CHAPTER 2

DATA & METHODOLOGY

2.1 Data

The photometric data obtained from the Optical Gravitational Lensing Experiment (OGLE) survey, Magellanic Cloud Photometric Survey (MCPS) and the Infrared Survey Facility (IRSF) Magellanic Cloud survey are used for our study. The details of each data set are given below.

2.1.1 Optical Gravitational Lensing Experiment Survey

The Optical Gravitational Lensing Experiment (OGLE) project is a long term project with the main goal of searching for the dark matter with microlensing phenomena. The first phase (OGLE I 1992-1995) was dedicated to detect statistically significant number of microlensing events towards the Galactic bulge. OGLE I observations were taken from the 1m Swope telescope of the Las Campanas Observatory, operated by the Carnegie Institution of Washington. The second phase (OGLE II, from 1997-2000) of the OGLE survey observed the bar regions of both the LMC and the SMC. The third phase (OGLE III, from 2001-2009) observed the bar as well as the surrounding central regions of the MCs. The observations of OGLE II and OGLE III were taken using the 1.3 m Warsaw telescope located at Las Campanas Observatory, Chile, which is operated by the Carnegie Institution of Washington. The IVth phase has started and covers a larger area of the LMC and the SMC in the sky compared to OGLE III coverage. One of the major outcomes of the OGLE II and OGLE III surveys is a large photometric database of stars in the MCs. We have utilised these catalogs in our study of the MCs.
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Figure 2.1: The red and black boxes are the OGLE II and OGLE III observed regions of the LMC.

During the second phase, OGLE II, the telescope was equipped with the first generation camera with a SITe 2048 x 2048 CCD detector working in drift scan mode. The pixel size was 24 µm which corresponds to 0.417 arcsec/pixel. For details of the instrumental setup refer to Udalski et al. (1997). The OGLE II survey scanned the central 5.7 square degrees of the LMC and presented a catalog (Udalski et al. 2000) of stars consisting of photometric data of 7 million stars in the B,V and I pass bands. The observed fields practically covered the entire bar of the LMC (21 fields), which covered 4.5 square degrees in the sky. Five additional fields in the North-West region of the LMC disk were also observed. The OGLE II survey of central region of the SMC contains photometric data of 2 million stars in the B,V and I pass bands. The catalog of the SMC (Udalski et al. 1998)
2.1 Data

Figure 2.2: The red and green boxes are the OGLE II and OGLE III observed regions of the SMC.

presented data of 11 fields which cover the central 2.5 square degrees of the SMC in the sky.

During the third phase, OGLE III, the telescope was equipped with the second generation camera consisting of eight SITe 2048 x 4096 CCD detectors with 15 µm pixels which corresponds to 0.26 arcsec/pixel scale. Details of the instrumental setup can be found in Udalski (2003). The OGLE III survey observed the central 39.7 and 14 square degrees of the LMC and the SMC, which cover the central bar as well as the surrounding regions, in V and I bands. Udalski et al. (2008a) presented the $VI$ photometric data of 35 million stars in the 116 LMC fields and Udalski et al. (2008b) presented the data of 6.2 million stars in the 40 SMC fields from this survey. The V and I band photometric data of the MCs obtained during the OGLE survey are used to identify and study the RC stars in the MCs. Along with the photometric maps, the OGLE survey identified and cataloged the variable stars (Cepheids, RRLS etc) in the MCs. The classical Cepheids and RR Lyrae
stars identified in the MCs during the OGLE III survey are used in our study. Soszynski et al. (2008) and Soszyński et al. (2010a) published the catalog of 3361 and 4630 classical Cepheids identified in the LMC and the SMC respectively during the OGLE III survey. The catalog of 24906 RRLS in the LMC and 2475 RR Lyrae stars in the SMC are given in Soszyński et al. (2009) and Soszyński et al. (2010b) respectively. The observed regions of the LMC and the SMC by this survey are shown in Fig. 2.1 and Fig. 2.2 respectively.

### 2.1.2 Magellanic Cloud Photometric Survey

The Magellanic Cloud Photometric Survey (MCPS) (Zaritsky et al. 1997) obtained the UBVI photometry of virtually all stars brighter than $V = 21$ mag in the MCs. The five year survey was conducted at the Las Campanas Observatory’s 1 m Swope telescope and the images were obtained using the Great Circle Camera (GCC, Zaritsky et al. 1996). The
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The MCPS observed region of the SMC.

thinned 2048 x 2048 CCD has 0.7 arcsec/pixel scale. The survey scanned 64 deg$^2$ of the LMC and 16 deg$^2$ of the SMC. The MCPS observed regions of the LMC and the SMC are shown in Fig. 2.3 and Fig. 2.4 respectively. Zaritsky et al. (2002) presented the UBVI photometric catalog of the SMC MCPS survey and Zaritsky et al. (2004) presented the data of the LMC MCPS survey. The RC stars in the MCs are identified and studied using this catalog also.

