CHAPTER 3

ANALYSIS OF RELIABILITY MODELS WITH IMPERFECT SWITCHING DEVICE, CONNECTION TIME AND TWO TYPES OF FAILURE
CHAPTER-III

ANALYSIS OF RELIABILITY MODELS WITH IMPERFECT SWITCHING DEVICE, CONNECTION TIME AND TWO TYPES OF FAILURE

3.1 Introduction

The previous chapter was presented to incorporate the concept of imperfect switching device and connection time with single repairman facility.

In the present chapter, a two-unit cold standby system with imperfect switching device having a single repairman is analysed and two types of failure, minor failure and total failure.

Taking the above facts in view, the present chapter is devoted to analyse two reliability models have been discussed i.e model-I and model II, useful in every day life.

Model-I consists of two-units, the main unit and the standby unit. It is assume that the main unit can work in normal mode i.e. with full efficiency. The single repairman facility is available with the system to repair the totally failed unit, minor failed unit and switch (first priority of the repairman is to repair the failed switch). For both the units, the distribution of time for failure is taken as negative exponential and the time taken for repairing is termed as general. Model-II also consists of two units, the main unit and the standby unit. In this model we consider the concept of disconnecting time and connecting time. The single repairman facility is available with the system to repair the totally failed unit, minor failed unit, disconnect the damaged switch and connect the new switch. For both the units, the distribution of time for failure is taken as negative exponential while the distribution of time for repairing is taken as general. The distribution of time taken to connect or disconnect the switch from the unit is assumed to be negative exponential and the distribution of time taken in replacement of the damaged switch with a new switch is general, the first priority of the repairman is to replace the damaged switch. Using
regenerative point technique, the following reliability characteristics of interest have been obtained.

(i) Distribution of time to system failure and its mean.
(ii) Pointwise and steady state availability of the system.
(iii) Expected busy period of repairman in \((0, t]\) and in steady state.
(iv) Expected number of visits by the repairman in \((0, t]\) and in steady state.
(v) Expected profit incurred in \((0, t]\) and in steady state.

Few characteristics such as MTSF, system availability and profit in steady state have also been studied through graphs.

### 3.2 Mathematica Treatment for Model-I

(i) A cold standby system consists of two-identical units-operative and standby. Each unit has three modes: normal \((N)\), minor failure \((MF)\) and total failure \((F)\). The standby unit cannot fail.

(ii) The system is assumed to be in the failed state whether the cause of failure is minor or total.

(iii) Failure is self-announcing.

(iv) A single repairman is available to repair a failed unit, minor failed unit and switch.

(v) Switching is imperfect in the transition from standby state to operating state.

(vi) First priority of the repairman is to repair the failed switch.

(vii) The failure time distribution of each unit is negative exponential while repair time distribution of the repairman is arbitrary.

(viii) The repair time distribution of the switch is general.

(ix) The detection of a failed unit is instantaneous and perfect.

(x) The unit works as a new one after repaired.

(xi) All the random variables are independent.

Symbols and notations used for the system states:

Various symbols used to represent the states of the system are –

\(N_0, N_s\) : Unit normal mode and operative/unit in normal mode and
standby.

F_r : Unit in failure mode and repair continued from the earlier state.
FR : Unit in failure mode and repair continued from the earlier state.
F_wr : Unit in failure mode and waiting for repair.
MF_r : Unit in minor failure mode and repair continued from the earlier state.
MF_wr : Unit in minor failure mode and repair continued from the earlier state.
SFN_s : Switch failed and unit in standby state.
Sr : Switch in failure mode and under repair

Following the above symbols, the possible states of the system model are

<table>
<thead>
<tr>
<th>Up states</th>
<th>Down states</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_0 = (N_0, N_s)</td>
<td>S_3 = (F_r, F_wr), S_4 = (MF_r, F_wr)</td>
</tr>
<tr>
<td>S_1 = (MF_r, N_0)</td>
<td>S_5 = (F_wr, SFN_s), S_6 = (F_r, MF_wr)</td>
</tr>
<tr>
<td>S_2 = (F_r, N_0)</td>
<td>S_7 = (MF_wr, SFN_s), S_8 = (MF_r, MF_wr)</td>
</tr>
</tbody>
</table>

Fig. 1 represents the state transition diagram of the system model, further let

p : Probability that switch is perfect
q : Probability that switch is imperfect.
\( \alpha \) : Constant failure rate of operative units from its normal to total failure mode.
\( \beta \) : Constant failure rate of operative units from its normal to minor failure mode.

G_1(t), G_1(t) : p.d.f. and c.d.f. of time to repair of a failed unit.
G_2(t), G_2(t) : p.d.f. and c.d.f. of time to repair of a minor failed unit.
G_3(t), G_3(t) : p.d.f. and c.d.f. is the repair time of the switch by the repairman.

P_{ij} : Transition probability from regenerative state S_i to S_j.
\( p_{ij}^{(k)} \) : Probability that the system transit from regenerative state S_i to S_j passing through the non-regenerative state k.
\( \mu_i \) : Mean Sojourn time in state S_i.
3.3 Transition Probabilities and Sojourn Times

The state transition diagram as in Fig. 3.1 states 3, 4, 5, 6, 7 and 8 are failed states. The epochs of entry into states 0, 1, 2, 5, 7 are regenerative points. The transition probabilities form the state $S_i$ to $S_j$ are given as follows:

$$Q_{01}(t) = \int_0^1 p\beta e^{-(\alpha+\beta)u} \, du$$

$$Q_{02}(t) = \int_0^1 p\alpha e^{-(\alpha+\beta)u} \, du$$

*Fig. 3.1*
The steady state transition probabilities are given as:

\[ P_{ij} = \lim_{t \to \infty} Q_{ij}(t) = \lim_{s \to 0} q_{ij}^*(s) \]
\[ P_{01} = \frac{p\beta}{\alpha + \beta}, \quad P_{10} = g_2^*(\alpha + \beta) \]

\[ P_{02} = \frac{p\alpha}{\alpha + \beta}, \quad P_{14} = P_{12}^4 = \frac{\alpha}{\alpha + \beta} (1 - g_2^* (\alpha + \beta)) \]

\[ P_{05} = \frac{q\alpha}{\alpha + \beta}, \quad P_{18} = P_{11}^8 = \frac{\beta}{\alpha + \beta} (1 - g_2^* (\alpha + \beta)) \]

\[ P_{07} = \frac{q\beta}{\alpha + \beta}, \quad P_{26} = P_{21}^6 = \frac{\beta}{\alpha + \beta} (1 - g_1^* (\alpha + \beta)) \]

\[ P_{20} = g_1^*(\alpha + \beta) \]

\[ P_{23} = \frac{p}{\alpha + \beta} (1 - g_1^* (\alpha + \beta)) \] \hspace{1cm} (3.16-3.25)

By these probabilities, we can verify:

\[ P_{01} + P_{02} + P_{05} + P_{07} = 1 \]

\[ P_{10} + P_{18} + P_{14} = 1 \]

\[ P_{10} + P_{11}^8 + P_{12}^4 = 1 \]

\[ P_{20} + P_{23} + P_{26} = 1 \]

\[ P_{20} + P_{22}^3 + P_{21}^6 = 1 \]

\[ P_{22} = P_{71} = 1 \] \hspace{1cm} (3.26-3.31)

