1.1 Preliminaries

Manpower planning is an attempt to match the supply of people with the jobs available for them. It can also be viewed as a process by which management determines how the organization should move from its current manpower structure to a desired one. Since it depends on the highly unpredictable behaviour of human beings and also the unpredictable environment, stochastic models of manpower planning have been proposed. These models incorporate several factors such as recruitment, training, promotion, demotion and wastage. These factors are interlinked and this analysis becomes highly essential due to the increasing costs related to recruitment, training, promotion, demotion etc., Hence proper manpower planning strives to have the right number of persons, the right kind of persons, at the right places and at the right time.

People, job, time and money are the basic ingredients of the manpower system. A decision maker must be aware of the interaction among the four ingredients in order to formulate and evaluate a suitable manpower policy. As already pointed out, manpower planning within an organization is for the purpose of producing the right number of persons of right type in the right jobs at the appropriate time with
optimum cost. Hence the role of mathematical modeling becomes inevitable in order to take a right decision at the right time.

Some basic concepts and definitions are given below which are used in the construction of mathematical models of recruitment policies.

1.1.1 Wastages

It refers to an act of the individual leaving the system over which an organization cannot have any control. The concept of wastage plays a vital role due to the following.

1. If the total number of persons (jobs) is fixed, wastages create vacancies and so promotion and recruitment opportunities arise.

2. Man power planning can be successful if and only if it is possible to predict the pattern of wastage in an organization.

3. If the pattern of wastage is evaluated, then the necessary number of promotions and recruitments can be decided.

4. Through the study of wastage, one can decide the health rate of the organization, the level of absenteeism and the industrial dispute.

5. On the whole, one can decide about the organization stability based on wastage rate.
The wastage can be voluntary or involuntary. Involuntary wastage comprises of the loss of persons from an organization due to death, ill-health, retirement and redundancy. Also, a person can withdraw from the existing job and can go in for a new job, one can get voluntary retirement through VRS and can go in for some other job. Hence the wastage patterns depends upon a number of factors such as personal and environmental factors.

1.1.2 Recruitment

The concept of recruitment also plays an important role in manpower planning. Recruitment is a part of flow. The number of recruits in each category is represented as a recruitment vector. In dealing with the problems of maintaining a given grade structure of an organization, recruitment control plays an important role and it is done by the proper choice of the recruitment vector \( r \) and \( p \), where \( p \) is the transition probability matrix describing the transitions of individuals between the grades of that organization.

1.1.3 Manpower System Costs

It may be observed that there is a very close similarity between the manpower system and the inventory system according to Grinold and Marshall (1977). The relevant costs in a manpower system consists of recruitment costs, overstaffing costs, understaffing costs, firing/retirement costs and retentions which are described below.
Recruitment Costs

This is the cost incurred in the process of recruitment. Recruitment costs can be broadly classified into two categories, viz. fixed and variable, which are proportional to the number of people recruited. The following are different components of recruitment costs, according to Poornachandra Rao (1993),

a. Cost of advertising.
b. Cost of administrating authority which determines recruitment policy.
c. Cost of manpower working on the processing of applications.
d. Cost of information processing.
e. Cost of conducting written tests.
f. Costs incurred in the form of payment to the interview committee members (if hired from outside) or the wages of the people on the interview committee (if the members are internal).
g. Traveling expenses paid to the candidate.
h. Cost of medical examination done by the organization.
i. Cost of training people.
j. Miscellaneous expenditure, including postage, telephone calls, etc.

While the above components of cost are only indicative, the actual components of recruitment cost depend upon the recruitment procedure followed by the organization. Often, the organization charges the applicants in the form of an application processing fee, but it has
been found that in many organizations the revenue generated from this fee is disproportionate to the actual recruitment costs. Of the various components of cost identified above, the cost of advertising and the cost of administrative authority form a fixed component, which is independent of the number of people recruited. Some costs, like those of interviewing people and traveling expenses paid to the candidate depend upon various factors such as the appearance of the candidates, the suitability of the candidates, etc. Moreover, these costs also depend upon the policy of the organization in determining the number of candidates to be interviewed, etc. If the management’s policy is to call people in a certain predetermined ratio, these costs will be proportional to the number of candidates called for interview and hence will be a constant. The cost of conducting a written test, the cost of manpower working on the processing of applications, information processing costs, the cost of medical examination and the cost of training people will have a certain fixed component and a variable component per recruit. In a typical military recruitment process where the selection process is in groups, the fixed costs will be higher.

**Overstaffing Costs**

Overstaffing costs are those incurred owing to an unutilized work force. These costs are analogous to the inventory costs in a production/inventory situation and are similar to holding cost.
**Understaffing Costs**

Understaffing costs are those resulting from decreased productivity and loss of good will (in a profit motive organization) as a result of the non-availability of the work force. This is similar to that of penalty/shortage cost in inventory problems.

**Firing/Retirement Costs**

These costs result from retrenchment or retirement of an employee.

**Retention costs**

In addition to the various costs identified above, there are certain costs which are involved in retraining an employee in an organization. We term these costs and they consist of (1) probation costs (2) training and development costs, (3) internal mobility costs.

