CHAPTER 4

ANALYSIS OF TWO IDENTICAL UNIT STANDBY SYSTEM WITH DISCRETE DISTRIBUTION AND TWO-TYPES OF FAILURE
CHAPTER-IV
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FAILURE

4.1 Introduction

In the field of reliability many researcher has worked on reliability modeling. In the literature of reliability, a large number of authors including Gupta and Kumar [1997], Kumar and Render [1993], Tuteja and Taneja [1992], Goyal and Kumar [2006], Gupta and Goel[1989], Kumar and Kumar[2007], Kumar, et al. [2008] and others have been analysed two and more units repairable system models assuming continuous distribution of time to failure and time to repair of a unit.

In some situation, discrete failure time distributions are appropriate to model “lifetimes”. For example, discrete distribution is appropriate when a piece of equipment operates in cycles and the number of cycles prior to failure is observed.

Discrete failure data arise in several common situations, for example: a) A device is monitored only once per time period (e.g, an hour, a day), and the observation is the number of time periods successfully completed prior to failure of the device. b) A piece of equipment operates in cycles and the experimenter observes the number of cycles successfully completed prior to failure.

In situations where the observed data values are very large (in thousands of cycles, etc.) a continuous distribution is an adequate model for the discrete random variable. However when the observed values are small, continuous distribution might not adequately describe a discrete random variable.

The purpose of the chapter is to incorporate the concept of discrete distribution. Generally consider that failure and repair times of a unit continuous distribution of time to failure and time to repair of a unit. However, in practice the situation exists when the failure and repair of a unit occur at discrete random
epochs so that the life time and repair time of a unit follows discrete distribution like geometric, negative binomial, poission etc.

Some examples of discrete life times are as follows:

"In a photo copy machine the bulb is lightened every time when a copy is taken. Thus, the life time of the bulb is a discrete random variable".

"In an on/off switching device, the life time of the switch is a discrete random variable".

"In a refrigerator, the bulb is lightened whenever the door of the refrigerator is opened. Thus, the life time of the bulb is the discrete random variable".

Thus, we see that many practical situations of importance are represented with the help of discrete life time models. Similarly, the repair time may be considered as discrete random variable as dividing the whole time interval into various small parts of time. Discrete time models are considered by several researchers including Padgett and Spurrier [1985], Salvia and Bollinger [1982], J.F.Lawless [1982], Kalbfleish and Prentice [1980] etc.

Keeping the above concept of discrete time modeling, we in the present chapter analyse two, identical unit redundant system models with geometric failure and repair time distribution. Initially one unit is operative and other is cold standby and total failure of the unit is via partial failure mode. The various important measures of system effectiveness are obtained by using the regenerative point technique.

4.2 System Description and Assumption

System is analysed under the following assumption

(i) A cold standby system consists of two-identical units-operative and standby. Each unit has three modes: normal (N), partial failure (PF) and total failure (F). The standby unit can not fail.

(ii) The system is assumed to be in the failed state whether the cause of failure is partial or total.

(iii) Failures are self-announcing.
(iv) A single repairman is available to repair a partially or totally failed unit.

(v) Switching for going to operating state from standby state is perfect and instantaneous.

(vi) The failure and repair time distribution are independent having geometric distribution with parameter $p$ and $r$ respectively.

(vii) The failure times of the units are taken as independent random variable.

(viii) The system may go to partially failed or totally failed states with probability “b or a” respectively.

The system effectiveness are obtain:

1. Transition probabilities and mean sojourn times in different states.
2. Reliability and mean time to system failure
3. Pointwise and steady state availabilities of the system.
4. Expected uptime of the system and expected busy period of the repairman during $(0, t]$ and in steady state.
5. Net expected profit incurred $(0, t)$ and in steady state.

4.3 Notations and States of the Systems

$a$ : Probability that system goes to failed state.

$b$ : Probability that system goes to partially failed state.

4.4 Symbols used for States of the System

$N_0$ : Unit is in normal mode and operative

$N_s$ : Unit is in normal mode and standby

$F/F_{wr}$ : Unit is in failure mode and under repair/waiting for repair.

