CHAPTER 5

RELIABILITY ANALYSIS OF TWO IDENTICAL UNIT PARALLEL SYSTEM WITH DISCRETE DISTRIBUTION AND TWO TYPES OF FAILURE
CHAPTER-V

RELIABILITY ANALYSIS OF TWO IDENTICAL UNIT PARALLEL SYSTEM WITH DISCRETE DISTRIBUTION AND TWO TYPES OF FAILURE

The previous chapter was presented to incorporate the concept of discrete distribution. In the previous chapter two identical unit redundant system models with geometric failure and repair time distribution. Initially one unit is operative and other is cold stand by and we consider two types of failure, partial failure and total failure.

In the present chapter, a two identical unit parallel system models with geometric failure and repair time distribution. Initial both unit are operative and we consider two types of failure partial failure and total failure. Various measures of system effectiveness are obtained by using the regenerative point technique. The following reliability characteristic of interest have been obtained

(i) Distribution of time to system failure (TSF) and its mean.
(ii) Pointwise and steady state availability of the system.
(iii) Expected busy period of the repairman in [0, t] and in steady state.
(iv) Expected profit incurred in [0, t] and in steady state.

Few characteristic such as MTSF, system availability and profit in steady state have also been studied through graphs.

5.1 System Description and Assumption

System is analysed under the following assumptions

(i) System consists of two identical unit arranged in a parallel network.
(ii) Each unit has three modes-normal (N), partial failure (PF) and total failure (F). Initially both the units are operative in normal mode and system failure occurs when the both units fail.
(iii) The system is assumed to be in the failed state whether the cause of failure is partial or total.
(iv) A single repairman is available to repair a partially or totally failed unit.
(v) A repaired unit works as good as new.
(vi) The failure and repair time distribution are independent having geometric distribution with parameter \( p \) and \( r \) respectively.
(vii) The failure times of the units are taken as independent random variable.
(viii) The system may go to partially or totally failed states with probability ‘b’ or ‘a’ respectively.
(ix) The failure times of the units are taken as independent random variable.

The system effectiveness are obtain:

(i) Transition probabilities and mean sojourn times in different states.
(ii) Reliability and mean time to system failure.
(iii) Pointwise and steady state availabilities of the system.
(iv) Expected uptime of the system and expected busy period of the repairman during \([0, t]\) and in steady state.
(v) Net expected profit incurred \([0, t]\) and in steady state.

5.2 Notations and States of the Systems

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>Probability that system goes to failed state.</td>
</tr>
<tr>
<td>( b )</td>
<td>Probability that system goes to partially failed state</td>
</tr>
</tbody>
</table>

5.3 Symbols Used for States of the System

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_0 )</td>
<td>unit is in normal mode and operative</td>
</tr>
<tr>
<td>( F_{fr}F_{wr} )</td>
<td>unit is in failure mode and under repair</td>
</tr>
<tr>
<td>( PF_{fr}PF_{wr} )</td>
<td>unit is in partial failure mode and under repair</td>
</tr>
</tbody>
</table>
5.4 Transition Probabilities and Sojourn Times

\[ Q_{01}(t) = \frac{2apq[1-q^{2(t+1)}]}{1-q^2} \]

\[ Q_{02}(t) = \frac{2bpq[1-q^{2(t+1)}]}{1-q^2} \]

\[ Q_{03}(t) = \frac{ap^2[1-q^{2(t+1)}]}{1-q^2} \]

\[ Q_{06}(t) = \frac{bp^2[1-q^{2(t+1)}]}{1-q^2} \]

\[ Q_{10}(t) = \frac{rq[1-(qs)^{t+1}]}{1-qs} \]
The steady state transition probabilities from state $S_i$ to $S_j$ can be obtained from