Probation costs are those incurred owing to the learning effect of an employee during a probationary period. The training and development costs indicated here are different from the one identified as component of recruitment costs, and are incurred owing to the development programs which an employee undergoes during the course of his service to the organization. Internal mobility costs are the costs involved in the promotion, demotion or transfer of an employee within the organization.
1.1.4 Shock Models And Cumulative Damage Process

The shock models deal with the life distribution of devices or components which are subjected to shocks. For example, the devices like an electric bulb, television picture tube etc., which are subjected to random fluctuations in the voltage of electricity. A sequence of shocks occur randomly in time and the instantaneous damage occurring at the random epochs cumulate to an unknown threshold value beyond which the system fails. The threshold level is by itself a random variable. At every shock, a random amount of damage is caused to the device and the damages in successive shocks get added together in the form of a cumulative damage. The rate at which the threshold is approached is also studied. There are various approaches to the rate of accumulation of the damages, but they all appear to be resolvable in terms of a stochastic process. If the damages caused by successive shocks, are independent and identically distributed random variables denoted as $X_i, i = 1,2,3,\ldots,n$ with common distribution function $F(.)$, and the random threshold level $Y$ is a random variable with distribution function $G(.)$ then the probability that the device survives $k$ damages is denoted by $P_k(x)$ which is given by

$$P_k(x) = \int_0^x F_k(x)[1 - G(x)]dx, \quad k = 1,2,3\ldots$$

where $F_k(x)$ is the $k$-fold convolution $F(x)$ with itself and $F_0(x) = 1$.

The reliability $R(t)$ of the device is given by
\[ R(t) = \sum_{k=0}^{\infty} P_k(t)V_k(t) \]

where \( V_k(t) \) is the probability that \( k \) damages are caused during \([0,t]\). The above model has been considered by Essary, Marshall and Proschan (1973) with the underlying process generating the shocks as Poisson.

**Definition 1.1.5 (control limit policy)**

A recruitment policy in which recruitment is made upon threshold crossing or upon satisfaction of some conditions, such as the accumulated loss of man hours exceeding a certain fixed value or the number of threshold crossings reaching a certain fixed number, etc., whichever happens earlier, is called a control limit policy.

**Definition 1.1.6 (Counting Process)**

A Stochastic process \( \{N(t), t \geq 0\} \) is said to be a counting process if \( N(t) \) represents the number of occurrences of an event up to time, \( t \). Hence a counting process must satisfy

i. \( N(t) \geq 0 \)

ii. \( N(t) \) is integer valued

iii. If set \( s < t \), then \( N(s) \leq N(t) \) and

iv. For \( s < t \), \( N(t) - N(s) \) equals the number of occurrences of an event in \( (s,t) \)
A counting process is said to have independent increments if the number of occurrences of an event in disjoint time intervals are independent. This means that the number of occurrences by time $t$ must be independent of the number of occurrences between times $t$ and $t + s$.

A counting process is said to have stationary increments if the distribution of the number of occurrences in any interval of time depends only on the length of the time interval. In other words, the process has stationary increments if the number of occurrences $N(t_2+s) - N(t_1+s)$ in $(t_1+s, t_2+s]$ has the same distribution as the number of occurrences $N(t_2) - N(t_1)$ in $(t_1, t_2)$ for all $t_1 < t_2$ and $s > 0$.

**Definition 1.1.7 (Poisson Process)**

A counting process $\{N(t), t \geq 0\}$ is said to be a Poisson process having rate $\lambda > 0$, if

i) $N(0) = 0$

ii) The process has stationary and independent increments,

iii) $P(N(h)=1) = \lambda h + o(h)$ and

iv) $P(N(h) \geq 2) = o(h)$

The inter-arrival times of occurrences in a Poisson process are independent and identically distributed exponential random variables.
Definition 1.1.8 (Renewal Process)

Let \((X_n, n=1,2,\ldots)\) be a sequence of non-negative independent random variables with a common distribution \(F\) with \(F(0) = P(X_n=0) < 1\). Let \(\mu = E(X_n)\) be the mean time between successive occurrences of an event. Write \(S_0 = 0, S_n = \sum_{i=1}^{n} X_i, n \geq 1\). Then the counting process \(N(t) = \text{Sup}\{n/S_n \leq t\}\) is called a Renewal process.

We say that a sequence of random variables \(\{Z_n, n=1,2,\ldots\}\) is stochastically decreasing if \(Z_n \geq Z_{n+1}\) for all \(n=1,2,\ldots\). Similarly we say that a sequence of random variables \(\{Z_n, n=1,2,\ldots\}\) is stochastically increasing if \(Z_n < Z_{n+1}\) for all \(n=1,2,\ldots\).

Definition 1.1.9 (Geometric process)

Given a sequence of non-negative random variables \(X_1, X_2,\ldots\) if for some \(a>0\), the sequence \((X_n, n=1,2,\ldots)\) forms a renewal process, then \((a^{n-1} X_n, n=1,2,\ldots)\) is called a geometric process and 'a' is called parameter of the geometric process.

A geometric process is a stochastically decreasing geometric process if \(a > 1\) and a stochastically increasing geometric process if \(a < 1\).

If \(a = 1\), then the geometric process reduces to the renewal process.
**Definition 1.1.10 (Survival Time Process)**

In manpower planning, survival time of the organization refers to the length of an interval between two consecutive decision points where threshold crossing is not taking place and hence no recruitment is done in this duration. The process of survival times is defined as the survival time process.

**1.1.11 An important Result on conditional Expectation**

If \( \beta \subset \beta' \) \((\sigma\text{-field})\), \( X \) is \( \beta \text{- measurable} \) and \( X' \) is \( \beta' \text{- measurable} \) then

\[
E(XX' / \beta) = E(X' E (X / \beta') / \beta)
\]

In particular, denoting by \( E(\bullet / X'Y) \) the conditional expectation given the \( \sigma \)-field \( \beta X'Y \) of event induced by \( (X',Y) \), we have,

\[
E(XX'/Y) = E(X'E (X/X', Y)/Y) \text{ a.s.}
\]

**1.1.12 Setting the clock Back to Zero (SCBZ) property**

A family of life distributions with survival functions \( \{S(x,\beta) : \beta \in \Omega\} \) is said to have the “Setting the Clock Back to Zero” property (or to be invariant) if for each \( \beta \in \Omega \) and \( x_0 > 0 \), the survival function satisfies the condition

\[
S(x + x_0, \beta) / S(x_0, \beta) = S(x,\beta^*) \text{ with } \beta^* = \beta^*(x_0, \beta) \in \Omega
\]
This means that the conditional distribution of the additional time of survival, given that it has survived \( x_0 \) units remains in the family. This property generalizes the lack of memory property of the exponential distribution for which the conditional distribution of additional survival time is exactly the same as the original distribution. In otherwords, the form of the original distribution remain unchanged under the following operations except for the values of its parameters.