Mean Sojourn time \( (\mu_i) \) in regenerative state \( i \) are

\[ \mu_i = \lim_{t \to \infty} \int_0^1 P(t) \left[ 0 < t < T \right] dt \]

\[ \mu_0 = \frac{1}{\alpha + \beta} \]

\[ \mu_1 = \frac{1}{\alpha + \beta} \{1 - g_2^* (\alpha + \beta)\} \]

\[ \mu_5 = \mu_7 = \int_0^\infty G_3(t) dt = -g_3^*(0) \] \hspace{1cm} (3.32-3.34)

The conditional mean time taken by the system transit from any regenerative state 'j', it is counted from epoch of entrance into the state 'i'.

\[
\int_0^\infty t \, d \, Q_0(t) = \left[ \frac{d}{ds} q_0^*(s) \right]_{s=0}
\]

Thus

\[
\begin{align*}
m_{01} + m_{02} + m_{05} + m_{07} &= \mu_0 \\
m_{10} + m_{14} + m_{18} &= \mu_1 \\
m_{10} + m_{12}^4 + m_{11}^8 &= K_2 \\
m_{20} + m_{23} + m_{26} &= \mu_2 \\
m_{20} + m_{22}^3 + m_{21}^5 &= K_1
\end{align*}
\]...(3.35-3.39)

3.4 Mean Time to System Failure

By probabilistic arguments, we obtain the following recursive relation for \( \phi_i(t) \):

\[
\begin{align*}
\phi_0(t) &= Q_0(t) \cdot \phi_1(t) + Q_{02}(t) \cdot \phi_2(t) + Q_{05}(t) + Q_{07}(t) \\
\phi_1(t) &= Q_{10}(t) \cdot \phi_0(t) + Q_{14}(t) + Q_{18}(t) \\
\phi_2(t) &= Q_{20}(t) \cdot \phi_0(t) + Q_{23}(t) + Q_{23}(t)
\end{align*}
\]...(3.40-3.42)

Taking Laplace-Stieltjes transforms (L.S.T.) of above relation and solving \( \phi_0**(s) \), the mean time to system failure when the system starts from the state ‘0’ is given by

\[
MTSF = \lim_{s \to 0} \frac{1 - \phi_0**(s)}{s} = \frac{N_1}{D_1}
\]...(3.43)

where

\[
\begin{align*}
N_1 &= \mu_0 + \mu_1 P_{01} + \mu_2 P_{02} \\
D_1 &= 1 - P_{01} P_{10} - P_{02} P_{20}
\end{align*}
\]

3.5 Availability Analysis

The availability \( A_i(t) \) is seen to satisfy the following recursive relations:

\[
\begin{align*}
A_0(t) &= M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) + q_{05}(t) \odot A_5(t) + q_{07}(t) \odot A_7(t) \\
A_1(t) &= M_1(t) + q_{10}(t) \odot A_0(t) + q_{11}(t) \odot A_1(t) + q_{11}(t) \odot A_1(t) \\
A_2(t) &= M_2(t) + q_{20}(t) \odot A_0(t) + q_{21}(t) \odot A_1(t) + q_{22}(t) \odot A_2(t) \\
A_5(t) &= q_{52}(t) \odot A_2(t) \\
A_7(t) &= q_{71}(t) \odot A_1(t)
\end{align*}
\]...(3.44-3.48)

where
Taking Laplace Transform (L.T) of the above equations, we get

\[
A_0^*(s) = \frac{N_2(s)}{D_2(s)}
\]

where,

\[
N_2(s) = \\
\begin{bmatrix}
M_0^*(s) & -q_{01}(s) & -q_{02}(s) & -q_{03}(s) & -q_{07}(s) \\
M_1^*(s) - q_{11}(s) & 1 - q_{12}^*(s) & -q_{14}(s) & 0 & 0 \\
M_2^*(s) - q_{21}^*(s) & 1 - q_{22}^*(s) & 0 & 0 \\
0 & 0 & -q_{52}(s) & 1 & 0 \\
0 & 0 & -q_{71}(s) & 0 & 0 & 1
\end{bmatrix}
\]

and

\[
D_2(s) = \\
\begin{bmatrix}
1 & -q_{01}(s) & -q_{02}(s) & -q_{03}(s) & -q_{07}(s) \\
-q_{10}(s) & 1 - q_{11}(s) & -q_{14}(s) & 0 & 0 \\
-q_{20}(s) & -q_{21}^*(s) & 1 - q_{22}^*(s) & 0 & 0 \\
0 & 0 & -q_{52}(s) & 1 & 0 \\
0 & 0 & -q_{71}(s) & 0 & 0 & 1
\end{bmatrix}
\]

\[
N_2(s) = M_0^*(s) \left(1 - q_{11}^*(s) \right) \left(1 - q_{22}^*(s) \right) - q_{12}^*(s)q_{21}^*(s) + q_{01}(s) \left(1 - q_{22}^*(s) \right)M_1^*(s) \\
+ q_{01}(s)q_{12}^*(s)M_2^*(s) + q_{02}(s)q_{21}^*(s)M_1^*(s) + q_{02}(s) \left(1 - q_{11}^*(s) \right)M_2^*(s) \\
+ q_{03}(s)q_{21}^*(s)q_{52}(s)M_1^*(s) + q_{03}(s)q_{52}(s) \left(1 - q_{11}^*(s) \right)M_2^*(s) \\
- q_{07}(s)q_{21}^*(s)q_{52}(s)M_1^*(s) - q_{07}(s)q_{52}(s) \left(1 - q_{11}^*(s) \right)M_2^*(s)
\]

\[
D_2(s) = \left(1 - q_{11}^*(s) \right) \left(1 - q_{22}^*(s) \right) - q_{12}^*(s)q_{21}^*(s) - q_{10}(s)q_{10}(s) \left(1 - q_{22}^*(s) \right) \\
- q_{12}^*(s)q_{20}(s)q_{10}(s) - q_{02}(s)q_{21}(s) - q_{02}(s)q_{20}(s) \left(1 - q_{11}^*(s) \right) \\
- q_{12}^*(s)q_{10}(s)q_{10}(s)q_{10}(s) - q_{12}^*(s)q_{21}(s) - q_{05}(s)q_{20}(s)q_{20}(s) \left(1 - q_{11}^*(s) \right) \\
- q_{05}(s)q_{10}(s)q_{21}(s)q_{52}(s) - q_{05}(s)q_{20}(s)q_{52}(s) \left(1 - q_{11}^*(s) \right) \\
- q_{07}(s)q_{71}(s)q_{10}(s) \left(1 - q_{22}^*(s) \right) - q_{07}(s)q_{12}(s)q_{20}(s)q_{71}(s)
\]

In steady state, the availability of the system is given by

\[
A_0 = \lim_{s \to 0} A_0^*(s) = \frac{N_2}{D_2} \quad \text{...(3.49)}
\]
where
\[ N_2 = \left( 1 - P_{22}^3 \right) \left[ \mu_1 P_{01}^* + \mu_0 (1 - P_{11}^8) \right] + \left[ (P_{02} + P_{05} - P_{07}) (\mu_1 P_{21}^6 + \mu_2 (1 - P_{11}^6)) \right] \]
\[ + \left[ P_{12}^4 (\mu_2 P_{01} - \mu_0 P_{21}^6) \right] \]

and
\[ D_2 = \mu_0 (P_{10} P_{21}^6 - P_{11}^8 P_{20} + P_{20}) + K_1 (1 - P_{11}^8 - P_{10} P_{01} - P_{10} P_{07}) \]
\[ + K_2 (1 - P_{22}^3 - P_{20} (P_{05} + P_{02}) - P_{20} P_{07}) \]