$PF_r/PF_{wr}$ : Unit is in partial failure mode and under repair/waiting for repair

Up States

$S_0 = (N_0, N_s), S_1 = (F_r, N_0), S_2 = (PF_r, N_0)$

Down State

$S_3 = (F_r, F_{wr}), S_4 = (PF_r, PF_{wr})$

$S_5 = (F_r, PF_{wr}), S_6 = (PF_r, F_{wr})$
4.5 Transition Probabilities and Sojourn Times

\[
Q_{01}(t) = ap \frac{1 - q^t}{1 - q}, \quad Q_{02}(t) = bp \frac{1 - q^t}{1 - q},
\]

\[
Q_{11}(t) = arp \frac{1 - (qs)^t}{1 - qs}, \quad Q_{13}(t) = aps \frac{1 - (qs)^t}{1 - qs},
\]

\[
Q_{10}(t) = rq \frac{1 - (qs)^t}{1 - qs}, \quad Q_{15}(t) = bps \frac{1 - (qs)^t}{1 - qs},
\]

\[
Q_{12}(t) = brp \frac{1 - (qs)^t}{1 - qs}, \quad Q_{20}(t) = rq \frac{1 - (qs)^t}{1 - qs},
\]

\[
Q_{22}(t) = brp \frac{1 - (qs)^t}{1 - qs}, \quad Q_{21}(t) = arp \frac{1 - (qs)^t}{1 - qs},
\]
The steady state transition probabilities from state $S_i$ to $S_j$ can be obtained from

$$P_{ij} = \lim_{t \to \infty} Q_{ij}$$

It can be verified that

$P_{01} + P_{02} = 1$, $P_{11} + P_{10} + P_{12} + P_{15} + P_{13} = 1$, $P_{22} + P_{20} + P_{21} + P_{24} + P_{26} = 1$

$P_{31} = P_{42} = P_{52} = P_{61} = 1$

4.6 Mean Sojourn Times

Let $T_i$ be the sojourn time in state $S_i$ ($i = 0, 1, 2$) then mean sojourn time in state $S_i$ is given by

$$\mu_i = \sum_{t=0}^{\infty} P(T_i > t)$$

So that

$$\mu_0 = \frac{1}{1-q}, \quad \mu_1 = \mu_2 = \frac{1}{1-q_s}, \quad \mu_3 = \mu_4 = \mu_5 = \mu_6 = \frac{1}{1-s}$$

(4.14-4.16)

Defining $m_{ij}$ as the mean sojourn time of the system in state $S_i$ when the system is to transit into state $S_j$ i.e.

$$m_{ij} = \sum_{t=0}^{\infty} t \, q_{ij}(t)$$

We obtained

$$m_{01} + m_{02} = q \mu_0 \quad m_{11} + m_{10} + m_{12} + m_{13} + m_{15} = (qs) \mu_1$$

$$m_{22} + m_{20} + m_{21} + m_{24} + m_{26} = (qs) \mu_2 \quad m_{31} = m_{52} = m_{42} = m_{61}$$

(4.17-4.20)

4.7 Mean Time to System Failure

Let $R_i(t)$ be the probability that the system works satisfactorily for at least $t$ epochs when it is initially started from operative regenerative state $S_i$ ($i = 0, 1, 2$)

$$R_0(t) = Z_0(t) + q_{01}(t) R_1(t-1) + q_{02}(t-1) R_2(t-1)$$

$$R_i(t) = Z_i(t) + q_{10}(t-1) R_0(t-1) + q_{11}(t-1) R_1(t-1) + q_{12}(t-1) R_2(t-1)$$

$$R_{01}(t) = Z_{01}(t) + q_{10}(t-1) R_0(t-1) + q_{11}(t-1) R_1(t-1) + q_{12}(t-1) R_2(t-1)$$

$$R_{02}(t) = Z_{02}(t) + q_{20}(t-1) R_0(t-1) + q_{21}(t-1) R_1(t-1) + q_{22}(t-1) R_2(t-1)$$

$$R_{11}(t) = Z_{11}(t) + q_{10}(t-1) R_0(t-1) + q_{11}(t-1) R_1(t-1) + q_{12}(t-1) R_2(t-1)$$

$$R_{12}(t) = Z_{12}(t) + q_{20}(t-1) R_0(t-1) + q_{21}(t-1) R_1(t-1) + q_{22}(t-1) R_2(t-1)$$

$$R_{22}(t) = Z_{22}(t) + q_{20}(t-1) R_0(t-1) + q_{21}(t-1) R_1(t-1) + q_{22}(t-1) R_2(t-1)$$
\[ R_2(t) = Z_2(t) + q_{20}(t-1) \circ R_0(t-1) + q_{22}(t-1) \circ R_2(t-1) + q_{21}(t-1) \circ R_1(t-1) \]

