CHAPTER 4

DEVELOPMENT AND SIMULATION OF STABILITY
MODEL FOR CUTTING TOOL SYSTEM

With the modern trend of machine tool development, accuracy and reliability are gradually becoming more prominent. To achieve higher accuracy and productivity it is not enough to design the machine tools from static considerations without considering the dynamic instability of the machine tools. If there are any relative vibratory motion present between the cutting tool and the job, it is obvious that the performance of the machine tool will not be satisfactory. Moreover, machine tool vibration has a detrimental effect on tool life, which in turn, lowers down the productivity and increases the cost of production.

During operations, machine tools are subjected to static or dynamic loads, these loads/forces may act in either of the following manners,

a) Dynamic behavior caused absolutely by the load acting during the action of the load (forced vibrations),

b) Dynamic behavior initiated by a load but persisting after load has ceased to act (free vibrations),

c) Dynamic behavior through an interaction between the structure and cutting process (Self-excited vibrations).
In almost all practical cases, finding the exact mathematical solution is impossible. For machine tools, the mathematical model will also become so complex that it is almost impossible to solve. Therefore, it is assumed that the machine tool can be exited only in one mode.

The dynamic forces that arise during cutting can cause the machine-tool structure to lose stability. When this happens, the machine tool vibrates and is said to chatter. Chatter occurs at the point when relative motion between the tool and workpiece results in a negative damping force that overcomes the dissipation inherent in the system. Chatter is a so-called self-excited oscillation because the vibration itself generates the energy that again creates the vibration.

The single degree of freedom chatter theory has been considered for only those cases where rigidity of the tool and support is relatively small in one direction which may allow the tool to vibrate in that direction. Otherwise, the tool motion will not be straight and two degree of freedom theory will have to be used for analyzing the problem.

The model proposed in this work is an analytical model used for the prediction of stability limit for hard turning systems during cutting process by using various modes, speeds, width of cut, damping ratios and stiffness.

4.1 STABILITY OF CUTTING TOOL SYSTEM

Some basic and important concepts and equations of the structural dynamics, which are used in the following sections, are overviewed here with the discussion of a single degree of freedom system. A viscously damped single degree of freedom system model is shown in Figure 4.1. Assuming that, any increment of force (P) due to regeneration effect occurring in y-
direction continues to act in \( \beta \)-direction and that is the only force acting on the system.

Figure 4.1 Single degree of freedom cutting tool system.

If the principal mode \( (x) \) is inclined to an angle \( \alpha \) to the direction of normal \( (y) \) and to the generated surface, the motion along \( y \) is related to motion along \( x \) by,

\[
y = x \cos \alpha
\]

Hence, the chip-thickness variation is:

\[
y = y(t) - y(t - T) = [x(t) - x(t - T)] \cos \alpha
\]

If the coupling coefficient between the force \( P(t) \) along \( y \) and \( z \) is given by \( r \),

\[
P(t) = -ry
\]
The force component along x is given by:

\[ P_x(t) = P(t) \cos(\alpha - \beta) \quad (4.3) \]

The equation of motion along x is:

\[ m\ddot{x} + c\dot{x} + kx = P_x(t) = P(t) \cos(\alpha - \beta) \quad (4.4) \]

where

\[ m = \text{Mass} \]
\[ c = \text{Damping coefficient} \]
\[ k = \text{Stiffness} \ (k = P / x) \]

The solution \( x(t) \) is given by:

\[ x(t) = \frac{P(t) \cos(\alpha - \beta)}{K} \left( \frac{1}{1 - \left( \frac{\omega}{\omega_0} \right)^2 + j2\xi \left( \frac{\omega}{\omega_0} \right)} \right) \quad (4.5) \]

where

\[ \xi = \text{damping ratio} = \frac{c}{c_c} \]
\[ \omega_0 = \text{natural frequency} = \frac{k}{m} \]
\[ \omega = \text{forcing frequency}, \]
\[ r = \text{frequency ratio} = \frac{\omega}{\omega_0} \]

Re-arranging the equation (4.5),
\[ P(t) = \frac{Kx(t)}{\cos(\alpha - \beta)}[(1 - r^2) + 2j\xi r] \]

Using the equation (4.2) the above equation can be re-written as

\[ x(t)\left\{ \frac{K}{\cos\alpha \cos(\alpha - \beta)} \right\}[(1 - r^2) + 2j\xi r] = -r[x(t) - x(t - T)] \quad (4.6) \]

Let

Coupling coefficient \( u = \cos\alpha \cos(\alpha - \beta) \) \quad (4.7)

Cross receptance \( \Theta = \frac{u}{K(1-r^2)+2j\xi r} \) \quad (4.8)

Combining equation (4.6) with equation (4.8),

\[ \frac{x(t)}{\Theta} = -r[x(t) - x(t - T)] \]

Hence,

\[ \frac{|x(t)|}{|x(t-T)|} = \frac{r}{\Theta(1/r) + r} = \frac{0}{\Theta(1/r) + r} = q \quad (4.9) \]

\( q < 1 \), the system is stable, i.e. the amplitude does not build up, but when \( q = 1 \), the system is at the threshold of stability.