1. truncating the original distribution at some point \( x_0 > 0 \);

2. considering the observable distribution for life times \( X > x_0 \); and

3. changing the origin by means of the transformation given by \( X_1 = X - x_0 \), so that \( X_1 > 0 \).

1.2 Review of Literature

In the area of manpower planning, research has been enormous with the result that a large number of research papers have been published since 1970. It would be a formidable task to give even a brief review of all such research contributions. The main area in which research articles have been published, may be classified as stochastic and mathematical models, relating to:

1. Recruitment

2. The flows in hierarchical manpower system

3. Promotion and wastage
4. Prediction about staff strength and recruitments for the future
5. Optimization

In this section, a brief survey of some selected research articles under the above classification is provided. The research papers relating to the optimization problems are of special interest because this thesis comprises of the results which are optimization problems in manpower planning as obtained by the author during her research studies.

Research papers which deal with the general concepts of manpower planning such as mathematical description of manpower system, historical developments of the subjects and mathematical aspects of manpower planning are given below in addition to the brief survey of research papers in specific areas mentioned above.

A brief idea of trends in manpower management has been discussed by Walker (1968) who has enlisted various problems in manpower management such as evolution of individual performance, predictive instruments within any organization to identify potential talents. The problem relating to manpower planning, compensation, organization planning etc., are also discussed.

A full fledged discussion about the statistical approach to manpower planning is by Bartholomew (1971) in which a description of manpower system along with the historical development of the subject, elementary theory of labour wastage and measures of wastage are given.
Completed Length of Service (CLS) distributions by making comparisons between different groups of employees are also given. Further, a Markov model for a graded organization and the prediction equation for stocks and flows are also given.

A descriptive survey of the manpower planning models and techniques can be seen in Bryant et.al., (1973), wherein a schematic exposition of the manpower planning techniques such as judgemental techniques, matrix models and quantitative techniques are discussed.

Mathematical aspects of manpower planning has been dealt in detail by Vajda (1975). In this paper, concepts of linear programming are used within discrete renewal model for the development of a graded population, when the transfer rates between the various grades are given, and wastage is replaced by suitable recruitment. The following questions are dealt with :

1. Which population structures (i.e. partitioned into grades) can be attained from a given structure after one or more steps?

2. From which structures can a given structure be attained in one or more steps?

3. If the present structure as well as a desired future structure are given, can the latter be attained from the former in one or more steps? If so, how?
4. The last problem is of special interest if starting structure is identical with that to be attained and is called re-attaining after more than one step or strictly maintaining after one step.

A number of numerical illustrations are also provided in this paper.

An interesting and elementary paper by Bernard (1976) gives the different ways of looking at manpower planning (i.e.) how rates of growth are more important than the number existing in different categories of a given manpower system.

1. **Models Relating to Recruitment**

Pollard (1967) has discussed some hierarchical population models with Poisson recruitment. The author considers an organization in which there are known number of employees \( n_j(t) \) in grade \( i \) at time \( t \) \((j=1,2,\ldots,k \text{ & } t=0,1,2,\ldots)\). At the time interval \((t, t+1)\) an employee from grade \( i \) moves to the grade \( j \) with fixed probability \( P_{ij} \). An employee in grade \( i \) leaves the system in the interval \((t, t+1)\) with the probability \( 1 - \sum_{j=1}^{k} P_{ij} \) and assumed to be non-zero. Under these conditions he has given the method of computing the expectations and central quadratic moments of the number of employees in various grades at discrete points of time.

The use of Operations research especially goal programming and network theory for a recruitment decision is by Charnes et.al.(1971)
taking problem as specific to Office of Civilians Manpower Management (OCMM). The model has been developed with multiple goals. The modeling strategies are discussed taking the constraints and objectives into consideration. Numerical examples are also provided.

A continuous-time population model with Poisson recruitment is discussed by Mclean (1976). In this paper, a continuous-time model of a multi-grade system is developed, which includes Poisson arrivals, interaction between grades and a leaving process. An expression is found for the first and second moments for each grade, and it is proved that the limiting distribution of grade sizes is Poisson. This model is appropriate to a manpower system in which the grades represent status in the company and also the grades are degrees of commitment to the firm.

An interesting paper by Abodunde and Mclean (1980) contains the discussions about the model where a manpower system with a constant level of recruitment is considered. It is related to the production planning in the development of telephone services and linking the same to the workforce. The constant level of recruitment necessary to bring the number of installations eventually up to their final levels is discussed. Also a stochastic model is developed which evaluates the effect of implementing the recruitment policies in terms of changing distribution of staff members, and the changing number of installations with time. Numerical results are also provided.
Davies (1982) has discussed a manpower model in which there are fixed promotion rates, no demotions and stochastic wastage. The author has considered a k-graded system of fixed size N and promotions from grade i to grade j with respective probabilities $p_{ij}$. Using the transitions in a Markov environment, the structural paths and their probabilities are defined.

Natarajan (1998) gives the analysis of a single grade manpower system. A single grade fixed size (size s) manpower system allowing wastages and recruitment is studied. Wastages occur according to a Poisson process. Recruitment is made instantaneously as and when the number of vacancies reaches a level say 's'. At the time of each recruitment, the number of vacancies filled up is a random variable following a binomial distribution with parameters $(s, p)$, $0 < p < 1$. The behaviour of the system is identified as a Markov renewal process and the distribution of the level of the system at any time $t$ is obtained. Also the mean time to reach the maximum system level is obtained. The stationary behaviour of the system is also considered.

