf}$.

![Figure 3.17 Normalised experimental pressure drop across the mixer](image)

Figure 3.17 and 3.18 reveals that the maximum pressure drop occurs at higher value of cross flow Reynolds number due to the increase in inertial effects for all the cases of initial injection angles. For a particular injection angle, the pressure drop increases as the cross flow Reynolds number increases due to the appearance of counter rotating vortex pair at the diffuser outlet.

The experimental result also reveals that the higher the value of cross flow Reynolds number $Re_{cf}$ and initial injection angle $\theta_o$ except 60°, the higher the pressure drop across the mixer. Thus, the initial injection angle and
cross flow Reynolds number appear to have a significant effect on the mixer pressure drop.

![Graph showing mixer pressure drop vs cross flow Reynolds number for inlet injection angles, 45° ≤ θ ≤ 135°](image)

**Figure 3.18** Mixer pressure drop Vs cross flow Reynolds number for inlet injection angles, 45° ≤ θ ≤ 135°

### 3.3.10 Measures of Mixing

The results from the experimental campaign provided data concerning mixing intensity and pressure drop in venturi-jet mixer. The concentration decay of jet was characterized and adopted as a reference to assess the effect of the jet injection angle on mixing behaviour. The main variables derived from the measurements to analyse the mixer performance were mixing index, spatial unmixedness and mixing length.

#### 3.3.10.1 Mixing index

To characterise the mixing performance by considering effects of arbitrary injection angle, pressure drop in the mixer and increasing inertia on flow and mixing of venturi-jet mixer, the mixing index can be used quantitatively (Wang et al. 2001).
The mixing efficiency can be calculated by Equation (3.24) as proposed by Jeon et al. (2000).

\[ m_{\text{eff}} = \left[ 1 - \frac{\int_{0}^{W} |c - c_{\text{avg}}| \, dx}{\int_{0}^{W} |c_{0} - c_{\text{avg}}| \, dx} \right] \times 100\% \]  

(3.24)

The pressure drop is a crucial factor to the design of a venturi-jet mixer device; therefore, the overall performance of venturi-jet mixer should include the evaluation of pressure drop. This overall mixing performance is termed mixing index, \( m_{\text{dx}} \), and used to evaluate the overall performance of a mixer. The mixing index is defined in Equation (3.25).

\[ m_{\text{dx}} = m_{\text{eff}} \times \frac{\Delta p_{\text{max}}}{\Delta p} \]  

(3.25)

The experimental results of mixing index of the venturi-jet mixer are presented in Figure 3.19 for all the cases of the present work. It is apparent that only a minor difference in the mixing index exists between the tested cross flows Reynolds number for a given injection angle.

The results (Figure 3.19) show that mixing index is greatest for the initial injection angle, \( \theta_{o} = 120^\circ \) and lowest for the case of \( 60^\circ \) injection. The increased mixing can be interpreted as the high injection angle causing the penetration of jet deeper into the cross flow and given more time for diffusion at the interface of the two liquids.

However, the improvement of mixing performance is not proportional to the increased pressure drop. The criterion for designing a passive mixer should comprise the capability of a pump to overcome pressure drop with an adequate mixing efficiency.
Therefore, the mixing efficiency has to compromise the pressure drop to optimize the design of a mixer (Paul and Muschelknautz 1982). The mixer with injection angle $\theta \geq 90^\circ$ has approximately the same pressure drop, and more mixing. These results show that the optimized mixer would be one with injection angle $\theta \geq 90^\circ$.

![Mixing index, $m_{idx}$ versus cross flow Reynolds number, $Re_{cf}$](image)

### 3.3.10.2 Spatial unmixedness

In a two-stream mixing problem the fully mixed concentration is defined by the jet-to-mainstream mass-flow ratio. A measure of the mixing rate can be obtained by comparing the jet mixture fraction distribution at any downstream plane to the fully mixed value. In a two-stream mixing problem the fully mixed concentration is defined by the jet-to-mainstream mass-flow ratio. In order to quantify mixing rates, Liscinsky et al. (1993) developed a measure of unmixedness based on the variance of the concentration distribution, defined as spatial unmixedness:

$$ U_s = \frac{c_{var}}{c_{avg} (1 - c_{avg})} \quad (3.26) $$
where \( c_{\text{var}} = \frac{1}{m} \sum_{i=1}^{m} ( \bar{c}_i - c_{\text{avg}} )^2 \)

Here \( c_{\text{var}} \) is the spatial concentration variance, \( \bar{c}_i \) is the time-average spatial concentration and \( c_{\text{avg}} \) is the fully mixed concentration. \( U_s = 0 \) corresponds to a perfectly mixed system, and \( U_s = 1 \) a perfectly segregated system. The denominator of Equation (3.26) denotes the maximum concentration fluctuation that can occur at the specified fully mixed concentration.

