The foundations of Fuzzy Mathematics were laid when Lotfi A. Zadeh, Professor of Electrical Engineering and Computer Science at Berkeley, USA, introduced the concept of fuzzy set in the year 1965, which has turned out to be a useful analytical device to analyze fruitfully those real world situations which are characterized by imprecision, vagueness and uncertainty. Since then many authors have expansively developed the theory of fuzzy sets and its applications to various branches of Mathematics such as algebra, topology, graph theory, probability theory, logic, linear programming and etc. The notion of fuzzy metric spaces was introduced in various ways by several authors such as I. Kramosil and J. Michalek (1975), M. A. Erceg (1979), Zi-ke Deng (1982), O. Kaleva and S. Seikkala (1984), N. H. Hsu (1992). In particular, I. Kramosil and J. Michalek introduced the notion of fuzzy metric spaces in the year 1975 by generalizing the concept of probabilistic metric spaces introduced by K. Menger to the fuzzy settings. In 1979, M. A. Erceg introduced a definition of fuzzy metric spaces using the concept of lattices. Zi-ke Deng introduced
fuzzy pseudo-metric spaces with the metric defined between two fuzzy points and developed its properties in the year 1982. In 1984, O. Kaleva and S. Seikkala introduced the notion of fuzzy metric spaces by setting the distance between two points to be a non-negative, upper semi continuous, normal and convex fuzzy number. A. George and P. Veeramani in 1994, further modified the notion of fuzzy metric spaces introduced by I. Kramosil and J. Michalek with the help of a continuous $t$-norm and obtained a Hausdorff topology for this kind of fuzzy metric spaces and proved that every metric $d$ on a non-empty set $X$ induces a fuzzy metric $M_d$ (the standard fuzzy metric) on $X$. Also it was proved by V. Gregori and S. Romaguera in the year 2000 that the topology induced by fuzzy metric in George and Veeramani's sense is actually metrizable. Many authors think that George and Veeramani's definition is an appropriate notion of metric fuzziness in the sense that it provides rich topological structures which can be obtained mainly from the classical theorems.

The concept of fuzzy norm on a linear space was introduced by several authors in various ways. In particular, A. K. Katsaras, while studying fuzzy topological vector spaces, introduced the notion of fuzzy norm in the year 1984. In fact, he was the first to introduce the notion of fuzzy normed linear spaces. In 1992, C. Felbin introduced the idea of a fuzzy norm on a
linear space by assigning a fuzzy real number to each element of a linear space so that the corresponding fuzzy metric is of Kaleva and Seikkala type. In 1994, S. C. Cheng and J. N. Mordeson introduced another idea of a fuzzy norm on a linear space in such a manner that the corresponding fuzzy metric is of Kramosil and Michalek type. In 2003, T. Bag and S. K. Samanta introduced the concept of a fuzzy norm in a slightly different way and proved some interesting fixed point theorems along with some other results in some of their papers. Further, the authors introduced $\alpha$-norms on a linear space corresponding to the fuzzy norm and proved the Decomposition Theorem which states: 'Let $(U, N)$ be a fuzzy normed linear space. Assume that

$$(N6) \forall t > 0, N(x, t) > 0 \Rightarrow x = 0.$$ 

Define $||x||_\alpha = \Lambda\{t: N(x, t) \geq \alpha\}, \alpha \in (0,1)$. Then $\{||x||_\alpha: \alpha \in (0,1)\}$ is an ascending family of norms on $U$ and they are called $\alpha$-norms on $U$ corresponding to the fuzzy norm on $U$.' The same authors further considered general $t$-norm in place of 'min' $t$-norm and observed that if $t$-norm is chosen other than 'min' then the Decomposition Theorem of a fuzzy norm into a family of crisp norm may not hold.

A self map $T$ of a non-empty set $X$ (i.e., $T: X \to X$) is said to have a fixed point $u \in X$ if $Tu = u$. For example, any continuous self map $f$ of the
closed interval \([-1, 1]\) has a fixed point. This may be easily verified by applying the Intermediate value Theorem to the function \(g(x) = x - f(x)\).