Taking Geometric Transform on both sides, we get

\[ R_0^*(h) = \frac{N_1(h)}{D_1(h)} \]

\[ D_1(h) = [1 - q_{11}^*(h)][1 - hq_{22}^*(h)] - h^2q_{12}^*(h)q_{21}^*(h) - h^2q_{01}^*(h)q_{10}^*(h) \]
\[ \times [1 - hq_{22}^*(h)] - h^2q_{12}^*(h)q_{20}^*(h)q_{01}^*(h) - h^2q_{02}^*(h)q_{12}^*(h)q_{21}^*(h) \]
\[ - h^2q_{02}^*(h)q_{20}^*(h)(1 - hq_{11}^*(h)) \]

\[ N_1(h) = Z_0^*(h)[1 - hq_{11}^*(h)][1 - hq_{22}^*(h)] - h^2q_{21}^*(h)q_{12}^*(h)Z_0^*(h) + hq_{01}^*(h) \]
\[ (1 - hq_{22}^*(h))Z_1^*(h) + h^2q_{01}^*(h)q_{12}^*(h)Z_2^*(h) + h^2q_{02}^*(h)q_{21}^*(h)Z_1^*(h) + hq_{02}^*(h)(1 - hq_{11}^*(h))Z_2^*(h) \]

Then,

\[ MTSF = \frac{N_1}{D_1} \]

\[ D_1 = (1 - P_{11}) (1 - P_{22}) - P_{12} P_{21} - P_{10} P_{10} (1 - P_{22}) - P_{12} P_{20} P_{01} - P_{02} P_{10} P_{21} \]
\[ - P_{02} P_{20} (1 - P_{11}) \]

\[ N_1 = (1 - \mu_0) [P_{12} P_{21} - (1 - P_{11}) (1 - P_{22})] + (\mu_1 + P_{10}) (P_{01} (1 - P_{22}) + P_{02} P_{21}) \]
\[ + [(\mu_2 + P_{20}) P_{02} (1 - P_{11}) + P_{12} P_{01}] \]

### 4.8 Availability Analysis

Let \( A_i(t) \) be the probability that the system is up at epoch \( t \) when it is initially started from regenerative state \( S_i \). By simple probabilistic argument the following recurrence relations are obtained

\[ A_0(t) = Z_0(t) + q_{01}(t-1) \circ A_1(t-1) + q_{02}(t-1) \circ A_2(t-1) \]
\[ A_1(t) = Z_1(t) + q_{10}(t-1) \circ A_0(t-1) + q_{11}(t-1) \circ A_1(t-1) + q_{13}(t-1) \circ A_3(t-1) \]
\[ + q_{15}(t-1) \circ A_5(t-1) + q_{12}(t-1) \circ A_2(t-1) \]
\[ A_2(t) = Z_2(t) + q_{20}(t-1) \circ A_0(t-1) + q_{21}(t-1) \circ A_1(t-1) + q_{23}(t-1) \circ A_3(t-1) \]
\[ + q_{24}(t-1) \circ A_4(t-1) + q_{26}(t-1) \circ A_6(t-1) \]
\[ A_3(t) = q_{31}(t-1) \circ A_1(t-1) \]
\[ A_4(t) = q_{42}(t-1) \circ A_2(t-1) \]
\[ A_5(t) = q_{52}(t-1) \circ A_2(t-1) \]
\[ A_6(t) = q_6(t-1) \oplus A_1(t-1) \]

By taking geometric transformation solving the equations.

\[ A_0^*(h) = \frac{N_2(h)}{D_2(h)} \]