$P_{01} = \frac{2aq}{1+q}$

$P_{02} = \frac{2bq}{1+q}$

$P_{05} = \frac{ap}{1+q}$

$P_{10} = \frac{rq}{1-qs}$

$P_{11} = \frac{arp}{1-qs}$

$P_{12} = \frac{brp}{1-qs}$

$P_{13} = \frac{bps}{1-qs}$

$P_{20} = \frac{rq}{1-qs}$

$P_{21} = \frac{arp}{1-qs}$

$P_{22} = \frac{brp}{1-qs}$

$Q_{11}(t) = \frac{arp[1-(qs)^{t+1}]}{1-qs}$

$Q_{12}(t) = \frac{brp[1-(qs)^{t+1}]}{1-qs}$

$Q_{13}(t) = \frac{bps[1-(qs)^{t+1}]}{1-qs}$

$Q_{15}(t) = \frac{aps[1-(qs)^{t+1}]}{1-qs}$

$Q_{20}(t) = \frac{rq[1-(qs)^{t+1}]}{1-qs}$

$Q_{21}(t) = \frac{arp[1-(qs)^{t+1}]}{1-qs}$

$Q_{22}(t) = \frac{brp[1-(qs)^{t+1}]}{1-qs}$

$Q_{24}(t) = \frac{bps[1-(qs)^{t+1}]}{1-qs}$

$Q_{26}(t) = \frac{aps[1-(qs)^{t+1}]}{1-qs}$

$Q_{02}(t) = Q_{31}(t) = Q_{32}(t) = Q_{33}(t) = \frac{r[1-s^{(t+1)}]}{1-s}$ (5.1 – 5.15)
5.5 Mean Sojourn Times

Let $T_i$ be the sojourn time in state $S_i (i = 0, 1, 2, 3, 4, 5, 6)$, then mean sojourn time in state $S_i$ is given by

$$\mu_i = \sum_{t=0}^{\infty} P(t_i > t)$$

$$\mu_0 = \sum_{t=0}^{\infty} q^{2t} s^t = \frac{1}{1 - q^2}$$

$$\mu_1 = \mu_2 = \sum_{t=0}^{\infty} q^t s^t = \sum_{t=0}^{\infty} (q s)^t = \frac{1}{1 - qs}$$

$$\mu_3 = \mu_4 = \mu_5 = \mu_6 = \frac{1}{1 - s}$$

(5.29 - 5.31)

Defining $m_j$ as the mean sojourn time of the system in state $S_i$ when the system is to transit into $S_j$, i.e.

$$m_j = \sum_{t=0}^{\infty} t q_{ij}(t)$$

$$m_{01} + m_{02} + m_{05} + m_{06} = q^2 \mu_0$$

$$m_{10} + m_{13} + m_{15} + m_{11} + m_{12} = qs \mu_1$$

$$m_{20} + m_{24} + m_{26} + m_{22} + m_{21} = qs \mu_2$$

$$m_{32} = m_{41} = m_{51} = m_{62} = S \mu_3$$

5.6 Reliability and Mean Time to System Failure

Let $R_i(t)$ be the probability that system works satisfactorily for atleast $t$ epochs ‘cycles’ when it is initially started from operative regenerative state $S_i (i = 0, 1, 2)$. To determine it we regard the failed state $S_3, S_4, S_5, S_6$ as absorbing state.

$$R_0(t) = z_0(t) + q_{01}(t-1) \odot R_1(t-1) + q_{02}(t-1) \odot R_2(t-1)$$

$$R_1(t) = z_1(t) + q_{10}(t-1) \odot R_0(t-1) + q_{11}(t-1) \odot R_1(t-1) + q_{12}(t-1) \odot R_2(t-1)$$
Taking geometric transformation on both sides, we get

$$R^*_0(h) = \frac{N_1(h)}{D_1(h)}$$

where

$$N_1(h) = \begin{vmatrix} z_0^*(h) & -hq_{01}^*(h) & -hq_{02}^*(h) \\ z_1^*(h) & 1-hq_{11}^*(h) & -hq_{12}^*(h) \\ z_2^*(h) & -hq_{21}^*(h) & 1-hq_{22}^*(h) \end{vmatrix}$$

and

$$D_1(h) = \begin{vmatrix} 1 & -hq_{01}^*(h) & -hq_{02}^*(h) \\ -hq_{11}^*(h) & 1-hq_{12}^*(h) & -hq_{12}^*(h) \\ -hq_{20}^*(h) & -hq_{21}^*(h) & 1-hq_{22}^*(h) \end{vmatrix}$$

$$N_1(h) = (1-hq_{11}^*(h))(1-hq_{22}^*(h))z_0^*(h) - h^2 q_{12}^*(h) q_{21}^*(h) z_0^*(h)$$

$$+ hq_{01}^*(h)(1-hq_{22}^*(h))z_1^*(h) z_1^*(h) + h^2 q_{12}^*(h) q_{01}^*(h) z_2^*(h)$$