3.6 Busy Period Analysis

By probabilistic arguments, we have the following recursive relation for \( B_i(t) \)

\[ B_0(t) = q_{01}(t) \odot B_1(t) + q_{02}(t) \odot B_2(t) + q_{05}(t) \odot B_5(t) + q_{07}(t) \odot B_7(t) \]

\[ B_1(t) = W_1(t) + q_{10}(t) \odot B_0(t) + q_{12}(t) \odot B_2(t) + q_{15}(t) \odot B_5(t) \]

\[ B_2(t) = W_2(t) + q_{20}(t) \odot B_0(t) + q_{21}(t) \odot B_1(t) + q_{22}(t) \odot B_2(t) \]

\[ B_5(t) = W_5(t) \]

\[ B_7(t) = W_7(t) + q_{71}(t) \odot B_1(t) \]

\[ \ldots (3.50-3.54) \]

where,
\[ W_1(t) = e^{-(\alpha + \beta) t} G_2(t) \]
\[ W_2(t) = e^{-(\alpha + \beta) t} G_1(t) \]
\[ W_7(t) = W_5(t) = G_3(t) \]

Taking Laplace transform of the above equation and solving them for \( B_0^*(s) \), we get

\[ B_0^*(s) = \frac{N_3(s)}{D_2(s)} \]

where
\[ N_3(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & -q_{05}^*(s) & -q_{07}^*(s) \\ W_1^*(s) & 1 - q_{11}^*(s) & -q_{12}^*(s) & 0 & 0 \\ W_2^*(s) & -q_{21}^*(s) & 1 - q_{22}^*(s) & 0 & 0 \\ W_5^*(s) & 0 & -q_{32}^*(s) & 1 & 0 \\ W_7^*(s) & -q_{71}^*(s) & 0 & 0 & 1 \end{vmatrix} \]

\[ N_5(s) = q_{01}^*(s) (1 - q_{32}^*(s)) W_1^*(s) + q_{12}^*(s) W_2^*(s) + q_{02}^*(s) q_{21}^*(s) W_1^*(s) \]
\[ + q_{02}^*(s) (1 - q_{11}^*(s)) W_2^*(s) + q_{05}^*(s) q_{21}^*(s) q_{32}^*(s) W_1^*(s) \]
\[ + q_{05}^*(s) q_{32}^*(s) (1 - q_{41}^*(s)) W_2^*(s) + q_{05}^*(s) (1 - q_{11}^*(s)) (1 - q_{32}^*(s)) W_1^*(s) \]
and $D_2(s)$ already specified.

In steady state, the total fraction of time which the system is under repair of the repairman, is given by

$$B_0 = \lim_{s \to 0} s B_0^*(s) = \frac{N_3}{D_2} \quad \ldots (3.55)$$

where

$$N_3 = (1 - P_{22}^4) \mu_2 + (P_{02} + P_{05}) \{\mu_2 (1 - P_{11}^8) - \mu_1 P_{20}\}$$

$$+ P_{12}^4 (\mu_2 + P_{07} - P_{07} P_{12}^4)$$

$$+ (1 - P_{11}^8) (1 - P_{22}^4) \{\mu_2 P_{07} + \mu_7 P_{07}\}$$

and $D_2(s)$ is already specified.

3.7 Expected Number of Visits by the Repairman

We have the following recursive relation

$$V_0(t) = Q_{01}(t) \cdot S \cdot (1 + V_1(t)) + Q_{02}(t) \cdot S \cdot (1 + V_2(t)) + Q_{05}(t) \cdot S \cdot (1 + V_5(t))$$

$$+ Q_{07}(t) \cdot S \cdot [1 + V_7(t)]$$

$$V_1(t) = Q_{10}(t) \cdot S \cdot V_0(t) + Q_{12}^4 \cdot S \cdot V_2(t) + Q_{11}^8 \cdot S \cdot V_1(t)$$

$$V_2(t) = Q_{20}(t) \cdot S \cdot V_0(t) + Q_{22}^2 \cdot S \cdot V_2(t) + Q_{21}^6 \cdot S \cdot V_1(t)$$

$$V_5(t) = Q_{52}(t) \cdot S \cdot V_2(t)$$

$$V_7(t) = Q_{71}(t) \cdot S \cdot V_1(t) \quad \ldots (3.56-3.60)$$

Taking Laplace Stieltjes transform (L.S.T.) of the above equation and solving them for $V_0^{**}(s)$, we get

$$V_0^{**}(s) = \frac{N_4(s)}{D_2(s)},$$

where

$$N_4(s) = 
\begin{bmatrix}
Q_{01}^{**}(s) + Q_{02}^{**}(s) + Q_{05}^{**}(s) + Q_{07}^{**}(s) & -Q_{05}^{**}(s) & -Q_{02}^{**}(s) & -Q_{07}^{**}(s) \\
0 & 1 - Q_{11}^{**}(s) & -Q_{12}^{**}(s) & 0 & 0 \\
0 & -Q_{21}^{**}(s) & 1 - Q_{22}^{**}(s) & 0 & 0 \\
0 & 0 & -Q_{52}^{**}(s) & 1 & 0 \\
0 & -Q_{71}^{**}(s) & 0 & 0 & 1
\end{bmatrix}$$

$$N_4(s) = \{Q_{01}^{**}(s) + Q_{02}^{**}(s) + Q_{05}^{**}(s) + Q_{07}^{**}(s)\} \{(1 - Q_{11}^{**}(s))(1 - Q_{22}^{**}(s))\}$$
and $D_2(s)$ already specified.

In steady state, the total number of visits by the repairman per-unit time are given by

$$V_0 = \lim_{t \to \infty} \left[ \frac{V_0(t)}{t} \right] = \lim_{s \to 0} [s V_0](s) = \frac{N_4}{D_2} \quad \ldots (3.61)$$

where,

$$N_4 = (1 - P_8^8)(1 - P_{22}^3) - P_{12}^4 P_{21}^6$$

and $D_2$ already specified.

3.8 Profit Analysis

The expected total profit earned by the system in Steady-state is given by

$$P_{21} = C_0 A_0 - C_1 B_0 - C_2 V_0 \quad \ldots (3.62)$$

where,

$C_0$ is the revenue per-unit up time

$C_1$ is the cost per-unit time for which the repairman is busy.

$C_3$ is the cost per-visits by the repairman.