On the other hand, the continuous self map \(f(x) = x + 1\) of \(\mathbb{R}\) has no fixed points. A fixed point theorem is a statement containing a set of conditions sufficient to ensure a fixed point of a self map in a metric space. Most of the common fixed point theorems involving a number of self maps mainly contain a commutativity condition, a condition on the ranges of the maps, some continuity conditions and a contractive condition, possibly a Lipschitz type or a Banach type or a Boyd and Wong type condition. Most of the authors used one or more of these conditions or their various generalized forms and obtained remarkable successes. In recent years, the study of fixed point theorems and common fixed point theorems satisfying some contractive-type conditions have been at the centre-stage of some intense research activity and a large number of research papers devoted to the development of the fixed point theorems and their applications have appeared in the literature.

Historically, the first theorem of this kind was established by L. E. J. Brouwer in 1912. It states that 'Every continuous map of a closed ball of \(\mathbb{R}^n\) to itself has a fixed point.' This result was not of much importance because of the finite dimension of the concerned space. This was extended to infinite dimensional space by G. D. Berkhoff and O. D. Kellogg in 1922.
proving that any compact, convex set $S$ in the spaces $L^2[0,1]$ and $C[a,b]$ has fixed point property for continuous functions. In 1930, J. Schauder, a Polish Mathematician, extended Brouwer's fixed point theorem to the case where $S$ is a compact, convex subset of a normed linear space. It states that: 'Let $C$ be a non-empty compact, convex subset of a normed linear space $X$. If $F$ is a continuous map from $C$ to $C$, then $F$ has a fixed point.'

In the year 1922, S. Banach established a fixed point theorem known as the **Banach Fixed point Theorem** or **The Banach Contraction Principle** which states that: 'If $T$ is a self map of complete metric space $(X, d)$ satisfying $d(Tx, Ty) \leq kd(x, y)$, for every $x, y \in X$ and some $0 < k < 1$, then $T$ has a unique fixed point.' Such a map $T$, called contraction, is always continuous. In 1961, M. Edelstein extended Banach Contraction Theorem which he called extended Contraction Principle and it states that: 'If $F$ is an $(\varepsilon, \lambda)$ uniformly locally contractive self map of a complete $\varepsilon$-chainable metric space, then $F$ has a unique fixed point $u \in X$ and $u = \lim_{n \to \infty} F^n x$, for every $x \in X$.' Further in 1962, he proved a fixed point theorem on convergence to a fixed point which states that: 'If $F$ is a contractive self map of a metric space $(X, d)$ and $x_0 \in X$ is such that $\{F^n x_0\}$ has a convergent subsequence converging to $u \in X$, then $u$ is the unique fixed point of $F$.' R. Kannan, an Indian Mathematician, proved a
fixed point theorem for a self map which is neither continuous nor contractive and thereby removing the continuity requirement from a fixed point theorem. He used in his proof an iterative method but his condition was entirely different. Renato Caccioppoli of Italy generalized the Banach Contraction Theorem with the help of a convergent series of positive real numbers and deduced it as a Corollary. In this connection further works of D. W. Boyd and J. S. W. Wong (1969), V. M. Sehgal (1969), L. B. Ciric (1971), J. Caristi (1976), P. P. Murthy (1993) are also of significant importance.

G. Jungck introduced the notion of compatibility in the year 1986 by generalizing the concept of weakly commuting mappings introduced by S. Sessa in 1982 and established some important common fixed point theorems in a series of his papers. R. P. Pant introduced the notion of $R$-weak commutativity by generalizing the notion of commutativity in the year 1994. In 1996, G. Jungck again generalized the notion of compatibility by introducing weak compatibility. The concept of non-compatibility is further weakened when M. Aamri and D. El. Moutawakil introduced a property known as $E.A.$ property in 2002.