$$+ h^2 q_{02}^*(h) q_{21}^*(h) z_1^*(h) + h q_{02}^*(h)(1-hq_{11}^*(h)) z_1^*(h)$$

$$D_1(h) = (1-hq_{11}^*(h))(1-hq_{22}^*(h)) - h^2 q_{12}^*(h) q_{12}^*(h) - h^2 q_{01}^*(h) q_{10}^*(h)(1-hq_{22}^*(h))$$

$$- h^3 q_{01}^*(h) q_{20}^*(h) q_{12}^*(h) - h^3 q_{02}^*(h) q_{10}^*(h) q_{21}^*(h) - h^2 q_{20}^*(h) q_{02}^*(h)(1-hq_{11}^*(h))$$

The mean time to system failure

$$\mu_i = \lim_{h \to 1} \frac{N_1(h)}{D_1(h)} - 1 = \frac{N_1}{D_1}$$

where,

$$N_i = \mu_i + (1-\mu_i) (P_{11} + P_{22} - 1) + P_{10}$$

$$D_i = (1 - P_{22}) - P_{11} - P_{10}$$

5.7 Availability Analysis

Let $A_i(t)$ be the probability that the system is up at epoch $t$ when it is initially started from regenerative state $S_i$ by simple probabilistic argument the following recurrence relations are obtained.

$$A_0^*(h) = z_0^*(h) + hq_{01}^*(h) A_1^*(h) + hq_{02}^*(h) A_2^*(h) + hq_{05}^*(h) A_5^*(h) + q_{06}^*(h) A_6^*(h)$$

$$A_1^*(h) = z_1^*(h) + hq_{13}^*(h) A_3^*(h) + hq_{10}^*(h) A_0^*(h) + hq_{11}^*(h) A_1^*(h) + hq_{12}^*(h)$$
\[ A_2^*(h) + hq_{13}(h)A_3^*(h) + hq_{15}(h)A_5^*(h) \]

\[ A_2^*(h) = z_2^*(h) + hq_{20}(h)A_0^*(h) + hq_{21}(h)A_1^*(h) + hq_{22}(h)A_2^*(h) + hq_{24}(h)A_2^*(h) + hq_{26}(h)A_6^*(h) \]

\[ A_3^*(h) = hq_{32}(h)A_2^*(h) \]

\[ A_4^*(h) = hq_{41}(h)A_1^*(h) \]

\[ A_5^*(h) = hq_{51}(h)A_1^*(h) \]

\[ A_6^*(h) = hq_{62}(h)A_2^*(h) \]  

(5.36 – 5.42)

By taking geometric transformation and solving the equation

\[ A_0^*(h) = \frac{N_2(h)}{D_2(h)} \]

where

\[
N_2(h) =
\begin{vmatrix}
 z_0^*(h) & -hq_{01}(h) & -hq_{02}(h) & 0 & 0 & -hq_{05}(h) & -hq_{06}(h) \\
 z_1^*(h) & 1-hq_{11}(h) & -hq_{12}(h) & hq_{13}(h) & 0 & -hq_{15}(h) & 0 \\
 z_2^*(h) & -hq_{21}(h) & 1-hq_{22}(h) & 0 & -hq_{24}(h) & 0 & -hq_{26}(h) \\
 0 & 0 & -hq_{32}(h) & 1 & 0 & 0 & 0 \\
 0 & -hq_{41}(h) & 0 & 0 & 1 & 0 & 0 \\
 0 & -hq_{51}(h) & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & -hq_{62}(h) & 0 & 0 & 0 & 1
\end{vmatrix}
\]

\[ N_2(h) = z_0^*(h)(l - hq_{11}(h)(l - hq_{22}(h)) - h^2 q_{26}(h)q_{62}(h)(l - hq_{11}(h))z_0^*(h) \]

\[ - h^2 q_{12}(h)q_{21}(h)z_0^*(h) - h^3 q_{24}(h)q_{41}(h)z_0^*(h) - h^3 q_{13}(h)q_{21}(h)q_{32}(h) \]

\[ z_0^*(h) = h^4 q_{13}(h)q_{24}(h)q_{32}(h)q_{41}(h)z_0^*(h) - h^2 q_{15}(h)q_{51}(h)(l - hq_{22}(h))z_0^*(h) \]

\[ + h^2 q_{51}(h)q_{62}(h)q_{15}(h)q_{26}(h)z_0^*(h) + hq_{01}(h)(l - hq_{22}(h))z_1^*(h) \]