Tlusty and Polacek assumed ‘r’ as real and hence they solved the stability criterion with only real part of \( \Theta \).

\[ \Theta = \frac{u}{K} \left[ \frac{(1-r^2)}{(1-r^2)^2 + 4r^2\xi^2} - j \frac{2\xi r}{(1-r^2)^2 + 4r^2\xi^2} \right] \quad (4.10) \]
Replacing $\varnothing$ by $\text{Re} (\varnothing)$ in equation (4.9),

$$q = \frac{|G|}{|G|^{1/2}} = 1 \quad \text{(for threshold of stability)}.$$

This equation is satisfied, if

$$|G| = |G| + 1 / r$$

From which

$$r^* = -1/2G^*$$

The limiting coupling coefficient is given by limiting value of the negative reciprocal of the real part of cross receptance for threshold criterion of stability.

$$G' = G \ast \cos \alpha \ast \cos (\alpha - \beta)$$

When $\alpha = \beta = 0$, $G' = G$.

The limiting coupling co-efficient is given by:

$$r_{\text{max}}^* = -\frac{1}{2G_{\text{min}}}$$

The governing equations of machine chattering can be derived from the general equation of vibration and the regenerative chatter equations.
In orthogonal cutting, the cutting force \( P(t) \) is proportional to the cutting area (the product of the chip width or depth of cut \( a_p \) and thickness \( h \))

\[
P(t) = k_s a_p h = k_s a_p \left[ x(t - T) - x(t) \right]
\]  

(4.14)

\( T \) is the time interval between the previous and current cuts.

Substituting equation (4.14) into the general equation of vibration:

\[
m\ddot{x} + c \dot{x} + kx = P(t)
\]  

(4.15)

The depth of cut is found from equation (4.14)

\[
a_p = -1/2k_s G
\]  

(4.16)

where \( G \) is the real part of the frequency response function (FRF) and from the equations (4.10) and (4.11)

\[
G = \frac{1}{k} \left[ \frac{\left(1-r^2\right)}{\left(1-r^2\right)^2 + 4r^2\zeta^2} \right]
\]

\( r = \) Ratio of chatter frequency to natural frequency \( (r = f / f_n) \).

\( f_n = \) Natural frequency of the machining system is also called the modal frequency.

\( \zeta = \) Ratio of the damping coefficient to the critical damping coefficient \( (\zeta = c / c_c) \).

\( c_c = \) Critical damping coefficient \( (c_c = 2\sqrt{km}) \)

Solving the equations (4.14) and (4.15) together, the depth of cut \( a_p \) is dependent on the frequency \( f \) of machine vibration or chatter through the
frequency ratio $r$. For each chatter frequency generated on a machining system, there is a corresponding critical chip width (minimum depth of cut) $a_p$. The cutting process is stable when its depth of cut is less than the critical value and unstable otherwise.

Roughness or waviness always exists on the machined surface of workpiece due to vibrations. According to regenerative chatter theory, chatter occurs whenever there is a shift of the phase angle $\epsilon$ between the current and previous surface waviness. Therefore, the ratio of chatter frequency $f$ to tooth-stroke frequency $f_t$ represents the number of surface waves between consecutive cutter teeth, and can be written as an integer $n$ (also called the lobe number. $n = 0, 1, 2, \ldots$) plus a fraction of $\epsilon / 2\pi$ radians

$$\frac{f}{f_t} = n + \frac{\epsilon}{2\pi}$$  \hspace{1cm} (4.17)

where $r_t$ is the ratio between the tooth frequency and natural frequency ($r_t = f_t / f_n$). The phase shift angle $\epsilon$ between the current and previous surface waviness may be expressed as

$$\epsilon = \pi + 2\tan^{-1} \frac{H}{G}$$  \hspace{1cm} (4.18)

Since the real and imaginary FRF’s $G$ and $H$ are both negative equation (4.10), using the principal range of $-\pi/2 < \tan^{-1}x < \pi/2$, we have $0 < \tan^{-1}(H / G) < \pi/2$, and the phase shift angle $\epsilon$ is between $\pi < \epsilon < 2\pi$.