Sathiyaamoorthi and Elangovan (1998) have obtained the mean and variance of the time for recruitment using shock model approach i) when loss of manpower is a non-negative integer valued random variable, ii) threshold for loss of manpower is a discrete random variable following geometric distribution and iii) the time between two
consecutive decisions form a sequence of independent and identically distributed random variables.

Mariappan and Srinivasan (2001, 2002) have obtained the mean and variance of the time for recruitment using shock model approach when (i) staff depletion are caused by decision making epochs and the inter-arrival time between consecutive decisions are exchangeable and constantly correlated exponential random variables (ii) the sequences associated with the commutative loss of manhours due to the exodus of personnel and the inter-decision times taken by the organization, form a correlated pair of renewal sequences. Sathiyamoorthy and Parathasarathy (2002) have obtained the expected time for recruitment when (i) loss of manpower is a continuous random variable (ii) threshold for loss of manpower is a continuous random variable having SCBZ property instead of exponential distribution which has lack of memory property and the inter-decision times form a sequence of independent and identically distributed random variables.

Saavithri and Srinivasan (2001) have obtained the expected time for recruitment using some univariate policies of recruitment when (i) the loss of man hours for each decision taken form a sequence of independent and identically distributed random variables, (ii) threshold for loss of manpower is a non-negative constant, (iii) Survival time process is a geometric process of independent random variables with state space \((0, \infty)\), (iv) survival time process and loss of manhours
process are independent with state space $(0, \infty)$. (v) threshold for loss of manpower is a non-negative constant.

Saavithri and Srinivasan (2003) have obtained the long run average cost per unit time for recruitment using some bivariate policies of recruitment when (i) the loss of man hours for each decision taken form a sequence of independent and identically distributed random variables, (ii) threshold for loss of manpower is a non-negative constant, (iii) survival time process is a geometric process of independent random variables with state space $[0, \infty)$, (iv) survival time process and loss of man hours process are independent with state space $[0, \infty)$.

2. **Models Relating to flows in a Hierarchical Manpower System.**

An interesting paper on an analysis of flows in a manpower system is by Butler (1971). This paper investigates the probability distribution of the number of leavers in a graded organization in which the grade sizes are fixed. Assuming the Poisson distribution for flows, he has shown that the number of leavers is approximately distributed in the Poisson form and this result is useful in estimating the prediction of errors.

A two dimensional Markov type manpower model for depicting the flow in a manpower system is made by Hayne and Marshall (1977). In this paper, a Markov model with 2-dimensional state space is considered. Under specific assumptions, the structure of the fractional
flow matrix is given as the Length of the Service Model (LOS), the
(Grade, LOS) model are also discussed. The second moment properties
are studied and also the fixed external flows, linear growth of external
flows and geometric growth of external flows are discussed. Examples
are also provided from military services.

Davies (1975) discussed about the maintenance of structures in
Markov chain model, which suffers losses and admits controlled
recruitment. The family of n-step maintainable structures is described
g eo metrically and examined.

Bartholomew (1976 a) has considered a firm as a system of stocks
and flows. The application of renewal theory to manpower planning
models is discussed in this paper.

Chandra (1989) has extended the two characteristics model, due
to Hayne and Marshall (1977) to three characteristic model. The basic
equations of manpower stocks and flows are analysed.

The problem of controlling the flow of people and maintaining a
desirable structure has received considerable attention for one-
dimensional flow from both deterministic and stochastic point of view.
Chandra (1990 a) gives parallel analysis for a two dimensional flow
using deterministic theory. An extension of the result for a partially
stochastic models has also been studied.
Attainability of two characteristic manpower structure is discussed by Chandra (1990 b). This paper deals with the stochastic behaviour of a two characteristic manpower model and calculates the probabilities of grade structure, being attainable in one step under control of recruitment using trinomial distribution. A geometric and probabilistic description of some structures is also discussed.

Nehru (1990) has discussed the methodology developed for a computerized manpower planning model for any hierarchical organization. The methodology employed in the study promises to be useful in identifying the exact manpower planning needs encompassing induction, promotion and wastages has been illustrated by an actual application to the problems of the Indian Navy.

Chandra (1991) has discussed the maintainability of grade structure in a two characteristic (Grade and Length of service) manpower models.

Mariappan and Srinivasan (2001) have considered a Markovian manpower flow model and obtained the basic stock and flow equations under different recruitment policies including promotion and demotion and related results for a two characteristic models. They (2001) have also obtained the basic stock and flow equations for a four characteristic manpower model by considering the aspect of promotion only.
3. **Models Relating to Promotion and Wastage:**

A non-linear model on the promotion of staff has been discussed by Young and Vassiliou (1974). In this paper, a non-linear hierarchically structured management staff in commercial and industrial organization is considered. The use of compartment model in manpower planning for promotions is quite common. Cardinas and Matis (1975), Parde (1975) and Thakur and Rescigno (1978) have discussed compartment models in manpower system. The firm is seen as a graded social system in which transition to the different states occur due to promotions, wastages and recruitments. They have suggested two principles which govern the behaviour of the hierarchical management systems.

1. Ecological principles, and

An ecological principle states that promotions should be proportional to the number of suitable staff available for promotion and the number of vacancies for promotions. The principle of fairness states that, whatever be the desired profile, the perceived profile depends upon the multiplicity of decisions on individual recommendations for promotions are made by a variety of people who attempt to make consistent judgement by fair comparisons with previous decisions. The model is used to predict the staff movements
and it has been shown that the non-linear model proves to be more accurate than the linear model for predictions.