\[ - h^3 q_{01}(h)q_{26}(h)q_{42}(h)z_1^*(h) + h^2 q_{01}(h)q_{12}(h)z_2^*(h) + h^3 q_{01}(h)q_{13}(h)q_{32}(h)z_2^*(h) \]

\[ + h^2 q_{02}(h)q_{21}(h)z_1^*(h) + h^3 q_{24}(h)q_{02}(h)q_{41}(h)z_1^*(h) + hq_{02}(h)(l - hq_{11}(h)) \]

\[ - h^3 q_{02}(h)q_{15}(h)q_{51}(h)z_2^*(h) - h^3 q_{05}(h)q_{21}(h)q_{62}(h)z_1^*(h) \]
\[ -h^4 q_{24}(h) q_{05}(h) q_{41}(h) q_{02}(h) z_1(h) - h^2 q_{05}(h) q_{02}(h) (1 - h q_{11}(h)) z_2(h) \]

\[ + h^4 q_{06}(h) q_{15}(h) q_{51}(h) q_{62}(h) z_2(h) + h^3 q_{06}(h) q_{42}(h) q_{21}(h) \]

\[ + h^2 q_{06}(h) q_{62}(h) (1 - h q_{11}(h)) z_2(h) + h^4 q_{51}(h) q_{62}(h) q_{06}(h) q_{15}(h) z_2(h) \]

and

\[ D_2(h) = \]

\[
\begin{pmatrix}
1 & -h q_{01}(h) & -h q_{02}(h) & 0 & 0 & -h q_{05}(h) & -h q_{06}(h) \\
-h q_{01}(h) & 1 - h q_{11}(h) & -h q_{12}(h) & h q_{13}(h) & 0 & -h q_{15}(h) & -h q_{16}(h) \\
-h q_{20}(h) & -h q_{21}(h) & 1 - h q_{22}(h) & 0 & -h q_{24}(h) & 0 & -h q_{26}(h) \\
0 & 0 & -h q_{32}(h) & 1 & 0 & 0 & 0 \\
0 & -h q_{41}(h) & 0 & 0 & 1 & 0 & 0 \\
0 & -h q_{51}(h) & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & -h q_{62}(h) & 0 & 0 & 0 & 1
\end{pmatrix}
\]

\[ D_2(h) = (1 - h q_{11}(h))(1 - h q_{22}(h)) - h^2 q_{26}(h)(1 - h q_{11}(h)) q_{62}(h) \]

\[ - h^2 q_{12}(h) q_{21}(h) - h^3 q_{12}(h) q_{24}(h) q_{41}(h) - h^3 q_{13}(h) q_{21}(h) q_{32}(h) \]

\[ - h^4 q_{13}(h) q_{24}(h) q_{32}(h) q_{41}(h) - h^2 q_{15}(h) q_{51}(h) (1 - h q_{22}(h)) \]

\[ + h^4 q_{15}(h) q_{26}(h) q_{51}(h) q_{62}(h) - h^2 q_{01}(h) q_{10}(h) (1 - h q_{22}(h)) \]

\[ + h^4 q_{01}(h) q_{10}(h) q_{26}(h) q_{62}(h) - h^3 q_{01}(h) q_{12}(h) q_{20}(h) \]

\[ - h^3 q_{01}(h) q_{13}(h) q_{20}(h) q_{32}(h) - h^2 q_{02}(h) q_{10}(h) q_{21}(h) \]

\[ - h^3 q_{02}(h) q_{10}(h) q_{24}(h) q_{41}(h) - h^2 q_{02}(h) q_{20}(h) (1 - h q_{11}(h)) \]

\[ + h^4 q_{02}(h) q_{13}(h) q_{20}(h) q_{51}(h) - h^2 q_{05}(h) q_{10}(h) q_{51}(h) (1 - h q_{22}(h)) \]

\[ + h^4 q_{05}(h) q_{10}(h) q_{26}(h) q_{51}(h) q_{62}(h) - h^4 q_{05}(h) q_{12}(h) q_{20}(h) q_{51}(h) \]

\[ - h^4 q_{05}(h) q_{13}(h) q_{20}(h) q_{32}(h) q_{51}(h) - h^4 q_{06}(h) q_{10}(h) q_{21}(h) q_{62}(h) \]

\[ - h^4 q_{06}(h) q_{20}(h) q_{62}(h) (1 - h q_{11}(h)) + h^4 q_{06}(h) q_{15}(h) q_{20}(h) q_{51}(h) q_{62}(h) \]

\[ - h^4 q_{10}(h) q_{24}(h) q_{06}(h) \]

and

\[ Z_0(t) = q^{z_1}, z_1(t) = z_2(t) = (q s)^t \]
Z_3(t) = z_4(t) = z_5(t) = z_6(t) = s'