2.1.3 IRSF Magellanic Cloud Point Source Catalog

The IRSF Magellanic Cloud Point Source Catalog (IRSF-MCPSC) (Kato et al. 2007) is an outcome of an imaging survey of the MCs in the Near Infrared (NIR) bands $J$ (1.25 $\mu$m), $H$ (1.63 $\mu$m) and $K_s$ (2.14 $\mu$m) during the period October 2001 to March 2006. The observations were made with the SIRIUS camera (Simultaneous three colour InfraRed Imager for Unbiased Survey) on the InfaRed Survey Facility (IRSF) 1.4 m tele-
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The relative distance of different regions of the MCs with respect to the mean distance to the galaxy and the extent along the line of sight are two important quantities which are required to understand the structure of the MCs. The basic methodology is to estimate the relative distances between regions in the MCs from the observed magnitudes, after correcting for extinction. All the tracers used in our study are standard candles. In the case of Cepheids, period-luminosity (PL) relation is used to estimate the individual distances to Cepheids and hence the relative distance of each Cepheid from the center of the galaxy. For the RC stars and RRLS their mean dereddened observed magnitude is a measure of the distances and the dispersion in their mean magnitude is an estimate of the line of sight depth. The R.A, Dec and relative distance between different regions are used to obtain a cartesian coordinate system. The structural parameters of different components of the MCs are obtained by applying appropriate methods, such as plane fitting procedure and inertia tensor analysis, on the cartesian coordinates.

2.2.1 Relative distances

- **Cepheids**: The methodology described below is similar to that used by Nikolaev et al. (2004) for the analysis of Cepheids. The analysis is based on the PL relation which is is given by

\[
\overline{M}_\lambda = \alpha_\lambda \log P + \beta_\lambda
\]

where \( \lambda \) denotes the photometric bands in which Cepheids are observed, \( \overline{M}_\lambda \) is the mean intrinsic magnitude, \( \alpha \) and \( \beta \) are the PL coefficients and \( P \) the pulsation period. The \( \overline{M}_\lambda \) can be converted into observed mean magnitude, \( \overline{m}_\lambda \) using the equation
\[ \overline{m}_\lambda = \mu + A_\lambda + \alpha_\lambda \log P + \beta_\lambda \]

where \( \mu \) and \( A_\lambda \) are the distance modulus and extinction, in the photometric band denoted by \( \lambda \), respectively. \( A_\lambda = R_\lambda E(B - V) \), where \( R_\lambda \) is the ratio of total to selective extinction and \( E(B - V) \) is the reddening. The quantities \( \mu \) and \( E(B - V) \) can be divided into two parts, one mean quantity corresponding to the entire galaxy and the other which varies from star to star. The index \( i \) in the below equations denotes individual star.

\[ \mu_i = \overline{\mu}_{\text{galaxy}} + \delta \mu_i \]

\[ E(B - V)_i = \overline{E(B - V)}_{\text{galaxy}} + \delta E(B - V)_i \]

The mean quantities can be incorporated in the quantity \( \beta \) and the equation for each star connecting the mean observed magnitude, relative distance and reddening is given as

\[ \overline{m} = \delta \mu_i + R_\lambda \delta E(B - V)_i + \alpha_\lambda \log P_i + \beta_\lambda \]

The above equation can be solved using linear least square method, to obtain the individual distances and reddenings. As we are interested in the relative distance (\( \Delta D \)) and relative reddening between different regions in the galaxy, the mean values of distance and reddening we take are not going to affect the determination of the structural parameters of the galaxy.

- **Red Clump stars**: The observed region of the MCs from different data sets are divided into several sub-regions. A sub-region of 1 arcmin\(^2\) area on the sky corresponds to an area of 14.5 pc\(^2\) and 17.5 pc\(^2\) in the LMC and the SMC respectively. For each sub-region \((V - I)\) vs I colour magnitude diagram (CMD) is plotted and the RC stars are identified. A sample CMD of both the LMC and the SMC are shown in Fig. 2.3 and Fig. 2.4 respectively. For all the regions in the MCs, the RC stars are found to be located well within the box shown in the CMD, with boundaries 0.65 - 1.35 mag in \((V - I)\) colour and 17.5 - 19.5 mag in I magnitude.