3.9 Particular Case

For graphical interpretation, the following particular case is considered:

$$g_1(t) = \gamma e^{-\gamma t}, \quad g_2(t) = \delta e^{-\delta t}, \quad g_3(t) = \lambda e^{-\lambda t}$$

Thus, we can easily obtained the following

$$P_{02} = \frac{p\alpha}{\alpha + \beta}, \quad P_{10} = \frac{1}{\alpha + \beta + \delta}, \quad P_{14} = \frac{p\alpha}{\alpha + \beta + \delta}$$

$$P_{01} = \frac{p\beta}{\alpha + \beta}, \quad P_{18} = \frac{p\beta}{\alpha + \beta + \delta}, \quad P_{12} = \frac{\beta}{\alpha + \beta + \delta}$$

$$P_{05} = \frac{q\alpha}{\alpha + \beta}, \quad P_{20} = \frac{1}{\alpha + \beta + \gamma}, \quad P_{26} = \frac{p\beta}{\alpha + \beta + \gamma}$$

$$P_{07} = \frac{q\beta}{\alpha + \beta}, \quad P_{23} = \frac{p\beta}{\alpha + \beta + \gamma}, \quad K_1 = \frac{1}{\gamma}, \quad K_2 = \frac{1}{\delta}$$
\[
\begin{align*}
\mu_0 &= \frac{1}{\alpha + \beta}, \quad \mu_1 = \frac{1}{\alpha + \beta + \delta}, \quad \mu_2 = \frac{1}{(\alpha + \beta + \gamma)}, \quad \mu_5 = \mu_7 = \frac{1}{\lambda}
\end{align*}
\]

Using the above equation (3.43), (3.49), (3.55), (3.61) and (3.62), we can have the expression MTSF, availability etc. for this particular case.

On the basis of the numerical values taken as:

\[
\alpha = 0.1, \quad \beta = 0.45, \quad \gamma = 0.25, \quad p = 0.65, \quad q = 0.35, \quad \delta = 0.89, \quad \lambda = 0.95
\]

The values of various measures of system effectiveness are obtained as:

- **Mean time to system failure (MTSF)** = 4.835294
- **Availability (Ao)** = 0.410121.
- **Busy period of analysis of repairman (Bo)** = 0.463868
- **Expected number of visits by the repairman (Vo)** = 0.182892

For the graphical interpretation, the mentioned particular case is considered.

Figs. 3.2 and 3.3 show the behaviour of MTSF and availability respectively with respect to failure rate (\(\alpha\)). It is clear from the graph that the MTSF and the availability both get decrease with increase in the value of failure rate.

Fig. 3.4 shows the behaviour of availability respectively with respect to repair rate (\(\gamma\)). It is clear from the graph that the availability gets increase with increase in the value of repair rate.
MTSF VS FAILURE RATE ($\alpha$)

\[ \delta = 0.89, \lambda = 0.95, \beta = 0.45, p = 0.65, q = 0.35. \]
AVAILABILITY VS FAILURE RATE (\(\alpha\))

\[
\delta = 0.89, \lambda = 0.95, \beta = 0.45, \\
p = 0.65, q = 0.35.
\]
AVAILABILITY VS REPAIR RATE ($\gamma$)

$\alpha = 0.25 \quad \alpha = 0.3 \quad \alpha = 0.4$

$\delta = 0.89, \lambda = 0.95, \beta = 0.45, p = 0.65, q = 0.35.$

Fig 3.4
Fig. 3.5 reveals the pattern of the profit with respect to failure rate (α) for different values of repair rate (γ). The profit decreases with the increase in the value of failure rate (α) and is higher for higher values of repair rate (γ).

\[ \delta = 0.89, \lambda = 0.95, \beta = 0.45, p = 0.65, \]
\[ q = 0.35, C_0 = 700, C_1 = 200, C_2 = 100. \]
Fig. 3.6 shows the pattern of the profit with respect to repair rate (γ) for different values of failure rate (α). The profit increases with the increase in the value of repair rate (γ) and is lower for higher values of failure rate (α).

\[ \delta = 0.89, \lambda = 0.95, \beta = 0.45, p = 0.65, q = 0.35, C_0 = 700, C_1 = 200, C_2 = 100. \]
3.10 Mathematical Treatment of Model-II
(i) A cold standby system consists of two-identical units-operative and standby. Each unit has three modes: normal (N), minor failure (MF) and total failure (F). The standby unit can not fail.

(ii) The system is assumed to be in the failed state whether the cause of failure is minor or total.

(iii) Failure is self-announcing.

(iv) A single repairman is available to repair a failed unit, minor failed unit and to connect/disconnect the switch to/from the unit.

(v) Switching is imperfect in the transition from standby state to operating state.

(vi) First priority of the repairman is to replace the damaged switch.

(vii) The failure time distribution of each unit is negative exponential while repair time distribution of the repairman is arbitrary.

(viii) The distribution of time taken to connect or disconnect the switch from the unit is assumed to be negative exponential and the distribution of time taken in replacement of the damaged switch with a new one is general.

(ix) The detection of a failed unit is instantaneous and perfect.

(x) The unit works as a new one after repaired.

(xi) All the random variables are independent.

3.11 Symbols for States of the System and Notations
\[ N_0, N_s \] : Unit normal mode and operative/unit in normal mode and standby.

\[ F_r \] : Unit in failure mode and under repair.

\[ F_R \] : Unit in failure mode and repair continued from the earlier state.

\[ F_{wt} \] : Unit in failure mode and waiting for repair.

\[ MF_r \] : Unit in minor failure mode and under repair.

\[ MF_R \] : Unit in minor failure mode and repair continued from the earlier state.
MF_{wr} : Unit in minor failure mode and waiting for repair.
SFN_s : Switch failed and unit in standby state.
S_d : Switch is damaged.
S_N : Switch is new.

Using these symbols for switching device and for both the units, the system may be in one of the following states.

**Up States**

\[ S_0 = (N_0, N_0); \quad S_1 = (MF_r, N_0); \quad S_2 = (F_r, N_0) \]

**Down States**

\[ S_3 = (F_R, F_{wr}); \quad S_4 = (MF_R, F_{wr}); \quad S_5 = (F_{wr}, SFN_3); \quad S_6 = (F_R, MF_{wr}); \]
\[ S_7 = (MF_{wr}, SFN_3); \quad S_7 = (MF_{wr}, SFN_5); \quad S_8 = (MF_R, MF_{wr}); \]
\[ S_9 = (MF_{wr}, SFN_3); \quad S_{10} = (MF_{wr}, SFN_5); \quad S_{11} = (F_{wr}, SFN_3); \quad S_{12} = (F_{wr}, SFN_5) \]

**Regenerative State** \( (S_0, S_1, S_2, S_5, S_7, S_9, S_{10}, S_{11}, S_{12}) \)

**Non-Regenerative States** \( (S_3, S_4, S_6, S_8) \)

### 3.12 Other Symbols

- \( p \) : The probability that switch is perfect
- \( q \) : The probability that switch is imperfect.
- \( \alpha \) : Constant failure rate of operative unit from its normal to total failure mode.
- \( \beta \) : Constant failure rate of operative unit from its normal to minor failure mode.
- \( \eta \) : The common parameter connecting/disconnecting time distribution of switch to/from the unit.
- \( G(\cdot) \) : c.d.f. of time taken by the repairman to replace the damaged switch with new switch.
- \( g_1(t), G_1(t) \) : p.d.f. and c.d.f. of time to repair of a failed unit.
- \( g_2(t), G_2(t) \) : p.d.f. and c.d.f. of time to repair of a minor failed unit.
- \( P_{ij} \) : Steady state transition probability from state from \( S_i \) to \( S_j \),
  
  \[ (S_j, S_i \in E) \]
  
  \[ = Q_{ij}(\infty) = \int_0^{\infty} q_{ij}(t)dt \]
3.13 Transition Probability and Sojourn Time

By simple probabilistic argument, one can obtain the one step transition probabilities as follows.

