Chapter 8

Conclusion

The present thesis mainly investigates some novel aspects of nonlinear plasma oscillations and wave breaking which have not been considered as yet.

In the second chapter of this thesis, we report on space time evolution of nonlinear oscillations in the lab frame, initiated by an arbitrary density perturbation which can be expressed as Fourier series in ‘$x$’. Before our solution, we were only aware of space time evolution of a pure sinusoidal density perturbation in the lab frame which is nothing but a very special case of our general solution. We have obtained this general solution as in realistic laser/beam plasma interaction experiments or simulations, instead of a single mode a bunch of modes get excited. We believe that space time evolution of these bunch of modes may be explained from our general solution. We have also shown the usefulness of our solution by giving examples of the space-time evolution of square wave, triangular wave and Dawson like initial density profiles. Moreover, we have obtained the breaking criteria for all the above mentioned profiles using the inequality as given in ref.[116]. It is found that square and triangular wave profiles break when their height becomes greater than or equal to 0.5. Our general solution provides the evolution of any arbitrary density profile only below the wave breaking amplitude as beyond the breaking amplitude transformation from Lagrange to Euler coordinates is no longer unique. We also studied the evolution and breaking of two mode case which is again a special case of our general solution. We found that addition of a second harmonic increases the breaking amplitude of the fundamental mode. Note here that a pure sine wave breaks when the amplitude of the normalized density perturbation be-
comes greater than or equal to 0.5. However, we found that if we add a very small perturbation to the second harmonic, oscillations do not break even when amplitude of the fundamental mode is greater than 0.5. Physically this happens because the second mode interfere with the fundamental mode in such a way that the inequality [116] does not satisfied anywhere. The breaking of two mode case has been further verified in 1-D particle in cell simulation. This result may have relevance in wake field acceleration experiments. Furthermore, we have studied the evolution and breaking of a more general two mode case where second mode need not be an integral multiple of the fundamental mode. From this solution we recover the case of Davidson et al. and commensurate mode case for different set of initial conditions.

In the chapter 3, we have studied the behavior of nonlinear oscillations in a cold viscous/hyperviscous and resistive plasma. Note here that the behavior of nonlinear oscillations in a cold viscous and resistive plasma has also been studied by Infeld et al. [118] for an unrealistic model of viscosity; they chose a viscosity coefficient which depends inversely on the density in order to obtain some simplification in the analytic treatment of this problem. They observed two new nonlinear effects: one is that oscillations do not break even beyond the critical amplitude and second one is that for larger value of viscosity coefficient density peak splits into two. It is to be noted here that in reality viscosity has a relatively weak dependence on density through Coulomb logarithm. Therefore we have first studied the evolution of these oscillations for more realistic case where viscosity coefficient is chosen to be independent of density. Later we have studied these oscillations for an alternative dissipative model by replacing viscosity by hyperviscosity. In both the cases, results are found to be qualitatively similar to Infeld et al. [118]. Physically, these nonlinear effects appears due to wave number dependent frequency and damping corrections that lead to interference effects between the various modes. We also found that resistivity alone do not show the splitting effect as the frequency shift introduced by resistivity is wave number independent. Moreover, we have given an analytical expression describing a relation between breaking amplitude and viscosity/hyperviscosity coefficient which clearly show that dissipative effects do not remove the wave breaking completely but enhance the critical amplitude.

In the chapter 4, we have included the relativistic effects in the cold plasma model in order to study the evolution and breaking properties of very large am-
amplitude plasma waves. As we have discussed in the previous chapters that the longitudinal relativistic plasma (AP) waves can be excited in the wake of the ultra-intense ultra-short laser pulse when it goes through underdense plasma [1]. It is the understanding till date that breaking amplitude of longitudinal relativistic plasma wave (AP wave) approaches to infinity as its phase velocity approaches to speed of light. Since these waves can have very large amplitude without breaking, they accelerate particles to very high energy in a distance much shorter than a conventional linear accelerator. Wave breaking formula for these waves is being used in recent particle acceleration experiments/simulations in order to interpret the observations [28, 30]. However, Infeld and Rowlands [107] have shown that all initial conditions, except the one which are needed to excite AP waves, lead to density burst (wave breaking) at an arbitrarily small amplitude. Thus in order to understand the connection between the theories of Akhiezer & Polovin [119] and Infeld & Rowlands [107], we have first obtained the initial conditions which excite AP waves when substituted in the solution of Infeld & Rowlands [107]. We have then loaded these initial conditions in a relativistic code based on Dawson sheet model to study the sensitivity of these waves with respect to some perturbations. We have done this because in a realistic wakefield acceleration experiment, there is always some noise (due to group velocity dispersion of the pulse, thermal effects etc. [132, 133, 134, 126]) along with the AP waves in the wake. We have observed the smooth propagating nature of AP wave in all physical variables up to thousands of plasma periods for pure AP type initial conditions. This was done to show that there is no numerical dissipation visible in our code at least up to thousands of plasma periods. We have then added a small sinusoidal perturbation to the large amplitude AP wave and found that AP wave breaks after a few plasma periods. We have noted here that amplitude of the AP wave was found to be well below the critical amplitude [119] even at the time of breaking.