Where

\[ N_2(h) = Z_6^*(h)[(1 - hq_{11}^*(h))(1 - hq_{22}^*(h))] - h^2q_{24}^*(h)(1 - hq_{11}^*(h))q_{42}^*(h)Z_6^*(h) \]

\[ - h^2q_{21}^*(h)q_{12}^*(h)Z_0^*(h) - h^3q_{26}^*(h)q_{12}^*(h)q_{61}^*(h)Z_0^*(h) - h^2q_{31}^*(h)q_{13}^*(h) \]

\[ (l - hq_{22}^*(h))Z_0^*(h) + h^4q_{42}^*(h)q_{31}^*(h)q_{24}^*(h)q_{13}^*(h) - h^3q_{21}^*(h) \]

\[ q_{15}^*(h)q_{52}^*(h)Z_0^*(h) - h^4q_{26}^*(h)q_{15}^*(h)q_{61}^*(h)q_{52}^*(h) + h(l - hq_{22}^*(h)) \]

\[ q_{01}^*(h)Z_1^*(h) - h^2q_{24}^*(h)q_{42}^*(h)q_{01}^*(h)Z_1^*(h) + h^2q_{01}^*(h)q_{12}^*(h)Z_2^*(h) \]

\[ + h^3q_{01}^*(h)q_{15}^*(h)q_{52}^*(h)Z_2^*(h) + h^2q_{21}^*(h)q_{02}^*(h)Z_1^*(h) + h^3q_{26}^*(h) \]

\[ q_{02}^*(h)q_{46}^*(h)Z_1^*(h) + hq_{02}^*(h)(1 - hq_{11}^*(h))Z_2^*(h) - h^3q_{13}^*(h)q_{31}^*(h) \]

\[ q_{02}^*(h)Z_2^*(h) \]

\[ D_2(h) = (1 - hq_{11}^*(h))(1 - hq_{22}^*(h)) - h^2q_{24}^*(h)q_{42}^*(h)(1 - hq_{11}^*(h)) - h^2q_{12}^*(h) \]

\[ q_{21}^*(h) - h^2q_{26}^*(h)q_{12}^*(h)q_{61}^*(h) - h^2q_{13}^*(h)q_{31}^*(h)(l - hq_{22}^*(h)) + h^4q_{42}^*(h)q_{31}^*(h)q_{24}^*(h)q_{13}^*(h) \]

\[ q_{24}^*(h)q_{13}^*(h)q_{31}^*(h) - h^3q_{15}^*(h)q_{21}^*(h)q_{52}^*(h) - h^4q_{15}^*(h)q_{26}^*(h) \]

\[ q_{52}^*(h) - h^2q_{01}^*(h)q_{01}^*(h)(1 - hq_{22}^*(h)) + h^4q_{01}^*(h)q_{10}^*(h)q_{24}^*(h) \]

\[ q_{42}^*(h) - h^3q_{12}^*(h)q_{01}^*(h)q_{20}^*(h) - h^4q_{15}^*(h)q_{01}^*(h)q_{20}^*(h)q_{42}^*(h) - h^3q_{02}^*(h)q_{40}^*(h)q_{21}^*(h) - h^4q_{02}^*(h)q_{10}^*(h)q_{26}^*(h)q_{61}^*(h) - h^2q_{02}^*(h)q_{20}^*(h) \]

\[ (1 - hq_{11}^*(h)) + h^4q_{31}^*(h)q_{02}^*(h)q_{13}^*(h)q_{20}^*(h) \]

Where,

\[ Z_0(t) = q^i, \quad Z_2(t) = Z_1(t) = (qs)^i, \quad Z_3(t) = Z_4(t) = Z_5(t) = Z_6(t) = s^i \]

Hence,

\[ Z_i^*(t) = \mu_i \]

The steady state availability of the system is given by

\[ A_0 = \lim_{t \to \infty} A_0(t) \]
Hence, Applying ’L’Hospital Rule, we get

\[ A_0 = -\frac{N_2(1)}{D_2'(1)} \]  

(4.32)

Where

\[ N_2(1) = (1 - p_{22} - p_{24}) (\mu_0 (1 - p_{11}) + \mu_1 p_{01}) + (p_{21} + p_{26}) (\mu_1 p_{02} - \mu_0 p_{12}) - p_{15} p_{21} \mu_0 - p_{13} (1 - p_{22}) \mu_0 + \mu_2 p_{01} (p_{12} + p_{15}) + \mu_2 p_{02} (1 - p_{11} - p_{13}) + p_{13} p_{24} - p_{26} p_{15} \]