For the system to reach state $S_1$ from $S_0$ on or before time $t$, we suppose that the system transits from $S_0$ to $S_1$ during the time interval $(u, u + du)$; $u < t$. It will happen only when the system has not transmitted to $S_2$, $S_5$ or $S_7$ upto time $u$. The probability of this event is $p\beta e^{-(\alpha+\beta)u} du$. Since $u$ varies from 0 to $t$, therefore

$$Q_{01}(t) = \int_0^t P\{\text{system transits from state } S_0 \text{ to } S_1 \text{ during } (u, u + du)\}$$

$$= \int_0^t p\beta e^{-(\alpha+\beta)u} du = \frac{p\beta}{\alpha + \beta} \{1 - e^{-(\alpha+\beta)t}\}$$

Following such steps, the other transition probabilities may be obtained as under:

$$Q_{02}(t) = \int_0^t p\alpha e^{-(\alpha+\beta)u} du, \quad Q_{02}(t) = \int_0^t q\alpha e^{-(\alpha+\beta)u} du$$

$$Q_{03}(t) = \int_0^t q\beta e^{-(\alpha+\beta)u} du, \quad Q_{14}(t) = \int_0^t \alpha e^{-(\alpha+\beta)u} G_2(u) du$$

$$Q_{16}(t) = \int_0^t e^{-(\alpha+\beta)u} dG_2(u), \quad Q_{16}(t) = \int_0^t \beta e^{-(\alpha+\beta)u} G_1(u) du$$

$$Q_{20}(t) = \int_0^t e^{-(\alpha+\beta)u} dG_1(u), \quad Q_{11}(t) = \frac{\beta}{\alpha + \beta} \int_0^t [1 - e^{-(\alpha+\beta)u}] dG_2(u)$$

$$Q_{21}(t) = \frac{\beta}{\alpha + \beta} \int_0^t [1 - e^{-(\alpha+\beta)u}] dG_1(u)$$
\[ Q_{12}^4(t) = \frac{\alpha}{\alpha + \beta} \int_0^1 (1 - e^{-(\alpha+\beta)u}) \, dG_2(u) \]

\[ Q_{22}^3(t) = \frac{\alpha}{\alpha + \beta} \int_0^1 (1 - e^{-(\alpha+\beta)u}) \, dG_1(u) \]

\[ Q_{79}(t) = \int_0^1 \eta e^{-\eta u} \, du = 1 - e^{-\eta t} = Q_{10,1}(t) = Q_{5,11}(t) = Q_{12,2}(t) \]

\[ Q_{11,12}(t) = \int_0^1 dG(u) = Q_{9,10}(t) \]

...(3.63-3.76)

Fig. 3.7
Considering \( t \to \infty \) in (1), the expressions for steady state transition probabilities can be obtained as follows:

\[
P_{79} = P_{01,1} = P_{5,11} = P_{12,2}
\]

\[
P_{01} = \frac{p \beta}{\alpha + \beta}; \quad P_{02} = \frac{p \alpha}{\alpha + \beta}; \quad P_{07} = \frac{q \beta}{\alpha + \beta}
\]

\[
P_{05} = \frac{q \alpha}{\alpha + \beta}
\]

\[
P_{14} = \frac{\alpha}{\alpha + \beta} \{1 - g_2^*(\alpha + \beta)\}; \quad P_{10} = g_2^*(\alpha + \beta)
\]

\[
P_{26} = \frac{\beta}{\alpha + \beta} \{1 - g_1^*(\alpha + \beta)\}; \quad P_{18} = \frac{\beta}{\alpha + \beta} \{1 - g_2^*(\alpha + \beta)\}
\]

\[
P_{20} = g_1^*(\alpha + \beta)
\]

\[
P_{23} = \frac{\alpha}{\alpha + \beta} \{1 - g_1^*(\alpha + \beta)\}
\]

\[
P_{1,1}^8 = \frac{\beta}{\alpha + \beta} \{1 - g_2^*(\alpha + \beta)\}
\]

\[
P_{2,1}^6 = \frac{\beta}{\alpha + \beta} \{1 - g_1^*(\alpha + \beta)\}
\]

\[
P_{1,2}^4 = \frac{\alpha}{\alpha + \beta} \{1 - g_2^*(\alpha + \beta)\}
\]

\[
\text{...(3.77-3.90)}
\]

The following relation hold good among the probabilities in (2)

\[
P_{01} + P_{02} + P_{07} + P_{05} = 1
\]

\[
P_{14} + P_{18} + P_{10} = 1
\]

\[
P_{26} + P_{23} + P_{20} = 1
\]

\[
P_{2,1}^6 + P_{2,2}^3 + P_{20} = 1
\]

\[
P_{1,2}^4 + P_{1,1}^8 + P_{10} = 1
\]

\[
\text{...(3.91-3.95)}
\]

If \( T \) is the random variable denoting survival time in state \( S_i \) then mean sojourn time \( \mu_i \) in state \( S_i \in E \) is

\[
\mu_i = \int \mathbb{P}(X_i > t) \, dt
\]

The mean sojourn times in various states are as follows:
\[
\mu_0 = \frac{1}{\alpha + \beta}; \mu_1 = \frac{1}{\alpha + \beta} \{1 - g_2^*(\alpha + \beta)\} \\
\mu_2 = \frac{1}{\alpha + \beta} \{1 - g_1^*(\alpha + \beta)\} \\
\mu_5 = \mu_7 = \mu_{10} = \mu_{12} = 1/\eta \\
\mu_9 = \mu_{11} = - g^*'(0) \\
\]

Defining \( m_{ij} \) as the mean sojourn time of the system in state \( S_i \) when the system is to transit into state \( S_j \), i.e.

\[
m_{ij} = - \left. \frac{d}{ds} Q^*_j(s) \right|_{s=0} = - \left. \frac{d}{ds} q^*_j(s) \right|_{s=0}
\]

or

\[
m_{ij} = \int_0^\infty t d Q_j(t) = \int_0^\infty t q_j(t) dt
\]

To calculate \( m_{ij} \), we first need to calculate \( q_j(t) \) which can be obtained from \( Q_j(t) \) on differentiating under the integral sign.