Hence,

z_i^*(t) = \mu_i

The steady state availability of the system is given by

A_0 = \lim_{t \to \infty} A_0(t)

we get

A_0 = \frac{-N_2(1)}{D_2(1)} \tag{5.43}

where

N_2(1) = (1-P_{11})(1-P_{22})+\mu_0\{P_{26}(P_{11}+P_{15})-9(P_{12}+P_{13})(P_{21}+P_{24})+P_{15}(P_{22}-1)\}

+ \mu_1\{1+P_{06}P_{15}-2P_{05}\}

and

D_2(1) = -\{\mu_0P_{20}q^2+\mu_1q_4(1-2P_{20}(P_{06}+P_{02}))\}

Now, the expected uptime of the system up epoch t is given by

\mu_{up}(t) = \sum_{x=0}^{t} A_0(x)

So that

\mu_{up}^*(h) = \frac{A_0^*(h)}{1-h}

5.8 Busy Period Analysis

By probabilistic arguments, we have the following recursive relation for

B_i(t)

B_0(t) = q_0(t-1) \odot B_1(t-1) + q_02(t-1) \odot B_2(t-1) + q_05(t-1) \odot B_3(t-1) + q_06(t-1) \odot B_6(t-1)

B_1(t) = z_1(t) + q_10(t-1) \odot B_0(t-1) + q_11(t-1) \odot B_1(t-1) + q_12(t-1) \odot B_2(t-1) + q_13(t-1) \odot B_3(t-1) + q_15(t-1) \odot B_5(t-1)

B_2(t) = z_2(t) + q_20(t-1) \odot B_0(t-1) + q_22(t-1) \odot B_2(t-1) + q_24(t-1) \odot B_4(t-1) + q_26(t-1) \odot B_6(t-1) + q_21(t-1) \odot B_1(t-1)
By taking geometric transformation and solving the equation

\[ B_6^*(h) = \frac{N_3(h)}{D_2(h)} \]

where,

\[
D_2(h) =
\begin{bmatrix}
0 & -h_{q_0}^*(h) & -h_{q_0}^*(h) & 0 & 0 & -h_{q_0}^*(h) & -h_{q_0}^*(h) \\
1-h_{q_1}^*(h) & -h_{q_1}^*(h) & h_{q_1}^*(h) & 0 & -h_{q_1}^*(h) & 0 & 0 \\
-1-h_{q_2}^*(h) & 1-h_{q_2}^*(h) & 0 & -h_{q_2}^*(h) & 0 & -h_{q_2}^*(h) & 0 \\
0 & -h_{q_3}^*(h) & 1 & 0 & 0 & 0 & 0 \\
-1-h_{q_4}^*(h) & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & -h_{q_6}^*(h) & 0 & 0 & 0 & 1 & 1 \\
\end{bmatrix}
\]