Now in order to get the scalings which describe the dependence of wave breaking time on the perturbation amplitude and AP wave amplitude respectively, we have further repeated the numerical experiment, first keeping the amplitude of the AP wave as fixed and varying amplitude of the perturbation, later keeping the perturbation amplitude as fixed and varying AP wave amplitude. We found that larger the amplitude of the perturbation or AP wave is, shorter the wave breaking time will be. Thus one has to reduce the noise or work at lower amplitude AP
wave in order to get maximum acceleration. Now, to understand the physics behind this phenomena, we have plotted frequency of sheets as a function of position with and without perturbation. We found that frequency of the system shows a flat dependence for pure AP wave case and acquires a spatial dependence for nonzero perturbation which gets stronger for larger perturbation amplitude. This is a clear signature of phase mixing. It is also shown that the scalings we discussed here, can be interpreted from the Dawson’s formula for phase mixing in inhomogeneous plasma [102]. We have thus shown that, although the ideal breaking amplitude of longitudinal AP waves is very high, they break at arbitrarily low amplitude via phase mixing when perturbed slightly. Thus all those experiments/simulations which use AP wave breaking formula may require revisiting. For example, Malka et al. [28] have observed the generation of 200 MeV electrons in their wake field acceleration experiment. Note here that the authors have used the formula [2] which valid as long as $eE / (m\omega_{pe}c) \leq 1$ in order to interpret their observation. However, in their experiment $eE / (m\omega_{pe}c)$ was approximately 3.8 which is much greater than unity and hence, one needed to use the energy gain formula for nonlinear waves [27]. If we do so, the energy gain would have been approximately 975 MeV. We believe that it is the phase mixing effect which damps the plasma wave well before the full dephasing length and is thus preventing electrons from gaining the full energy.

In chapter 5, we have looked at the phenomena that occur on the long time scale where the effect of ion motion can not be ignored anymore. It is the understanding till date that if we allow ions to move, plasma oscillations phase mix and break at arbitrarily small amplitude due to nonlinearly driven ponderomotive forces only. However, we have shown that it is not only the nonlinearly driven ponderomotive force but also the naturally excited zero frequency mode (which is nothing but the ion acoustic mode in a zero temperature cold plasma) which could be responsible for phase mixing. Actually, if we choose an arbitrary initial condition, solution will be a mixture of high frequency oscillations due to “$\omega_p$” and, DC and secular terms due to ion acoustic mode “0”. However, we can adjust the initial conditions in such a way that only one of the two modes get excited. In this chapter we have first shown how to choose initial conditions such that the zero frequency mode does not get excited and we see pure oscillation in the first order and then phase mixing occurs due to nonlinearly driven ponderomotive forces only. We have also
shown that although the breaking amplitude of cold plasma BGK waves is high $keE/(m\omega_{pe}^2) \sim 1$, they break at arbitrarily small amplitude via phase mixing if ions are allowed to move. This result has been further verified in PIC simulation. Here zero frequency mode of the system is found to be the only candidate responsible for phase mixing because ponderomotive force for waves is zero. Moreover, we have reported nonlinear traveling wave solutions in an arbitrary mass ratio cold plasma which is correct up to second order. These waves do not exhibit phase mixing as for waves ponderomotive force is zero and zero frequency mode is absent here.

In chapter 6, we have studied the physics of plasma oscillations beyond wave breaking. It is the understanding till date that after the wave breaking plasma becomes warm and all energy of the wave goes to the random kinetic energy of the particles [102, 122]. We have studied, a long time evolution of plasma oscillation in the wave breaking regime using 1D PIC simulation and demonstrated that all the coherent ESE does not convert to random energy of particles but a fraction which is decided by the Coffey criterion [121], always remains with the wave which support a trapped particle distribution in the form of oppositely propagating BGK waves. These BGK waves have also been seen in warm plasma [137, 138, 139, 140] using Vlasov simulation well below the breaking amplitude. The randomized energy distribution of the particles is found to be characteristically non-Maxwellian with a preponderance of energetic particles.