Thus we have

\[
q_{01}(t) = p\beta e^{-(\alpha + \beta)t}; \quad q_{02}(t) = p\alpha e^{-(\alpha + \beta)t} \\
q_{05}(t) = q\alpha e^{-(\alpha + \beta)t}; \quad q_{07}(t) = q\beta e^{-(\alpha + \beta)t} \\
q_{14}(t) = \alpha e^{(\alpha + \beta)t} \bar{G}_2(t) \\
q_{10}(t) = g_2(t) e^{-(\alpha + \beta)t} \\
q_{1a}(t) = \beta e^{-\beta t} e^{-\alpha t} \bar{G}_2(t) = \beta e^{-(\alpha + \beta)t} \bar{G}_2(t) \\
q_{20}(t) = \beta e^{-(\alpha + \beta)t} \bar{G}_1(t); \quad q_{22}(t) = g_1(t) e^{-(\alpha + \beta)t} \\
q_{23}(t) = \alpha e^{(\alpha + \beta)t} \bar{G}_1(t) \\
q_{1,2}(t) = [\alpha e^{-(\alpha + \beta)t} \odot 1] g_2(t) \\
q_{2,1}(t) = [\beta e^{-(\alpha + \beta)t} \odot 1] g_2(t) \\
q_{2,1}(t) = [\beta e^{-(\alpha + \beta)t} \odot 1] g_1(t) \\
q_{2,2}(t) = [\alpha e^{-(\alpha + \beta)t} \odot 1] g_1(t) \\
q_{79}(t) = q_{10,1}(t) + q_{12,2}(t) = \eta e^{-\eta t} q_{11,12}(t) = q_{9,10}(t) = g(t)
\]

\[
\ldots(3.101-3.115)
\]
In view of above results

\[ m_{01} = \frac{p \beta}{(\alpha + \beta)^2}; \quad m_{14} = \frac{\alpha}{(\alpha + \beta)} [1 - g_2^* (\alpha + \beta)] + \frac{\alpha}{\alpha + \beta} g_2^* (\alpha + \beta) \]

\[ m_{02} = \frac{p \alpha}{(\alpha + \beta)^2}; \quad m_{10} = -g_2^* (\alpha + \beta) \]

\[ m_{05} = \frac{q \alpha}{(\alpha + \beta)^2}; \quad m_{18} = \frac{\beta}{(\alpha + \beta)^2} [1 - g_2^* (\alpha + \beta)] + \frac{\beta}{\alpha + \beta} g_2^* (\alpha + \beta) \]

\[ m_{07} = \frac{q \beta}{(\alpha + \beta)^2}; \quad m_{20} = -g_2^* (\alpha + \beta) \]

\[ m_{12} = \frac{\alpha}{\alpha + \beta} [K_1 + g_1^* (\alpha + \beta)] m_{23} = \frac{\alpha}{(\alpha + \beta)} [1 - g_1^* (\alpha + \beta)] + \frac{\alpha}{\alpha + \beta} g_1^* (\alpha + \beta) \]

\[ m_{22}^6 = \frac{\beta}{\alpha + \beta} [K_1 + g_1^* (\alpha + \beta)] m_{26} = \frac{\beta}{(\alpha + \beta)^2} [1 - g_1^* (\alpha + \beta)] + \frac{\alpha}{\alpha + \beta} g_1^* (\alpha + \beta) \]

\[ m_{5,11} = m_{10,1} = m_{12,2} = m_{7,9} = 1/\eta \]

\[ m_{11,12} = \int_0^\infty t g(t) dt = K \ldots (3.116-3.129) \]

The following relations among \( m_i \)'s are also observed

\[ m_{01} + m_{02} + m_{05} + m_{07} = \mu_0 \]
\[ m_{10} + m_{14} + m_{18} = \mu_1 \]
\[ m_{20} + m_{26} + m_{23} = \mu_2 \]
\[ m_{10} + m_{11}^4 + m_{12}^4 = K_2 \]
\[ m_{20} + m_{22}^3 + m_{21}^6 = K_1 \ldots (3.130-3.134) \]

3.14 Mean Time to System Failure

To obtain the distribution of the time to system failure we regard the failed states as absorbing states. By employing the arguments used for the regenerative process, the following recursive relations for \( \pi_i(t) \) are obtained

\[ \phi_0(t) = Q_{01}(t) S \phi_1(t) + Q_{02}(t) S \phi_2(t) + Q_{03}(t) + Q_{07}(t) \]
\[ \phi_1(t) = Q_{10}(t) S \phi_0(t) + Q_{14}(t) + Q_{19}(t) \]
\[ \phi_2(t) = Q_{20}(t) S \phi_0(t) + Q_{26}(t) + Q_{23}(t) \ldots (3.135-3.137) \]
By taking the L.S.T of relations (3.135-3.137) and solving for \( \phi_0(s) \), we have

\[
\phi_0^{**}(s) = \left. \frac{N(s)}{D(s)} \right|_{s=0}
\]

Where,

\[
N(s) = Q_{05}^{**}(s) + Q_{07}^{**}(s) + Q_{01}^{**}(s)Q_{14}^{**}(s) + Q_{01}^{**}(s)Q_{18}^{**}(s) + Q_{02}^{**}(s)Q_{26}^{**}(s)
\]
\[+ Q_{02}^{**}(s)Q_{23}^{**}(s) \]
\[
D(s) = 1 - Q_{01}^{**}(s)Q_{10}^{**}(s) - Q_{02}^{**}(s)Q_{20}^{**}(s) \]

Here, for brevity, we have omitted the argument(s) in \( Q_j^{**}(s) \). We can easily verify that

\[ \phi_0^{**}(s) = 1 \Rightarrow \text{distribution is proper.} \]

By differentiating equation (3), it is found that

\[
\text{MTSF} = - \left. \frac{d}{ds} \phi_0^{**}(s) \right|_{s=0} = \frac{N_1}{D_1} = \frac{D'(0) - N'(0)}{D'(0)} \]

where

\[
N_1 = \mu_0 + \mu_1 P_{01} + \mu_2 P_{02}
\]
\[
D_1 = 1 - P_{01} P_{10} - P_{02} P_{20}
\]

3.15 Availability Analysis

Using the arguments of the theory of regenerative process, the availability

\[
A_i(t) \]

is seen to satisfy following recursive relations:

\[
A_0(t) = M_0(t) + q_{01}(t) \otimes A_1(t) + q_{02}(t) \otimes A_2(t) + q_{05}(t) \otimes A_5(t) + q_{07}(t) \otimes A_7(t)
\]
\[
A_1(t) = M_1(t) + q_{10}(t) \otimes A_0(t) + q_{12}^4(t) \otimes A_2(t) + q_{11}(t) \otimes A_1(t)
\]
\[
A_2(t) = M_2(t) + q_{20}(t) \otimes A_0(t) + q_{21}^6(t) \otimes A_1(t) + q_{22}^7(t) \otimes A_2(t)
\]
\[
A_3(t) = q_{5,11}(t) \otimes A_{11}(t)
\]
\[
A_{11}(t) = q_{11,12}(t) \otimes A_{12}(t)
\]
\[
A_{12}(t) = q_{12,2}(t) \otimes A_{2}(t)
\]
\[
A_7(t) = q_{7,9}(t) \otimes A_9(t)
\]
\[
A_9(t) = q_{9,10}(t) \otimes A_{10}(t)
\]
By definition, $M_j(t)$ is the probability that the system initially up in a regenerative state $S_i \in E$ continues to be up till time $t$, without transiting to any other regenerative state or returning to the same state via one or more non-regenerative states. Thus

$$M_0(t) = e^{-(\alpha+\beta)t}$$

$$M_1(t) = \overline{G}_2(t) e^{-(\alpha+\beta)t}$$

$$M_2(t) = \overline{G}_1(t) e^{-(\alpha+\beta)t}$$

$$M_0^*(0) = \int_0^\infty e^{-(\alpha+\beta)t} dt = 1/(\alpha+\beta) = \mu_0$$

$$M_1^*(0) = \int_0^\infty \overline{G}_2(t) e^{-(\alpha+\beta)t} dt = \frac{1}{\alpha+\beta} \{1 - \overline{g}_2^*(\alpha+\beta)\} = \mu_1$$

$$M_2^*(0) = \int_0^\infty \overline{G}_1(t) e^{-(\alpha+\beta)t} dt = \frac{1}{\alpha+\beta} \{1 - \overline{g}_1^*(\alpha+\beta)\} = \mu_2$$