\[
N_3(h) = h_{q_0}^*(h)(1-h_{q_2}^*(h))z_1^*(h) - h_{q_0}^*(h)q_{26}^*(h)q_{62}^*(h)z_6^*(h) \\
+ h_{q_0}^*(h)q_{12}^*(h)z_2^*(h) + h_{q_0}^*(h)q_{12}^*(h)q_{24}^*(h) + h_{q_0}^*(h)q_{12}^*(h)q_{26}^*(h)z_6^*(h) \\
+ h_{q_0}^*(h)q_{13}^*(h)(1-h_{q_2}^*(h))z_3^*(h) + h_{q_0}^*(h)q_{13}^*(h)q_{24}^*(h)q_{32}^*(h)z_4^*(h) \\
- h_{q_0}^*(h)q_{13}^*(h)q_{26}^*(h)q_{62}^*(h)q_{32}^*(h)z_4^*(h) \\
+ h_{q_0}^*(h)q_{15}^*(h)q_{26}^*(h)q_{62}^*(h)q_{26}^*(h)q_{32}^*(h)z_6^*(h) \\
+ h_{q_0}^*(h)q_{21}^*(h)z_1^*(h) + h_{q_4}^*(h)q_{24}^*(h)q_{26}^*(h)z_1^*(h) \\
+ h_{q_0}^*(h)(1-h_{q_1}^*(h))z_2^*(h) + h_{q_0}^*(h)q_{24}^*(h)(1-h_{q_1}^*(h))z_4^*(h) \\
+ h_{q_0}^*(h)q_{26}^*(h)(1-h_{q_1}^*(h))z_6^*(h) + h_{q_0}^*(h)q_{13}^*(h)q_{21}^*(h)z_5^*(h) \\
+ h_{q_0}^*(h)q_{13}^*(h)q_{26}^*(h)q_{13}^*(h)q_{21}^*(h)z_5^*(h) \\
+ h_{q_0}^*(h)q_{15}^*(h)q_{21}^*(h)z_5^*(h) - h_{q_0}^*(h)q_{15}^*(h)q_{45}^*(h)q_{51}^*(h)z_2^*(h) \\
+ h_{q_0}^*(h)q_{15}^*(h)q_{24}^*(h)q_{41}^*(h)z_3^*(h) - h_{q_0}^*(h)q_{15}^*(h)q_{45}^*(h)q_{51}^*(h)z_4^*(h) \\
+ h_{q_0}^*(h)q_{15}^*(h)q_{24}^*(h)q_{41}^*(h)z_5^*(h) - h_{q_0}^*(h)q_{15}^*(h)q_{45}^*(h)q_{51}^*(h)z_5^*(h)
\]
+ h^2 q_{05}(h)(1 - hq_{22}(h))q_{31}(h) - h^4 q_{05}(h)q_{26}(h)q_{51}(h)q_{62}(h)z_1(h)
+ h q_{05}(h)(1 - hq_{11}(h))(1 - hq_{22}(h))z_5(h) - h^3 q_{05}(h)q_{26}(h)q_{62}(h)
(1 - hq_{11}(h))z_5(h) + h^4 q_{05}(h)q_{12}(h)q_{41}(h)z_5(h) - h^3 q_{05}(h)q_{12}(h)q_{21}(h)q_{5}(h)
+ h^4 q_{05}(h)q_{24}(h)q_{12}(h)q_{41}(h)z_5(h) + h^4 q_{05}(h)q_{12}(h)q_{41}(h)z_5(h)
+ h^4 q_{02}(h)q_{12}(h)q_{26}(h)q_{51}(h)z_6(h) + h^4 q_{05}(h)q_{13}(h)q_{32}(h)q_{51}(h)z_2(h)
+ h^4 q_{05}(h)q_{13}(h)q_{32}(h)q_{51}(h)z_5(h) + h^3 (1 - hq_{22}(h))q_{51}(h)q_{65}(h)q_{13}(h)z_5(h)
+ h^5 q_{05}(h)q_{13}(h)q_{24}(h)q_{32}(h)q_{51}(h)z_5(h) - h^5 q_{05}(h)q_{13}(h)q_{24}(h)q_{32}(h)q_{41}(h)z_5(h)
+ h^5 q_{05}(h)q_{13}(h)q_{24}(h)q_{32}(h)q_{51}(h)z_5(h) + h^5 q_{05}(h)q_{13}(h)q_{51}(h)z_6(h)
+ h^2 q_{02}(h)q_{21}(h)z_1(h) + h^3 q_{02}(h)q_{24}(h)q_{41}(h)z_1(h)
+ h q_{02}(h)(1 - hq_{11}(h))z_1(h) + h^2 q_{02}(h)q_{24}(h)(1 - hq_{11}(h))z_4(h)
+ h^2 q_{02}(h)q_{26}(h)(1 - hq_{11}(h))z_6(h) + h^3 q_{02}(h)q_{13}(h)q_{21}(h)z_3(h)
+ h^4 q_{02}(h)q_{13}(h)q_{24}(h)q_{41}(h)z_3(h) + h^3 q_{02}(h)q_{13}(h)q_{51}(h)z_5(h)
- h^4 q_{02}(h)q_{15}(h)q_{24}(h)q_{51}(h)z_5(h) + h^4 q_{02}(h)q_{15}(h)q_{24}(h)q_{41}(h)z_5(h)
- h^4 q_{02}(h)q_{15}(h)q_{26}(h)q_{51}(h)z_6(h)
and

D_2(h) is the same as in availability analysis.