Substitute equation (4.16) into equation (4.18) and then into equation (4.17), we obtain the equation of regenerative chatter

$$\frac{f}{f_t} = n + \frac{1}{2} + \frac{1}{\pi} \tan^{-1} \frac{-2\xi r}{(1 - r)}$$  \hspace{1cm} (4.19)
Equation (4.19) represents the relationships among the chatter frequency $f$, the tooth frequency $f_t$, and the lobe number $n$. Together with equation (4.16), they form the governing relationship between the depth of cut $a_p$ and spindle speed $N$. The spindle speed $N$ can be related to the tooth frequency $f_t$ ($f_t = n_t \frac{N}{60}$), where $n_t$ is the number of teeth on the cutter.

### 4.2 STABILITY ANALYSIS

Regenerative chatter theory creates a relationship between spindle speeds and the critical chip width or depth of cut. The theory produces a stability lobe diagram and makes it possible to achieve the highest applicable metal removal rate (MRR) for a machining process. A method is presented to create the stability lobe diagram using a MATLAB. The characteristics and limitations of the stability lobe diagram are discussed.

The implementation of MATLAB in chatter suppression involves the design of where parameters like mass, stiffness of tool, rake angle, damping of tool and speed have to be designed suitably. The methodology is developed to help those small and medium sized enterprises for which it is difficult to acquire specific equipment such as impact hammers, piezoelectric transducers, specific software, and sensors for chatter detection.

#### 4.2.1 Effect of Mode Orientation Angle on Stability

The effect of mode orientation angle $\alpha$ on real cross receptance is an interesting exercise to study. From the equation (4.7), the effect of mode orientation angle $\alpha$ on cross receptance is simulated with the help of software MATLAB program (A4.1). The function $u$ for $\beta = 60^\circ$ is plotted in Figure 4.2.
From Figure 4.2, it is observed that for $\alpha = (\pi/2)$ and $\alpha = [(\pi/2) + \beta]$, the coupling coefficient, $u$ becomes zero and an unconditional stability is achieved. This is clearly shown in Figure 4.3. A stability plot for different values of $\alpha$ keeping $\beta$ invariant shows that the stability limits are affected by the mode orientation angle $\alpha$ as indicated in Figure 4.3.
Figure 4.4 Effect of $\alpha$ on stability for different damping ratio

The limit of stability, connoted by coupling coefficient, $r^*$, is minimum for $\alpha = (\beta/2)$.

From the Figures 4.3 and 4.4, it is indicated that the permissible stability value $r^*$ is minimum at $\alpha_1 = 30^\circ$. Figure 4.4 shows the effect of mode coupling angle $\alpha$ on stability limit for finding various damping ratio and keeping $\beta$ invariant. The higher damping ratio has then higher stability region. The stability limit is increased when the damping ratios increase.

4.2.2 Effect of Cutting Speed on Stability for Various Damping Ratios

From the equation (4.12), it is indicated that the effect of speed on stability is simulated with the help of software MATLAB program (A4.2). Figure 4.5 shows the stability plot of conventional characteristic equation method. The stability plot approximated on this pattern is shown that, mass
0.467 kg, stiffness is 4218.2 N/mm$^2$ and damping ratios $\zeta = 0.014285$, 0.01905, 0.03754, 0.05254, 0.0861.

Figure 4.5 Effect of speed on system stability for different damping ratio

Variation of stability as function of speed for different damping ratio by keeping mass and stiffness invariant is clearly shown in the Figure 4.5. The chatter frequency is greater than the resonant frequency (Sweeney 1971). Hence, the stability plot is discussed after the resonant frequency. In this model, resonance takes place at the speed of 960 rpm. Higher damping ratio moves the stability boundary upward and stability region become wider. From the Figure 4.5, it understood that the PTFE coated carbide shim tool has a wider stability region.

4.2.3 Effect of Cutting Speed on Stability for Various Stiffness

The variation of stability as function of speed for different stiffness of cutting tool is clearly shown in the Figure 4.6.
Figure 4.6 Effect of speed on system stability for different stiffness

Figure 4.6 shows the stability plot of conventional characteristic equation method. The stability plot approximated on this pattern is shown that the mass as 0.467 kg, damping ratio of 0.0861 and stiffness as $k = 3800$ kN/m, 4000 kN/m, 4200 kN/m, 4400 kN/m. When stiffness increases, the natural frequency is also increased. If the natural frequency increases then the boundary curve moves right. Moreover, if it decreases on the contrary then the curve moves left.