Normalizing \( c_{\text{var}} \) by the denominator allows \( U_s \) values to be compared regardless of the jet to mainstream mass-flow ratio of the system (Liscinsky et al. 1993). Therefore, this parameter allows comparison of the relative mixing effectiveness of each configuration reported in this study for different jet-to-mainstream momentum ratios and jet inlet angles.

The experimental results of spatial unmixedness (\( U_s \)) in the venturi-jet mixer were calculated and plotted in Figure 3.20(a-f) for jet inlet angles 45\(^\circ\), 60\(^\circ\), 120\(^\circ\), 135\(^\circ\) and for Reynolds number value of 31917 and 57451. The experimental results show that spatial unmixedness decreases along the downstream direction and mixing is rapid up to a distance of \( x/d = 10 \) for all the jet inlet angles.

The initial rapid mixing rate is attributed to the development of the pair of counter-rotating vortices which in turn causes shear between the jet and mainstream leading to rapid local entrainment (Vranos et al. 1991). The increased mixing in the downstream may be because of the conversion of parabolic velocity distribution to a more even uniform distribution flow in the divergent portion and thereby allowing better mixing of fluids.
Also, the increased surface contact area of the two streams allowed for greater viscous interaction and thus contributed to complete mixing. There is little effect on the mixing rate in the near field for Reynolds number due to entrainment effects.

The observation that the effect of Reynolds number on the mixing changed with the area ratio was not expected. The change is within the uncertainty in the measurements; however, the trend is graphically quite consistent despite the numerical uncertainty. Figure 3.20 (c & d) show inlet jet angles above 90° significantly increase the mixing rate at x/d > 5 for all the Reynolds number used in the present study.

3.3.10.3 Mixing (flame) length

Another global measure to quantify the quality of mixing is the flame length. The flame length represents the distance for the jet to become colourless when it entrains and mixes with a certain mass of cross flow.
(Broadwell and Breidenthal 1984). The flame length is an important quantity in several existing models of mixing using passive mixer.

For the present study, flame length $x_f$ is considered as the axial length of jet for 95% mixing. It can be easily obtained from the measurement of tracer concentration decay. It is well known that, the shorter the flame length is, the shorter the mixing tube length can be. This will be beneficial to the size and weight of the mixing systems.

![Figure 3.21](image)

**Figure 3.21 Normalised downstream flame length** (— : Broadwell and Breidenthal, €: 45°, ș: 60°, □: 90°, Δ: 120°, ⦿: 135°)

Broadwell and Breidenthal (1984) studied the structure and mixing of a transverse jet in incompressible flow using a simple mixing model. They studied the effect of velocity ratio $0<1/R<0.6$ and equivalence ratio $\phi$, $1.1<\phi<11$ on flame length of the jet. The equivalence ratio $\phi$, can be defined as the mass of cross flow required to completely mix from a unit mass of jet fluid.
The correlation for the flame length with equivalence ratio and velocity ratio is written as (Broadwell and Breidenthal 1984),

$$x_f = 4.76 \left( \frac{v}{u_o} \right)^{1/2} \left( \phi + 1 \right)^{3/2} A_o^{1/2}$$

(3.27)

A typical normalised flame length of the venturi-jet mixer with arbitrary injection angle is compared with the simple model describing the mixing of a transverse jet in incompressible flow shown in Figure 3.21 (Broadwell and Breidenthal 1984). From Figure 3.21, it can be seen that: the normalised flame length of the venturi-jet mixer is about $6.8116 - 4.2494$, which is slightly high for $45^\circ$ initial injection and then reduces as the injection angle is increased in step indicating very rapid mixing at $\theta_o > 90^\circ$. 