An analytical model for company manpower planning has been developed by Keenay et al., (1977). In a model for manpower hierarchy, by expressing algebraically the relationship between its size, age distribution, wastage rates, the age distribution of the recruits and the proportion in the different grades for each age, are obtained. It is assumed that the demand for manpower grows exponentially at a rate $p$. The wastage rate $W(x,t)\delta t$ of staff age $x$ at time $t$ will leave the organization in $(t, t+\delta t)$. Regarding the stocks it is assumed that $N(t)$ is the total size of the system at time $t$ and $g(x,t)$ is the age distribution at $t$. $q(x,t,t)$ is defined as the proportion of all employees aged $x$, at time $t$, who have been promoted to the top grade and $p(t)$ be the proportion of employees in the top grade. $R(t)\delta t$ denotes the total number of recruits in the small interval $(t,t+\delta t)$ and $r(x,t)$ is the age distribution of recruits at time $t$. In addition to these assumptions some constraints are also proposed. First of all, the over all proportion in the top grade, $p(t)$ is related to proportion promoted at each age by relation.

$$p(t) = \int_0^t q(x,t) g(x,t) \, dt$$

Since the employees are not normally demoted it is assumed that $q(x+\alpha, t+\alpha) \geq q(x,t)$ for all $x, t, \alpha$. The values of the decision variable are obtained subject to the constraints.
Vassiliou (1978) has considered a high order non-linear Markovian model for promotion in manpower systems. Here again the ecological principles of promotion are considered. The purpose of this model is to estimate the staff strength in the different grades and the rate of flows between grades.

Among the papers which are directed towards the study of wastage, the paper by Leeson (1979) is an interesting one. He considers a single graded manpower system in which the wastage rates, promotion rates, demotion rates, length of service are all specified. Assuming these rates, the author examines whether the associated transaction probabilities are stationery or not. Suitable numerical examples are also provided.

In the analysis of wastage, the paper by Agrafiotis (1984) is worth mentioning because of the deviation of this model from the conventional models relating to the analysis of wastage in manpower systems. The author of this paper has presented a model which is designed to investigate the effect on wastage of the internal structure of the company and the promotion of its staff. Also a stochastic model has been developed to depict the probability that the number of promotions to an employee in the interval (0, t). The estimation method for calculating these probabilities is also discussed.

Davies (1985) examined Markov manpower system in continuous time where demotion rates are 0 and promotion rates are time-
dependent. The transient and limiting behaviours of the model are discussed and illustrated with examples.

A model responding to promotion blockages is discussed by Kalamatianou (1988). This model is proposed for manpower system in which promotion probabilities are functions of the seniority within grades. He investigated the adequacy of a strategy which appeared to be used by the management of the nurses in an Athens hospital. It is shown that the strategy is capable of restoring the system to normal state but that the problem will usually recur.

Ragavendra (1991) has discussed a bivariate model for Markov manpower planning systems. Most of the Markov manpower models concentrate either on estimating the gradewise distribution of future manpower structure, given the existing structure and promotion policies, or on delivering policies towards promotion, given the required failure structure. However in many large organizations agreements between employee unions and management result in the training of policies towards promotion based either on seniority (length of service in the grade) or on performance (as in the case of high fliers). In this paper, these two criteria are considered in a bivariate distribution framework. The transition probabilities for promotion obtained from the Markov models are further translated into required seniority and performance rating. The procedure is illustrated through an example.
4. Models relating to Prediction about Staff Strength and Recruitment for the Future.

Young and Almond (1961) have discussed a mathematical model which is used for the purpose of predicting the number and distribution of staff among various grades in future. The author considers the concepts like pattern of recruitment, promotion and withdrawals using which the mathematical equations have been formulated. These equations are in terms of probability values. Defining a matrix $Q$ on the basis of the compound probabilities $q_{rs} = p_r + W_s p_{ro}$, it has been shown that the matrix $Q$ has at least one latent root equal to unity. Also, a difference equation of the form

$$(E - Q)\delta t = \delta I_p,$$

is solved to represent the distribution of staff.

Two mathematical models of the personnel movement through hierarchical organisations are discussed by Marshall (1973). The comparison of two mathematical models is also discussed in this paper. The first model is a Markov model and the second model is Cohort model. Defining $X_i(t), X_j(t)$ as the number in grade $i$ and $j$ at time $t$, the covariance matrix $C(t)$ and the structure of $B(t)$ is also discussed. Here

$C(t) = (C_{ij}(t), \text{ where } C_{ij}(t) = \text{Cov } [X_i(t), X_j(t+1)]$ and

$B(t) = (b_{ij}(t), \text{ where } b_{ij}(t) = \text{Cov } (X_i(t), X_j(t)))$

Numerical illustrations for giving forecast of enrollment is also given in this paper.
Bartholomew (1976 b) discusses the importance of predictions in manpower planning. In this paper, taking into consideration the stochastic models, the methods of predictions regarding stocks and flows are discussed. The Markov control theory is explained and numerical illustrations are also given.

5. Models involving Optimization.

There are many methods of approach to solving optimization problems in manpower planning. Mostly the calculus of maxima and minima is used. In addition to the calculus approach, techniques of operations research such as linear programming, goal programming, dynamic programming and network analysis are also used.

In the determination of desirable long term manpower policies, the stationary behaviour of the system is given due importance. The methods of the demography and actuarial science are used in this approach. The theory of stationarity is closely linked with that of control and optimization which is one of the main areas of research according to Bartholomew and Forbes (1973).

An optimum manpower utilization model using mathematical programming models is discussed by Schneider and Kilpatrick (1975). This model is with particular reference to Health Maintenance Organization (HMO). The interaction between effective manpower utilization, faculty requirements and available capital is discussed in
two basic models, one is an overall planning model and the other is a subscriber maximization model. The objective used in the models pertains to either minimum cost or minimum feasible use of physicians through the substitution of physician extenders. It is discussed in this paper that two objectives in preparing a model for medical care process are

1. Minimize the medical manpower needs for a given number of subscribers, and

2. Maximize the number of subscribers by a given number of medical personnel.

They have derived several results taking into consideration the different type of costs involved.