S. Gähler investigated 2-metric spaces in 1963 and 2-normed linear spaces in 1964 in a series of his papers, which opened up a new direction
for the development of fixed point theory in these spaces. It may be recalled that a 2-metric $d$ on a non-empty set $X$ is a function $d: X^3 \to \mathbb{R}$ satisfying some conditions that are analogous to the area function in Euclidean spaces. More explicitly: Let $X$ be a non-empty set. A mapping $d: X^3 \to \mathbb{R}$ is said to be a 2-metric and the pair $(X, d)$, a 2-metric space, if

(i) Given distinct elements $x, y \in X$ there exists an element $z \in X$ such that $d(x, y, z) \neq 0$,

(ii) $d(x, y, z) = 0$, when at least two of $x, y, z \in X$ are equal,

(iii) $d(x, y, z) = d(x, z, y) = d(y, z, x)$, for all $x, y, z \in X$,

(iv) $d(x, y, z) \leq d(x, y, u) + d(x, u, z) + d(u, y, z)$, for all $x, y, z, u \in X$.

P. L. Sharma, B. K. Sharma and K. Iseki investigated, for the first time, the contractive-type mappings in 2-metric spaces in the year 1976. This motivated the idea of studying 2-metric spaces and 2-normed linear spaces in the fuzzy structures.

Recently, researchers are seemed to be interested in extending and generalizing the various results of the classical fixed point theory to the fuzzy metric spaces, fuzzy 2-metric spaces and fuzzy normed linear spaces. In 1988, M. Grabiec extended the two fixed point theorems of S. Banach and M. Edelstein to contractive mappings of complete and compact fuzzy metric spaces respectively. The works of Y. J. Cho, S. H.
Cho and J. H. Jung, A. George and P. Veeramani and etc. along this line are also significant. S. Sharma, in 2002, extended the results of B. Fisher involving three mappings to fuzzy metric spaces and fuzzy 2-metric spaces. J. Han proved a common fixed point theorem in fuzzy 2-metric spaces by extending the results of S. H. Cho which are generalizations of the results due to S. Sharma. The works of K. S. Dersanambika and Aswathy M. R. in fuzzy 2-metric spaces are also interesting.

In 1999, R. Vasuki has extended the common fixed point theorem of \(R\)-weakly commuting maps due to R. P. Pant to fuzzy metric spaces. In 2000, B. Singh and M. S. Chouhan extended the concept of compatibility and semi-compatibility to fuzzy metric spaces and proved some interesting common fixed point theorems in fuzzy metric spaces in the sense of A. George and P. Veeramani. S. H. Cho and J. H. Jung, in the year 2006 established some common fixed point theorems for four weakly compatible maps of an \(\epsilon\)-chainable fuzzy metric space. U. Mishra, A. S. Ranadive and D. Gopal, in the year 2008, used reciprocal continuity while proving some common fixed point theorems in fuzzy metric spaces and showed that the notion of reciprocal continuity can widen the scope of many interesting common fixed point theorems in fuzzy metric spaces.
Applications of fuzzy set theory have touched almost every fields of human endeavour. Fixed point theorems are the most important tools for providing the existence and uniqueness of the solutions of various Mathematical models (differential, integral, partial differential equations and variational inequalities) representing the phenomena suitable for different fields such as steady state temperature distribution, chemical reactions, neutron transport theory, economic theories, epidemics and flow of fluids. Fixed point theorems and common fixed point theorems have also been found useful in the fields of differential equations, algebra, economics, game theory, dynamics, optimal control theory and broadly a major theoretical tool of functional analysis. The iterative techniques of fixed point theory can be applied for solving optimization problems. At present, this field has been recognized as one of the most active fields of research.