In the long run, the probability that the repair facility is busy in repair of
failed unit is given by

\[ B_0(t) = \lim_{t \to \infty} B_0(t) \]

Hence, by applying 'L' Hospital Rule

\[ B_0(t) = -\frac{N_3(1)}{D_2(1)} \quad (5.51) \]

where

\[ N_3(1) = \{P_{01} + P_{05}(P_{12} + 2P_{15} - P_{26}) + P_{05}(P_{21} + P_{12} + 2P_{24} +1) + P_{13}P_{01}\} \mu_1 \]
+ \{P_{01}(1 + P_{15} + 2P_{13}) - P_{01}P_{22}(P_{13} + P_{15}) - P_{22}P_{01} + 2P_{021}P_{24} \]
+ P_{02} P_{26} + P_{05} - P_{05} P_{22} - P_{02} P_{24} P_{11} - P_{11} P_{05} + P_{12} P_{26} P_{02} \\
- P_{05} P_{21} P_{13} + P_{24} + P_{24} P_{05} P_{12} \mu_3 + P_{05} (1 + P_{24}) - P_{02} P_{15} P_{26}

and $D'_2(l)$ is the same as in availability analysis.

### 5.9 Profit Function Analysis

The expected total profit in steady-state is

$$P = C_0 A_0 - C_1 B_0$$

(5.52)

where

- $C_0$ : be the per unit up time revenue by the system
- $C_1$ : be the per unit down time expenditure on the system

### 5.10 Particular Case

$$P_{01} = \frac{2aq}{1+q}, \quad P_{02} = \frac{2bq}{1+q}$$

$$P_{05} = \frac{ap}{1+q}, \quad P_{06} = \frac{bp}{1+q}$$

$$P_{10} = P_{20} = \frac{rq}{(1-qs)}, \quad P_{32} = P_{41} = P_{51} = P_{62} = 1$$

$$P_{13} = P_{26} = \frac{bps}{1-qs}, \quad P_{11} = P_{21} = \frac{arp}{1-qs}$$

$$P_{12} = P_{22} = \frac{brp}{1-qs}, \quad P_{15} = P_{24} = \frac{aps}{1-qs}$$

$$\mu_0 = \frac{1}{1-q^2}, \quad \mu_1 = \mu_2 = \frac{1}{1-qs}, \quad \mu_3 = \mu_5 = \mu_6 = \mu_4 = \frac{1}{1-s}$$

Using the above equation and equation (5.35), (5.43), (5.51) and (5.52) we can have the expression for MTSF, availability etc. for this particular case.

On the basis of the numerical values taken as:

- $P = 0.01$, $r = 0.25$, $a = 0.45$, $b = 0.55$.

The values of various measures of system effectiveness are obtained as:

- Mean time to system failure (MTSF) = 3432.836
- Availability ($A_0$) = 0.71897.
- Busy period of analysis of repairman ($B_0$) = 0.12597

For the graphical interpretation, the mentioned particular case is considered.
Figs. 5.2 show the behaviour of MTSF with respect to failure rate (p). It is clear from the graph that the MTSF get decrease with increase in the value of failure rate.

Fig. 5.3 shows the behaviour MTSF with respect to repair rate (r). It is clear from the graph that the MTSF gets increase with increase in the value of repair rate.

Fig. 5.4 reveals the pattern of the profit with respect to failure rate (p) for different values of repair rate (r). The profit decreases with the increase in the value of failure rate (p) and is higher for higher values of repair rate (r). For $r = 0.25$, $P \geq 0$ according as $p \leq 0.535$. So, the system is profitable only if failure rate is less than 0.535.
MTSF Vs FAILURE RATE(p)

- \( r = 0.25 \)
- \( r = 0.5 \)
- \( r = 0.75 \)

\( a = 0.45, b = 0.55 \)

Fig. 5.2
MTSF Vs REPAIR RATE (r)

- \( p = 0.25 \)
- \( p = 0.50 \)
- \( p = 0.75 \)

\[ a = 0.45, b = 0.55 \]

Fig 5.3
PROFIT Vs FAILURE RATE (p)

$r=0.5$  \hspace{1cm}  $r=0.25$

$a=0.45, b=0.55, c_0=500, c_1=100$

Fig. 5.4