To obtain \( D_2'(1) \)

\[ D_2'(1) = -[q_3 \mu_1 (1 - p_{22} - p_{24} - p_{20} p_{02}) + q_4 \mu_0 (p_{20} (p_{12} + p_{15}) - p_{10} (p_{24} - p_{22})) + q_3 \mu_2 (1 - p_{11} - p_{13} - p_{01} p_{10}) + q_3 \mu_3 [p_{24} (1 - p_{11} - p_{01} p_{10}) + p_{15} (p_{01} p_{20}) + (p_{21} + p_{26}) + p_{13} (1 - p_{22} - p_{24} - p_{02} p_{20}) + p_{26} (p_{12} + p_{02} p_{10})] \]

Now, the expected uptime of the system up epoch t is given by

\[ \mu_{up}(t) = \sum_{x=0}^{t} A_0(x) \]

So that

\[ \mu_{up}^*(h) = \frac{A_0^*(h)}{1-h} \]

4.9 Busy Period Analysis

Let \( B_i(t) \) be the probability that the repair facility is busy in repair of failed unit when the system initially starts from regenerative state \( S_i \). Using simple probabilistic arguments, as in case of reliability and availability analysis the following recurrence relations can be easily developed.

\[ B_0(t) = q_{01}(t-1) \circ B_1(t-1) + q_{02}(t-1) \circ B_2(t-1) \]
\[ B_1(t) = Z_1(t) + q_{10}(t-1) \circ A_0(t) + q_{11}(t-1) \circ B_1(t-1) + q_{12}(t-1) \circ A_2(t-1) + q_{13}(t-1) \circ B_3(t-1) + q_{15}(t-1) \circ A_5(t-1) \]
\[ B_2(t) = Z_2(t) + q_{20}(t-1) \circ B_0(t-1) + q_{22}(t-1) \circ B_2(t-1) + q_{23}(t-1) \circ B_3(t-1) + q_{24}(t-1) \circ B_4(t-1) + q_{26}(t-1) \circ B_6(t-1) \]
\[ B_3(t) = Z_3(t) + q_{31}(t-1) \circ B_1(t-1) \]
\[ B_4(t) = Z_4(t) + q_{42}(t-1) \circ B_2(t-1) \]
\[ B_5(t) = Z_5(t) + q_{52}(t-1) \circ B_2(t-1) \]
\[ B_0(t) = Z_0(t) + q_{61}(t-1) \otimes B_1(t-1) \]  
(4.33 - 4.39)

By taking geometric transformation and solving the equations

\[
B_0^*(h) = \frac{N_3(h)}{D_2(h)}
\]

Where,

\[
N_3(h) = h q_{01}^*(h)(1 - h q_{22}^*(h)) - h^3 q_{01}^*(h) q_{24}^*(h) q_{42}^*(h) Z_1^*(h) + h^2 q_{01}^*(h) q_{12}^*(h)
\]

\[
Z_2^*(h) + h^3 q_{01}^*(h) q_{12}^*(h) q_{24}^*(h) Z_4^*(h) + h^3 q_{01}^*(h) q_{12}^*(h) q_{26}^*(h) Z_6^*(h) + h^2 q_{02}^*(h) q_{24}^*(h) Z_2^*(h) + h^2 q_{02}^*(h) q_{26}^*(h) Z_4^*(h) + h^2 q_{02}^*(h) q_{61}^*(h) Z_6^*(h) + h^2 q_{02}^*(h) q_{61}^*(h) Z_7^*(h)
\]

\[
q_{01}^*(h) q_{13}^*(h)(1 - h q_{22}^*(h)) Z_3^*(h) - h^4 q_{01}^*(h) q_{13}^*(h) q_{24}^*(h) q_{42}^*(h) Z_3^*(h) + h^2 q_{02}^*(h) q_{21}^*(h) Z_2^*(h) + h^2 q_{02}^*(h) q_{26}^*(h) Z_4^*(h) + h^2 q_{02}^*(h) q_{61}^*(h) Z_6^*(h) + h^2 q_{02}^*(h) q_{61}^*(h) Z_7^*(h)
\]

and

\[
D_2(h) \text{ is the same as in availability analysis.}
\]

In the long run, the probability that the repair facility is busy in repair of failed unit is given by

\[
B_0(t) = \lim_{t \to \infty} B_0(t)
\]