On being highly motivated by the works of M. Grabiec, G. Jungck, R. P. Pant, S. Sharma, T. Bag and S. K. Samanta, we have taken up the works of establishing some fixed point theorems as well as common fixed point theorems in fuzzy metric spaces, fuzzy 2 - metric spaces and fuzzy normed linear spaces. We have extended the works of M. Grabiec to fuzzy 2 - metric spaces. We have extended the fixed point theorems of R. Caccioppoli and M. Edelstein to fuzzy metric spaces as well as fuzzy
2 - metric spaces. In the process, we have introduced the notions of \( \varepsilon \) - chain and \((\varepsilon, \lambda)\) uniformly locally contractive mappings in fuzzy 2 - metric spaces. We also have introduced the notions of sequentially convergent mappings and subsequentially convergent mappings in fuzzy metric spaces and established some fixed point theorems using these concepts together with extensions of V. M. Sehgal, D. W. Boyd and J. S. W. Wongs' fixed point theorems to fuzzy metric spaces. Further we have studied common fixed point theorems in fuzzy metric spaces by extending the common fixed point theorems due to P. P. Murthy, G. Jungck of classical metric spaces to fuzzy metric spaces using the concepts of compatibility, reciprocal continuity, \( E.A. \) Property. We have proved a Theorem which establishes the existence of a unique common fixed point of an infinite family of self maps when all the maps may be non-commuting, may be discontinuous and even many may not satisfy the compatibility conditions. Towards the end, we have established some fixed point theorems in fuzzy normed linear spaces. We have incorporated sufficient examples to illustrate our results and deduce Corollaries.

The thesis 'The Fixed Point Theory in Fuzzy Metric and Normed Linear Spaces' is furnished with nine Chapters.
The first Chapter of the thesis is a collection of some basic concepts and known results without proofs which are collected from various sources on fuzzy sets, fuzzy metric spaces, fuzzy 2-metric spaces and fuzzy normed linear spaces.

Chapter two briefly sketches the relevant parts of the historical developments of contraction mapping principle as well as fixed point theorems in metric spaces. Here we review the fixed point theory in metric spaces, 2-metric spaces and the important lemmas, theorems and examples which are useful in our works.

Chapter three is furnished with the establishment of Banach and Edelstein's fixed point theorems of complete and compact fuzzy 2-metric spaces respectively. Here we also prove two lemmas.

In Chapter four, we establish Caccioppoli Contraction Theorem and Edelstein Contraction Theorem with the help of an \((\varepsilon, \lambda)\) uniformly locally contractive mapping in a complete \(\varepsilon\)-chainable fuzzy metric space. We also deduce the fuzzy Banach Contraction Theorem as a Corollary and illustrate our results with an example.

In Chapter five, we introduce the notions of \(\varepsilon\)-chain and \((\varepsilon, \lambda)\) uniformly locally contractive mapping in fuzzy 2-metric spaces and establish Caccioppoli’s Contraction Theorem and Edelstein’s Contraction Theorem.
in fuzzy 2 - metric space. We also construct one example here to illustrate our results.

In Chapter six, we introduce the notions of sequentially convergent mappings and subsequentially convergent mappings in a fuzzy metric space defined in a slightly different way and show that the notions of continuity and sequential convergence are independent concepts and generalize the fuzzy Banach Contraction Theorem with these concepts. Here we also prove the fuzzy analogues of Boyd and Wong's Theorem and V. M. Sehgal's Theorem. We deduce Corollaries and illustrate each of our results with suitable examples.

In Chapter seven, we use the concepts of compatibility, semi - compatibility and reciprocal continuity to establish the fuzzy analogues of the common fixed point Theorems due to P. P. Murthy and G. Jungck and deduce Corollaries.

In Chapter eight, our works contain the establishment of a common fixed point Theorem of an infinite family of maps when all the maps may be non - commuting, may be discontinuous and even many may not satisfy the compatibility conditions. Here we also establish a common fixed point Theorem involving four mappings. We deduce Corollaries and illustrate our results with suitable examples.
In Chapter nine, our last Chapter, we study fixed point Theorems and common fixed point Theorems in fuzzy normed linear spaces defined in a slightly different way. Our results include extensions of the Contraction Theorems mainly due to S. Banach, R. Caccioppoli, R. Kannan of classical metric spaces to fuzzy normed linear spaces. We further prove a common fixed point Theorem in fuzzy normed linear spaces which generalize the results of R. Vasuki in fuzzy metric spaces and the fuzzy analogue of a fixed point Theorem due to G. Jungck. We further add a General Conclusion towards the end of the Chapter.