A longitudinal model for calculating optimal accession policies is given by Grinold (1976). The model is based on the demand and recruitment of naval aviators. The input into the training process is the qualified aviators and the demands for the aviators are uncertain. The author has classified the individuals in the system according to their length of service upto the time point t. The basic manpower flow process is described taking into consideration the stock of manpower at time t. The author has also discussed a stochastic demand process and the related deterministic supply process assuming the demand process \{Z_t\} as Markovian. The continuation rates for a 6-state structure is
obtained as a numerical illustration. Finally, an optimal accession policy based on the Markov transition probabilities, the continuation rates and the effectiveness coefficient are discussed.

A mathematical model of a military manpower system with a view to determine the optimal steady state wage rate and force distribution by length of service is by Jaquette and Nelson (1976). In this paper, it is assumed that the cost of hiring personnel is determined by military manpower supply functions which relate enlistment and reenlistment rates to military pay. The optimal force is defined as that force which provides the greatest military capability for a given budget cost. Optimal rates of pay are determined by maximizing the productivity index subject to a budget constraint. Assuming the basic flow process as Markovian, the optimal rates of pay are determined. The steady state optimal policy for the Cobb-Douglas type function is obtained using the Lagrangian multiplier technique. Numerical results are also discussed.

The concept of optimization models is given in Grinold and Marshall (1977). In evolving manpower policies the use of optimization model is discussed. The concept of long term horizons involved with manpower decisions and the uncertainty in future manpower requirements is discussed. Optimization problems form a part of planning process. A decision maker gives the data and assumptions as the input and conceptualizes a model for the purpose of obtaining the
optimal, specification of future system performance. The input data consists of projected legacies, future requirements, budget conditions, costs, discount rates, utilization factors and the co-efficient governing the flow process. The optimal policy is derived with regard to the requirements etc. The optimal long term policies are derived by the use of linear programming methods taking the discounted cost as the objective and minimization of the same with regard to inflows as decision variables. Examples and problems are indicated using specific cost, stock levels and flows.

The optimization of state patrol manpower allocation has been discussed by Lee et.al., (1979). One of the most difficult tasks of state highway patrol administrators is allocation of manpower, (i.e) determining the most effective level of operational manpower for patrol tasks. In this paper, an integer goal programming model for allocating highway patrol men to road segment within a patrol region is presented. For the purpose of optimization, various constraints such as total patrol constraints, enforcement constraint, minimum shift requirement, minimum patrol men per road segment per shift, maximum patrol men per road segment per shift, traffic density etc., are taken into account. The goal programming model provides the optimal solution to the problem.

The use of dynamic programming to decide the optimal recruitment and transition strategies for manpower systems is by
Mehlmann (1980). In this paper, it is assumed that an organization with different grades of employees, and the transitions between the grades take place according to a discrete time Markov chain. The concepts of dynamic programming techniques having a finite horizon are used for the purpose of determining the optimal recruitment size in each grade and the transitions between the grades. A numerical illustration with an organization having three grades is also discussed.

According to Malcolm Bennison and Jonathan Casson (1984), any manpower planning enquiry probably requires a number of parallel runs to be undertaken to evaluate alternative options as a basis for subsequent decision making. In contrast, the body of optimization models set out to establish, for a given set of manpower goals, what the optimal manner of achieving them is. Most of the work on optimization model are centered on the application of mathematical programming techniques. The usual objective function in terms of cost is minimized taking into consideration the constraints such as the promotion rules. Goal programming models are also introduced. These methods are usually very complex having a large number of variables and equations.

An optimal recruitment policy has been discussed by Mukherjee and Chattopadhyay (1985). They consider an organization in which a group of \( n \) persons is recruited at some point of time. Each recruit can be in service for a period of \( t \) years at the most. It is also assumed that the efficiency of each recruit may be adversely affected by a long
duration of service. A recruitment policy which involves a planned recruitment of the entire staff required at intervals of time $T$ along with the forced retirement of those in service till the end of each such interval has been considered. The optimal value of $T$ which minimized the total cost of unfilled vacancies and forced retirements had been worked out, assuming exponential and Pearsonian type XI distributions for time at which recruits are withdrawn from service. In two other models, optimal values of $T$ have been determined assuming exponential withdrawal times so as to maximize the expected net gain per unit of time and the expected net discounted gain per unit of time.

An optimal planning of manpower training programmes has been discussed by Goh et.al., (1987). In fact, two different models are discussed in this paper by assuming the finite planning period and an infinite planning period. A finite state Markov chain is used to model the manpower state for the finite planning period and the optimum solution is computed using the dynamic programming technique. A non linear integer programming problem is used to model the manpower state for an infinite planning period. They consider an organization which has a total of $N$ employees. Training programmes are conducted for a fixed period with a view to increase the skill of the workers and to raise their productivity. The return from a training program is also assessed, and the problem solving involves, finding a training policy, such that the expected total return over the entire
planning period is maximized. This paper suggests that the expected return in terms of productivity, due to the training of personnel should be maximized by adopting a training policy or the number and extent of training programmes which would involve the minimum cost. This is precisely called the optimal planning of training policies.

The use of dynamic programming to determine the optimal manpower recruitment policies is discussed by Poornachandra Rao (1990). This paper deals with the objective of minimizing the manpower system costs. The author has taken some basic assumptions while formulating the manpower problem to determine the optimal recruitment policies. It is interesting to note that the concept of inventory control theory is used as a basis of the dynamic programming formulation and the algorithm is provided in this paper.