In chapter 7, we have studied a full nonlinear treatment of the formation and collapse of double layers in the long scale length limit, using the method of Lagrange variables, and analytically described the early work, using harmonic and void like initial conditions.

**Future scope**

We know that a sine wave, square wave and triangular wave density profiles do not break as long as $keE/(m\omega_{pe}^2) \geq 0.5$. On the other hand, Dawson like initial density profile breaks only when $keE/(m\omega_{pe}^2)$ becomes greater than or equal to unity. Therefore, one needs to understand what is so special about this profile that it does not break even though $keE/(m\omega_{pe}^2)$ is greater than 0.5. We tried to understand this via a two modes case which does not show breaking even when
However, in order to make a more clear connection among the
works carried out by Davidson et al., by us and by Dawson, a better understanding
needs to be developed. Although, we have verified the evolution and breaking
criteria of single mode case, two mode case and Dawson’s case in PIC simulation,
the evolution and breaking of square wave and triangular wave cases still need to
be examined numerically in PIC or sheet simulation.

By taking an example of sinusoidal initial density perturbation, we have shown
that after the wave breaking plasma becomes warm but some fraction of initial
energy always remains with the remnant wave and the final distribution is found
to be non-Maxwellian. This behavior may also be studied for the above mentioned
profiles after verifying their evolution and breaking criteria. We know that the cold
plasma BGK waves [120] breaks when $keE/(m\omega^2_{pe})$ becomes greater than or equal
to unity. Physics of these waves beyond wave breaking needs to be explored as it
may have direct relevance in particle acceleration experiments. It is also interesting
to know the type of distribution functions that will be formed after the breaking
of these waves.

First thing one should note here that we have argued in the study beyond cold
wave breaking that during the evolution of the distribution in the breaking regime
when amplitude of the wave becomes smaller than the breaking amplitude in warm
plasma by Coffey, wave stops breaking and converting coherent energy into heat.
However, Coffey has derived the maximum amplitude in warm plasma for a water
bag distribution function and in our case we found the distribution function bi-
Maxwellian. Thus the wave breaking criteria in warm plasma needs revisiting for
an arbitrary distribution function.

Second thing is that we have given a qualitative interpretation for the process
of acceleration from multistream motion to coherent states. However, for a better
understanding a quantitative analysis, which explains the stochastic acceleration
of particles, is still needed.

We have studied the physics of nonrelativistic plasma oscillations in the presence
of viscosity and resistivity and have derived an analytical formula for Dawson
like initial density profile which describes how the critical amplitude depends on
viscosity and resistivity coefficient. This formula needs to be verified numerically
and one may ask whether this formula is valid for the realistic case also where vis-
cosity coefficient is chosen to be independent of density. Besides, a general formula
for an arbitrary initial condition is still required. Moreover one may explore the effects of viscosity and resistivity on relativistic plasma oscillations. For example, one may ask whether the splitting effects, in the density profile, persist for the relativistic case also. We know that relativistic plasma oscillations always phase mix away. Therefore, the effects of dissipative terms on the phase mixing time may also have some relevance in the particle acceleration experiments.

We have obtained the electron-ion traveling wave solutions correct up to second order in an arbitrary mass ratio cold plasma. An exact solution may be more interesting and then one may study the sensitivity of these waves with respect to some perturbations as these waves also (like AP waves) seem to phase mixed if perturbed slightly. Moreover, for a better understanding, phase mixing of plasma oscillations and waves in an arbitrary mass ratio cold plasma need to be examined experimentally.

We have shown that ideal breaking criteria of relativistic traveling waves in cold plasma does not hold in the presence of perturbations (due to noise). If we perturb these waves slightly they break at arbitrary amplitude after a finite time which is decided by both amplitude of the wave and the perturbation. We have proposed that if the wave breaking time is longer than the dephasing time it will not affect the acceleration process. However, if it is shorter one can never achieve the full expected energy. Therefore in order to support our theory one needs to do a qualitative analysis numerically as well as experimentally. For the relativistic studies, we have kept the ions fixed as we have looked only at short time scales phenomena. Hence one may include the effect of ion motion on the relativistic plasma oscillations and waves for the long time scale studies.

We have studied the behavior of relativistic plasma waves only up to the phase mixing time. Therefore, the behavior of these relativistic plasma waves beyond wave breaking (phase mixing time) needs to be understood. Do they also lead to coherent structures after long time evolution? What kinds of distributions are formed after their breaking? These questions are still unanswered.

We have studied the evolution and collapse of double layers using method of Lagrange variables. When double layer collapse i.e.; near the singularity, the electron and ion density tends to zero, which makes the electron relativistic. Therefore it will be of interest to explore the consequences of relativistic electron nonlinearities.