4.2.4 Characteristics of Stability Lobe According to Natural Frequency and Damping Ratio

The characteristics of the stability curve with respect to the dynamic parameters, damping ratio and stiffness are combined and presented in the Figure 4.7.
Figure 4.7  Architecture of the stability lobe behavior according to natural frequency and damping ratio

From the Figure 4.7 it is seen that the stability lobe boundary moves upward and the stability region becomes wide when the damping ratio increased. However, the boundary region moves downward for lower damping ratio and the size of stable region becomes smaller and narrower. If the natural frequency increased then the boundary curve moves right. On the contrary, if the natural frequency decreases then, the curve moves left.

4.2.5 Stability Lobe for Cutting Tool System

The relationships among the chatter frequency $f$, and the tooth frequency $f_t$, and the lobe number $n$, together with equation (4.12), they form the governing relationship between the depth of cut $a_p$ and spindle speed $N$. Using MATLAB program (A4.4) relationship between depth of cut $a_p$ and spindle speed $N$ are plotted in Figure 4.8. Since the series of relationships curve in Figure 4.8 is shaped like lobes, the graph is usually called a stability lobe diagram. The stability plot approximated on this pattern is shown that the mass 0.467 kg, damping ratio of 0.0164 and stiffness $k = 4200$ kN/m. A stability lobe diagram shows the relationship between chip width (or depth of cut) and spindle speed, with the lobe number as a parameter.
Usually, the variable on its x-axis is represented as spindle speed \( N \), tooth frequency \( f_t \), the variable on its y-axis is represented as depth of cut \( a_p \).

![Stability Lobe Diagram](image)

**Figure 4.8 Stability Lobe Diagram for hard turning**

By changing, the damping ratio of the tool material from 0.0164 to 0.0861, it is observed that the stability lobe moves upward as shown in Figure 4.9. There is an optimal depth of cut for every cutting speed, which gives maximum cutting speed boundary value. There is an optimal cutting speed, which gives the best margin of cutting speed.

![Stability Lobe Diagram](image)

**Figure 4.9 Stability lobes for carbide and PTFE coated carbide shim tool holders**
Higher MRR without chatter can be achieved by machining the workpiece with the cutting condition selected from the stability boundary.

4.2.6 Properties of Stability Lobe Diagram

Some interesting characteristics exist in the stability lobe diagram, which may be utilized to optimize the chip width or depth of cut and obtain the maximum material rate (MRR) in machining processes.

In a stability lobe diagram, a series of scallop shaped lobes intersects with each other. These lobes form the limits for chattering. Locally, for each lobe, it is stable below the lobe and unstable above the lobe. Since the lobes intersect, a point located below one lobe could be above the neighboring lobe. This point must be treated as unstable. Therefore, globally we must consider the relationship between adjacent lobes in determining the stability. The upper portion of any two adjacent lobes above their intersection point should be trimmed off. The intersection point connects all the lobes into ‘chained’ chatter lines. All points above the chatter line are unstable, and below are stable. As spindle speed increases, the lobes become wider with larger intervening spaces between consecutive lobes, and intersection points are higher. This phenomenon creates a desirable situation for machining at both higher speed and deeper cut simultaneously and a wider speed range as well.

The above simulation revealed the following:

- The stability limit of cutting tool with different dampers can be predicted using MATLAB.
- The permissible stability value $r^*$ is minimum at $\alpha = \beta / 2 = 30$. And between $60 < \alpha < 180$ the self-excited vibrations due to mode-coupling would not occur.
• PTFE coated carbide shim lifts the stability lobe upward and widen the stability region. However, the boundary region moves downward for lower damping ratio and the size of stable region becomes smaller and narrower.

• By increasing the stiffness of system, the natural frequency is increased and the boundary curve moves right. Similarly, if the stiffness is decreased then the curve moves left.

• Higher damping ratio PTFE coated carbide shim has higher stability limit than the conventional carbide shim.

An interesting inference has been drawn from stability analysis. The degree of stability against mode-coupled self-excited vibrations can be increasing not only by increasing the stiffness or damping of the system but also by changing the mutual tuning of the mode with regard to their direction of orientation. This finding confirms that the dynamic stability can be increased without any modification in structure and the weight of the existing machine tool.