Taking laplace transform (L.T) of the above equation and solving for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

where

\[
N_2(s) = \begin{bmatrix}
M_0^*(s) & -q_{01}^*(s) & -q_{02}^*(s) & -q_{05}^*(s) & -q_{07}^*(s) & 0 & 0 & 0 & 0 \\
M_1^*(s) & 1 - q_{11}^*(s) & -q_{12}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
M_2^*(s) & -q_{21}^*(s) & 1 - q_{22}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{511}^*(s) & 0 \\
0 & 0 & 0 & 0 & 1 & -q_{7,9}^*(s) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -q_{9,10}^*(s) & 0 & 0 \\
0 & -q_{10,1}^*(s) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{11,12} & 0 \\
0 & 0 & -q_{12,2} & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
In steady state, the availability of the system is given by

\[ A_0 = \lim_{s \to 0} s A_0^*(s) = \frac{N_2}{D_2} \quad \text{(3.148)} \]

where

\[ N_2 = (1 - P_{22}^S) [\mu_0 (1 - P_{11}^S) - P_{07} P_{10} + \mu_1 P_{01} + (1 - P_{11}^S)\{\mu_2 P_{02} + P_{20} P_{07}\}] \]
97
t + P_{21} \left\{ \mu_1 P_{02} + \mu_1 P_{05} - \mu_0 P_{12} \right\} + P_{12}^4 \left\{ \mu_2 P_{01} - P_{20} P_{07} \right\}
\]
D_2 = \mu_0 \left\{ P_{10} - P_{10} P_{22}^3 + P_{12}^4 - P_{21}^6 P_{10} \right\} + K_2 \left\{ 1 - P_{22}^3 - P_{20} P_{20} - P_{02} P_{22}^4 \right\}
+ K_1 \left\{ P_{10} P_{02} + P_{12}^4 \right\}

3.16 Busy Period Analysis of Repairman

By probabilistic argument, we have the following recursive relation for B_i(t)
B_0(t) = q_{0,1}(t)B_1(t) + q_{0,2}(t)B_2(t) + q_{0,5}(t)B_5(t) + q_{0,7}(t)B_7(t)
B_1(t) = W_1(t) + q_{1,0}(t)B_0(t) + q_{1,2}(t)B_2(t) + q_{1,8}(t)B_1(t)
B_2(t) = W_2(t) + q_{2,0}(t)B_0(t) + q_{2,1}(t)B_1(t) + q_{2,2}(t)B_2(t)
B_3(t) = W_3(t) + q_{3,11}(t)B_{11}(t)
B_7(t) = W_7(t) + q_{7,0}(t)B_0(t)
B_{11}(t) = W_{11}(t) + q_{11,2}(t)B_{12}(t)
B_{12}(t) = W_{12}(t) + q_{12,2}(t)B_2(t)
B_9(t) = W_9(t) + q_{9,10}(t)B_{10}(t)
B_9(t) = W_9(t) + q_{9,10}(t)B_{10}(t)
B_{10}(t) = W_{10}(t) + q_{10,1}(t)B_1(t)

where,
W_1(t) = e^{-(a+\beta)t} \bar{G}_2(t)
W_2(t) = e^{-(a+\beta)t} \bar{G}_1(t)
W_3(t) = W_7(t) = W_{10}(t) = W_{12}(t) = e^{-nt}
W_{11}(t) = W_9(t) = \bar{G}_1(t) \quad W_5^*(0) = \mu_5
W_i^*(0) = \frac{1}{\alpha + \beta} \{ 1 - g_2^*(\alpha + \beta) \} = \mu_i \quad W_{i0}^*(0) = \mu_{i0}
W_2^*(0) = \frac{1}{\alpha + \beta} \{ 1 - g_1^*(\alpha + \beta) \} = \mu_2 \quad W_{i2}^*(0) = \mu_{i2}
W_{11}^*(0) = \int_0^\infty \bar{G}(t) \, dt = -g^*(0) = \mu_{11} \quad W_9^*(0) = \mu_9

Taking Laplace transform (L.T) of the above equation and solving for B_0*(s), we get
\[ B_0^*(s) = \frac{N_3(s)}{D_2(s)} \]

where

\[
N_3(s) = \begin{bmatrix}
0 & -q_{01}(s) & -q_{02}(s) & -q_{05}(s) & -q_{07}(s) & 0 & 0 & 0 & 0 \\
W_1^*(s) & 1 - q_{11}^*(s) & -q_{12}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
W_2^*(s) - q_{21}^*(s) & 1 - q_{22}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
W_3^*(s) & 0 & 0 & 1 & 0 & 0 & 0 & -q_{5,11}(s) & 0 \\
W_7^*(s) & 0 & 0 & 0 & 0 & 0 & 1 - q_{7,9}(s) & 0 & 0 \\
W_8^*(s) & 0 & 0 & 0 & 0 & 0 & 1 - q_{9,10}(s) & 0 & 0 \\
W_{10}^*(s) & -q_{10,1}(s) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
W_{11}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
W_{12}^*(s) & 0 & -q_{12,2} & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
D_2(s) \text{ already specified.}
\]

In steady-state, the total fraction of the time for which the system is under replacement is given by

\[ B_0 = \lim_{t \to \infty} B_0(t) \]
\[ \lim_{s \to 0} s B_0(s) = \frac{N_3}{D_2} \]  

where,

\[ N_3 = \mu_1(1 + P_{07}) (1 - P_{22}^3) + P_{21}^6 (P_{02} + P_{07}) \]
\[ + (1 - P_{11}^8)(1 - P_{22}^3) \{(P_{05} + P_{07})(\mu_9 + 2\mu_5)\} \]
\[ + (1 - P_{11}^8)(P_{02} + P_{05})\mu_2 - P_{02}^4 P_{21}^6 \mu_{11} + P_{07}^4 P_{12}^4 \]

and \( D_2 \) is already specified.