Hence, by applying 'L'Hospital Rule

\[
B_0 = \frac{N_3(l)}{D_2'(l)}
\]  
(4.40)

Where

\[
N_3(l) = \rho_{01}(1 - \rho_{22}) + \mu_1 (\rho_{02} (p_{21} + p_{26}) - \rho_{01} p_{24}) + \mu_2 (\rho_{01} p_{12} - \rho_{02} (p_{11} + p_{13} - 1))
\]

\[
+ \mu_3 \{(p_{26} + p_{24}) (p_{01} p_{12} + p_{02} (1 - p_{11})) + p_{02} (p_{21} + p_{26}) (p_{13} + p_{15})\}
\]

and \( D_2'(l) \) is the same as in availability analysis.

### 4.10 Profit Function Analysis

The expected total profit in steady-state is
\[ P = C_0A_0 - C_1B_0 \]  

(4.41)

where

\[ C_0 \] be the per unit up time revenue by the system and \[ C_1 \] be the per unit down time expenditure on the system.

### 4.11 Particular Case

\[ P_{01} = a, \quad P_{02} = b, \quad P_{11} = P_{21} = \frac{arp}{1 - q}, \quad P_{13} = P_{26} = \frac{aps}{1 - qs} \]

\[ P_{10} = P_{20} = \frac{rq}{1 - qs}, \quad P_{12} = P_{22} = \frac{brp}{1 - qs}, \quad P_{15} = P_{24} = \frac{bps}{1 - qs} \]

Using the above equation and equation (4.24), (4.36), (4.40) and (4.41) we can have the expression for MTSF, availability etc. for this particular case.

On the basis of the numerical values taken as:

\[ P = 0.1, \quad r = 0.25, \quad a = 0.45, \quad b = 0.55. \]

The values of various measures of system effectiveness are obtained as:

**Mean time to system failure (MTSF)** = 52.33

**Availability** \( (A_0) \) = 0.23758.

**Busy period of analysis of repairman** \( (B_0) \) = 0.754556

For the graphical interpretation, the mentioned particular case is considered.

**Figs. 4.2** show the behaviour of MTSF with respect to failure rate \( (p) \). It is clear from the graph that the MTSF get decrease with increase in the value of failure rate.

**Fig. 4.3** shows the behaviour MTSF with respect to repair rate \( (r) \). It is clear from the graph that the MTSF gets increase with increase in the value of repair rate.
Fig. 4.4 reveals the pattern of the profit with respect to failure rate \((p)\) for different values of repair rate \((r)\). The profit decreases with the increase in the value of failure rate \((p)\) and is higher for higher values of repair rate \((r)\). Following can also be observed from the graph:

(i) For \(r = 0.5\), \(P > \text{ or } = \text{ or } < 0\) according as \(p < \text{ or } = \text{ or } > 0.7356\). So, the system is profitable only if failure rate is less than 0.7356.

(ii) For \(r = 0.75\), \(P > \text{ or } = \text{ or } < 0\) according as \(p < \text{ or } = \text{ or } > 0.8098\). Thus the system is not profitable when \(> 0.8098\).

So, the companies using such systems can be suggested to purchase only those system which do not have failure rates greater than those discussed in points (i) to (iii) above in this particular case.

Fig. 4.5 shows the pattern of the profit with respect to repair rate \((r)\) for different values of failure rate \((p)\). The profit increases with the increase in the value of repair rate \((r)\) and is lower for higher values of failure rate \((p)\).
MTSF Vs FAILURE RATE (p)

- \( r=0.25 \)
- \( r=0.5 \)
- \( r=0.75 \)

\[ a = 0.45, \, b = 0.55 \]

Fig. 4.2
MTSF vs Repair Rate ($r$)

- $p = 0.25$
- $p = 0.50$
- $p = 0.75$

$a = 0.45, b = 0.55$

Fig 4.3
PROFIT Vs FAILURE RATE (p)

- $r=0.5$
- $r=0.75$

$a = 0.45, b = 0.55, C_0 = 1000, C_1 = 200$. 

Fig. 4.4
PROFIT VS REPAIR RATE ($r$)

- $p=0.25$
- $p=0.5$

$a = 0.45$, $b = 0.55$, $C_0 = 500$, $C_1 = 50$. 

Fig. 4.5