Subramanian (1996) has discussed an optimum promotion policy taking into account the cost of promotion of a person belonging to the grade $i$, ($i = 1, 2, 3, \ldots, n$) at time $t$ as a function of the number of employees in the grade $i$ at time $t$. In doing so the total cost involved in this case is given by

$$C = \int_0^T C_i(t)S_i(t)P_i(t)\,dt$$

Using the Euler Lagrange's method the solution which is the promotion rate from grade $i$ at time $t$ as

$$P_i(t) = \left(\frac{K}{S_i} \times C(S_i^*)\right)$$
where $S_i^*$ is the value of $S_i$ when the cost is maximum. $C(S_i^*)$ is the cost of promoting a person from grade $i$ to grade $(i+1)$ when the size of the grade $i$ at time $t$ is $S_i(t)$.

In addition to the above, many models have been developed for the purpose of manpower management in specific organizations. In many of the models the concept of stochastic processes have been applied. One could find the application of Markov chains in the study of transitions between the different states of the graded manpower system. Semi-Markov process and its properties have also been widely used.

The application of replacement strategies to manpower planning is discussed by David Robinson (1974). The author considers a two stage replacement policy. The individual replacement costs for the two stages are different and the sizes of the two stages are given to be $N_1$ and $N_2$ with $N_1+N_2 = N$. It is also assumed that the individual replacement is costlier in stage I than in stage II. The life expectancy of an item decreases with age. All failures in stage II are replaced by the components already operating in stage I. Failures in stage I are replaced by new items. This concept is extended to the case of a $K$ stage model. These results are applied to the manpower system with multi grades. The mean time to promotion has been obtained.
The limiting behaviour of discrete time Markovian manpower models with non-homogeneous independent Poisson input has been discussed by Mehlmann (1977). The author considers an organization in which the population of persons are divided into K grades. The individual transition at epochs \( t=0,1,2,\ldots \) takes place between the grades according to a time homogeneous Markov chain. It is also assumed that the input is Poisson, time homogeneous and independent. An asymptotic relation for the population numbers in the various grades is derived.

Another paper by Mehlmann (1977) discusses the problem of determining the asymptotic form of stock vector of the different grades in a continuous time Markovian manpower model. The recruitment is assumed to be exponential. The transition probability matrix is found out on the basis of transition between the grades on a discrete time scale \( t=0,1,2,\ldots \). The asymptotic form of the stock vector \( n(t) \) relating to the different grades assuming asymptotically exponential recruitment function \( R(t) \) is discussed. In addition to this the career prospectus in a Markovian manpower model is also discussed.

A discrete time Markov population model in which the class corresponds to the length of service has been considered by Mcclean (1977). In this model the different classes correspond to the length of service (LOS) and at every point of time, each member of the system either moves on to the next class or leaves and is replaced by a recruit.
in the first class. The classes are denoted as \( C_1, C_2, \ldots, C_k \) with \( N_i, i=1,2,\ldots,k \), the number of people in class \( C_i \). The probability of a transition from \( C_i \) to \( C_{i+1} \) is denoted as \( p_i \) and \( q_i \) is the probability of leaving from \( N_i \) and being replaced by a recruit to \( N_1 \) of the first class so \( p_i + q_i = 1 \), \( i=1,2,\ldots,k \) and \( q_k = 1 \). Under these assumptions the mean vector and variance covariance matrix of the class size at time \( t \) are defined and it has been proved that the steady state class sizes are independent Poisson. An important observation that can be made in this connection is that leaving depends upon the length of service (LOS) only.

Mcclean (1978) has discussed some continuous time stochastic model of a multi grade population, assuming a \( k \) grade system. A person leaves the \( r \)th grade with probability \( q_r \) and is promoted with probability \( p_r \). The length of time \( x \) of stay in the grade \( r \) has a probability density function \( g_r(x) \) and a survivor function \( G_r(x) \), in case he leaves the grade \( S_r \). In case, the person is promoted, the corresponding probability density function of the length of time of stay which is \( y \) is \( f_r(y) \) with the corresponding survivor function \( F_r(y) \). Under these assumptions the semi-Markov transition between grades are discussed. The mean grade size at time \( t \) is obtained.

Mehlmann (1979) has discussed a semi-Markovian manpower model in continuous time wherein he considers a continuous time scale \( t \) and the individual transition between the \( k \) grades of an organization
taking place according to a time homogeneous Semi-Markov process. The states of the process are the k transient grades and the absorbing state 0, which represents the world outside the organization. $Z_{ij}(u)$ is taken to be the force of transition from state i to state j (i≠j) in the duration u and $Z_{ij}(u) \delta u + o(\delta u)$ is the probability of a transition to state j within time $\delta u$, given a current stay in i of duration u. The limiting behaviour of the grade sizes in terms of the exponential input rates, and the probability of an individual holding time in a state are obtained.

A semi-Markov model for a multigrade population with Poisson recruitment is due to Mcclean (1980). The author considers a multi grade population with semi-Markov transition between grades, Poisson arrival to each grade, and departure from each grade. The semi-Markov transitions facilitate taking into account the existing knowledge about the distribution of length of service until an individual leaves the firm. The distribution of the number of the members in each grade at any time t resulting from a particular immigration pattern and also the limiting distribution of the grades sizes have been obtained. The situations where the recruitment is time dependent Poisson is also discussed.

A paper by Mcclean (1991) deals with manpower planning models and their estimation. The author has given the need for manpower planning technique for the modern manager, especially in a climate of economic recession and government cut backs. The supply
and demand models of manpower planning, the use of Markov chain formulation for the purpose of manpower system in the future is also discussed in this paper. The modeling of manpower system is dealt in detail.

Ledermann (1992) has given the procedure for obtaining an estimate of the rate of convergence to its limit when the structure of the different grade size in a multigrade organization at time $t$ with proportional wastage compensated by recruitment at the lowest rank is in the form of a stochastic matrix. The state of convergence is discussed and supported by a numerical illustration.

Savithri and Srinivasan (2002) have obtained the long run average cost per unit time for recruitment using some univariate policies of recruitment when (i) the loss of manhours for each decision taken form a sequence of independent and identically distributed random variables, (ii) threshold for loss of manpower is a non-negative constant. (iii) Survival time process is a geometric process of independent random variables with state space $[0,\infty)$ (iv) survival time process and loss of manhours process are independent with state space $[0,\infty)$.