### 3.17 Expected Number of Visits by the Repairman

By the probabilistic arguments, we have the following recursive relations:

\[
V_0(t) = Q_{01}(t) \iff (1+V_1(t)) + Q_{02}(t) \iff (1+V_2(t)) + Q_{05}(t) \iff (1+V_5(t)) + Q_{07}(t) \iff (1 + V_7(t))
\]
\[
V_1(t) = Q_{10}(t) \iff V_0(t) + Q_{12}^4(t) \iff V_2(t) + Q_{11}^6(t) \iff V_1(t)
\]
\[
V_2(t) = Q_{20}(t) \iff V_0(t) + Q_{21}^4(t) \iff V_2(t) + Q_{21}^6(t) \iff V_1(t)
\]
\[
V_5(t) = Q_{5,11}(t) \iff V_{11}(t)
\]
\[
V_7(t) = Q_{7,9}(t) \iff V_{9}(t)
\]
\[
V_9(t) = Q_{9,10}(t) \iff V_{10}(t)
\]
\[
V_{10}(t) = Q_{10,1}(t) \iff V_{1}(t)
\]
\[
V_{11}(t) = Q_{11,12}(t) \iff V_{12}(t)
\]
\[
V_{12}(t) = Q_{12,2}(t) \iff V_{2}(t)
\]

Taking Laplace-Stieltjes transformations of the above equation and solving them for \( V_0^{**}(s) \) we get

\[
V_0^{**}(s) = \frac{N_4(s)}{D_2(s)}
\]

where
\[ N_4(s) = \]
\[
\begin{array}{cccccccc}
Q^{**}(s) + Q^{**}(s) + Q^{**}(s) + Q^{**}(s) - Q^{**}(s) - Q^{**}(s) - Q^{**}(s) - Q^{**}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -Q^{**}(s) - Q^{**}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -Q^{**}(s) - Q^{**}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -Q^{**}(s) \\
0 & 0 & 0 & 0 & 1 - Q^{**}(s) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 - Q^{**}(s) & 0 & 0 & 0 \\
0 & -Q^{**}(s) & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 - Q^{**}(s) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{array}
\]

\[ N_4(s) = (Q^{**}(s) + Q^{**}(s) + Q^{**}(s) + Q^{**}(s)) \{ (1 - Q^{**}(s))(1 - Q^{**}(s)) - Q^{**}(s)Q^{**}(s) \} \]

and \( D_2(s) \) is already specified.

In steady-state the total number of visits by the repairman per unit time is given by

\[ V_0 = \lim_{t \to \infty} \{ v_0(t)/t \} \]

\[ = \lim_{s \to 0} v_0^{**}(s) \]

\[ = \frac{N_4(0)}{D_2(0)} \quad \text{or} \quad \frac{N_4}{D_2} \quad \ldots (3.169) \]

where

\[ N_4 = (1 - P_1^4)(1 - P_2^2) - P_1^4 P_2^6 \]

and \( D_2 \) is already specified.

3.18 Cost-Benefit Analysis

The expected total profit in steady-state is

\[ P = K_0 A_0 - K_1 B_0 - K_2 V_0 \quad \ldots (3.170) \]

where \( K_0 \) = the revenue per unit uptime of the system.
\( K_1 \) = is the cost per unit time for which repairman is busy.

\( K_2 \) = is the cost per visit for repairman.

Using the above equation (3.138), (3.148), (3.159), (3.169) and (3.170), we can have the expression MTSF, availability etc. for this particular case.

### 3.19 Particular Case

For graphical interpretation, the following particular case is considered:

\[ g_1(t) = \gamma e^{-\gamma t}, \quad g_2(t) = \delta e^{-\delta t}, \quad g(t) = \lambda e^{-\lambda t} \]

Thus, we can easily obtained the following

\[
\begin{align*}
  P_{02} &= \frac{p\alpha}{\alpha + \beta}, \\
  P_{10} &= \frac{\delta}{(\alpha + \beta + \delta)}, \\
  P_{14} &= P_{12} = \frac{\alpha}{(\alpha + \beta + \delta)}, \\
  P_{01} &= \frac{p\beta}{\alpha + \beta}, \\
  P_{18} &= P_{11} = \frac{\beta}{(\alpha + \beta + \delta)}, \\
  P_{05} &= \frac{q\alpha}{\alpha + \beta}, \\
  P_{20} &= \frac{\gamma}{(\alpha + \beta + \gamma)}, \\
  P_{26} &= P_{21} = \frac{\beta}{(\alpha + \beta + \gamma)}, \\
  P_{07} &= \frac{q\beta}{\alpha + \beta}, \\
  P_{23} &= P_{22} = \frac{\alpha}{(\alpha + \beta + \gamma)}, \\
  \mu_0 &= \frac{1}{\alpha + \beta}, \\
  \mu_1 &= \frac{1}{\alpha + \beta + \delta}, \\
  \mu_2 &= \frac{1}{(\alpha + \beta + \gamma)}, \\
  \mu_5 &= \mu_7 = \mu_10 = \mu_{12} = \frac{1}{\eta},
\end{align*}
\]

\[
\begin{align*}
  K_1 &= \frac{1}{\gamma}, \\
  K_2 &= \frac{1}{\delta}, \\
  \mu_{11} &= \mu_9 = K = \frac{1}{\lambda}.
\end{align*}
\]

On the basis of the numerical values taken as:

\[
\begin{align*}
  \alpha &= 0.1, & \beta &= 0.45, & \gamma &= 0.25, & p &= 0.65, \\
  q &= 0.35, & \delta &= 0.85, & \lambda &= 0.95, & \eta &= 0.089
\end{align*}
\]

The values of various measures of system effectiveness are obtained as:

- **Mean time to system failure (MTSF)** = 3.285211
- **Availability** \((A_0)\) = 0.747633.
- **Busy period of analysis of repairman** \((B_0)\) = 0.886019
- **Expected number of visits by the repairman** \((V_0)\) = 0.283111

For the graphical interpretation, the mentioned particular case is considered.
Figs. 3.8 and 3.9 show the behaviour of MTSF and availability respectively with respect to failure rate \((\alpha)\). It is clear from the graph that the MTSF and the availability both get decrease with increase in the value of failure rate.

Fig. 3.10 shows the behaviour of availability respectively with respect to repair rate \((\gamma)\). It is clear from the graph that the availability gets increase with increase in the value of repair rate.

Fig. 3.11 reveals the pattern of the profit with respect to failure rate \((\alpha)\) for different values of repair rate \((\gamma)\). The profit decreases with the increase in the value of failure rate \((\alpha)\) and is higher for higher values of repair rate \((\gamma)\).

Fig. 3.12 shows the pattern of the profit with respect to repair rate \((\gamma)\) for different values of failure rate \((\alpha)\). The profit increases with the increase in the value of repair rate \((\gamma)\) and is lower for higher values of failure rate \((\alpha)\).
MTSF VS FAILURE RATE ($\alpha$)

\[ \begin{align*}
\gamma &= 0.25 \\
\gamma &= 0.3 \\
\gamma &= 0.4 \\
\end{align*} \]

\[ \lambda = 0.95, \delta = 0.89, \beta = 0.35, \]
\[ p = 0.65, q = 0.35. \]

Fig 3.8
Fig 3.9

$\lambda = 0.95, \delta = 0.89, \beta = 0.35,$
$p = 0.65, q = 0.35.$
AVAILABILITY VS REPAIR RATE ($\gamma$)

$\lambda = 0.95, \delta = 0.89, \beta = 0.35,$
$p = 0.65, q = 0.35.$

Fig 3.10
PROFIT VS FAILURE RATE ($\alpha$)

$\gamma = 0.3 \quad \text{and} \quad \gamma = 0.4$

$\lambda = 0.95$, $\delta = 0.89$, $\beta = 0.35$,
$\rho = 0.65$, $\eta = 0.35$, $C_0 = 500$,
$C_1 = 150$, $C_2 = 100$.

Fig 3.11
PROFIT VS REPAIR RATE ($\gamma$)

- $\alpha=0.25$
- $\alpha=0.3$

$\lambda = 0.95$, $\delta = 0.89$, $\beta = 0.35$,
$p = 0.65$, $q = 0.35$, $C_0 = 600$,
$C_1 = 550$, $C_2 = 450$.

Fig 3.12