Savithri and Srinivasan (2003) have obtained the long run average cost per unit time for recruitment using some bivariate policies of recruitment when (i) the loss of man hours for each decision taken
form a sequence of independent and identically distributed random variables (ii) threshold for loss of manpower is a non-negative constant, (iii) survival time process is a geometric process of independent random variables with state space \([0, \infty)\) (iv) survival time process and loss of man hours process are independent with static space \([0, \infty)\).

1.3. CURRENT WORK

In any organization, the exit or wastage of personnel is quite common. The exit of personnel in marketing organization is very frequent, especially when policies regarding incentives and targets are revised. There are certain special problems associated with the organization engaged in sales and marketing. Frequent exits and recruitment are very common in such organizations. Frequent recruitment is also expensive due to the cost of recruitment and training. Whenever the organization announces revised policies regarding sales target, revision of wages, incentives and perquisites, the exodus of personnel is possible. This inturn produces loss in man hours which adversely affects the sales turnover of the organization.

If the loss in man hours due to the exit of personnel crosses a particular level, known as threshold, the organization reaches an uneconomic status which otherwise be called the breakdown point and the recruitment is to be done at this point. This is the idea behind recruitment through shock model approach.
The main objective of the thesis is to construct different mathematical models on manpower planning to obtain: (i) the mean and variance of the time for recruitment for a single and two graded manpower system using shock model approach under different conditions on the threshold distribution and inter-decision times of exits, (ii) the long-run average cost per unit time for a single graded system associated with an univariate policy of recruitment (iii) optimum cost for promotion in manpower planning and (iv) optimum time interval between screening tests for promotion. The abstract of the research work done is given below chapterwise:

CHAPTER 2: EXPECTED TIME FOR RECRUITMENT WITH CORRELATED INTER-DECISION TIMES OF EXITS WHEN THRESHOLD DISTRIBUTION HAS SCBZ PROPERTY

In this chapter, the author has considered a single graded organization in which the inter-decision times are exchangeable and constantly correlated exponential random variables. Assuming that the threshold distribution has "Setting the Clock Back to Zero (SCBZ) property, a mathematical model is constructed and the mean and variance of the time for recruitment associated with an univariate policy of recruitment is obtained through shock model approach. The results are numerically illustrated by assuming specific distribution.
CHAPTER 3: EXPECTED TIME FOR RECRUITMENT IN A TWO GRADED MANPOWER SYSTEM WITH GEOMETRIC THRESHOLDS AND CORRELATED INTER-DECISION TIMES.

In this chapter, a two graded manpower system is considered in which the inter-decision times are correlated. Assuming that the thresholds are geometric random variables, two mathematical models are considered and the mean and variance of the time for recruitment associated with an univariate policy of recruitment is obtained for both the models using shock model approach. Numerical examples are provided and the relevant conclusions are made.

CHAPTER 4: EXPECTED TIME FOR RECRUITMENT IN A TWO GRADED MANPOWER SYSTEM WITH EXPONENTIAL THRESHOLDS AND CORRELATED INTER-DECISION TIMES.

In this chapter, a two graded manpower system having two independent exponential thresholds is considered. Two mathematical models are considered and the mean and variance of the time for recruitment associated with an univariate policy of recruitment for both the models are obtained when the inter-decision times are correlated as in Chapter 2. Numerical examples are presented for a better understanding of the model.

CHAPTER 5: EXPECTED TIME FOR RECRUITMENT IN A TWO GRADED MANPOWER SYSTEM ASSOCIATED WITH CORRELATED
INTER-DECISION TIMES WHEN THRESHOLD DISTRIBUTION HAS SCBZ PROPERTY.

Assuming that the inter-decision times are exchangeable and constantly correlated exponential random variables and the threshold distribution has SCBZ property, a mathematical model for a two graded manpower system is constructed in Chapter-5 and the mean and variance of the time for recruitment associated with an univariate policy of recruitment is obtained through shock model approach. Numerical illustrations are provided by assuming specific distribution.

CHAPTER 6 : COST ANALYSIS ON UNIVARIATE POLICY OF RECRUITMENT IN MANPOWER PLANNING ASSOCIATED WITH A CORRELATED PAIR OF RENEWAL SEQUENCES – A SHOCK MODEL APPROACH.

In this chapter, a single graded manpower system is considered and a mathematical model involving an univariate policy of recruitment associated with a correlated pair of renewal sequences based upon shock model approach is constructed. An explicit expression for the long-run average cost per unit time for this policy is obtained.

CHAPTER 7 : COST ANALYSIS ON A UNIVARIATE POLICY OF RECRUITMENT IN MANPOWER PLANNING ASSOCIATED WITH A CORRELATED PAIR OF RENEWAL SEQUENCES – A CUMULATIVE SHOCK MODEL APPROACH.
In this chapter, a single graded manpower system is considered and a mathematical model involving an univariate policy of recruitment associated with a correlated pair of renewal sequences based upon cumulative shock model approach is constructed. An explicit expression for the long-run average cost per unit time under the above setup is obtained. A numerical example is provided for a better understanding of the model.

CHAPTER 8 : OPTIMUM COST FOR PROMOTION IN MANPOWER PLANNING.

In this chapter, an organization in which promotions are effected to fill up the vacancies created, is considered. A mathematical model is constructed and the optimum rate at which vacancies should be created so as to minimize the total cost incurred in the process, is estimated. Numerical illustrations are also provided.
In this chapter, an organization with three grades is considered where the vacancies are filled either by promotion in the form of screening tests or by direct recruitment. A mathematical model is constructed and the optimal time interval between successive screening tests is obtained so as to minimize the expected total cost. The analytical results are numerically illustrated, by assuming specific distribution.