The RC stars occupy a compact region in the CMD and they have a constant characteristic I band magnitude and \((V - I)\) colour. A spread in magnitude and colour
of red clump stars is observed in the CMDs of both the LMC and the SMC. Their number distribution profiles resemble a Gaussian. The peak values of their color and magnitude distributions are used to obtain the dereddened RC magnitude.

To obtain the number distribution of the RC stars in each region, the data are binned in colour and magnitude. The obtained distributions in colour and magnitude are fitted with a function, Gaussian + quadratic polynomial. The Gaussian represents the RC stars and the other terms represent the red giants in the region. A non linear least square method is used for fitting and the parameters are obtained. In Fig. 2.5, Fig. 2.6, Fig. 2.7 and Fig. 2.8 the colour and magnitude distributions of the RC stars in the LMC and the SMC are shown respectively. The best fit profile is also
2.2 Methodology

Figure 2.6: A sample CMD of a sub-region in the SMC. The box used to identify the RC stars is also shown

shown in all the plots. The parameters obtained are the coefficients of each term in the function used to fit the profile, error in the estimation of each parameter and reduced $\chi^2$ value. The errors in the estimated parameters are calculated using the covariance matrix. The parameters that we are interested in the estimation of the dereddened $I_0$ magnitude are the peak $I$ mag and $(V - I)$ colour.

The peak values of colour, $(V - I)$ mag in each location is used to estimate the reddening. The reddening is estimated using the formula

$$E(V - I) = (V - I)_{observed} - (V - I)_{intrinsic}$$
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Figure 2.7: A typical colour distribution of the RC stars in the LMC. The best fit to the distribution is also shown.

where \((V - I)_{\text{intrinsic}}\) is the intrinsic colour of the RC stars in the MCs. The \((V - I)_{\text{intrinsic}}\) for the LMC is taken as 0.92 mag (Olsen & Salyk 2002). The intrinsic colour of the RC stars in the SMC is chosen as 0.89 mag to produce a median reddening equal to that measured by Schlegel et al. (1998) towards the SMC. The interstellar extinction is estimated using the formula, \(A_I = 1.4 \times E(V - I)\) (Subramaniam 2005).

After correcting the mean I mag for interstellar extinction, \(I_0\) mag for each region is estimated. The error in \(I_0\) is estimated as
2.2 Methodology

Figure 2.8: A typical magnitude distribution of the RC stars in the LMC. The best fit to the distribution is also shown.

\[ \delta I_0^2 = (\text{avg error in peak } I)^2 + (1.4 \times \text{avg error in peak } (V - I))^2. \]

The variation in the I$_0$ mag between the sub-regions is assumed only due to the difference in the relative distances. Thus the difference in I$_0$ between the sub-regions is a measure of the relative distance ($\Delta D$), using the distance modulus formula given below

\[ (I_0 \text{ mean } - I_0 \text{ of each region}) = 5 \log_{10}(D_0/(D_0 \pm \Delta D)), \]
2.2 Methodology

![Graph showing a typical colour distribution of the RC stars in the SMC. The best fit to the distribution is also shown.](image)

Figure 2.9: A typical colour distribution of the RC stars in the SMC. The best fit to the distribution is also shown.

where $D_0$ is the mean distance to the galaxy.

- **RR Lyrae stars**: The ab type stars among the RRLS could be considered to belong to a similar class and hence assumed to have similar properties. The mean magnitude of these stars in the I band, after correcting for the metallicity and extinction effects, can be used for the estimation of distance. The reddening obtained using the RC stars (described in the previous sub-section) is used to correct the extinction to individual RRLS. Stars within each bin of the reddening map are assigned a single reddening and it is assumed that reddening does not vary much within the bin. Thus the extinction corrected $I_0$ magnitude for all the RRLS are estimated.
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2.2.2 Line of sight depth

- Red Clump stars: The spread in the distribution of the RC stars in the CMD includes the information of the line of sight depth of the region. The width in the colour and magnitude distributions of the RC stars can be used to estimate the line of sight depth. The width of the Gaussian in the distribution of colour ($\sigma_{\text{colour}}$) is due to the internal reddening ($\sigma_{\text{internal reddening}}$), apart from observational error ($\sigma_{\text{error}}$) and population effects ($\sigma_{\text{intrinsic}}$). The width in the distribution of magnitude ($\sigma_{\text{mag}}$) is due to population effects ($\sigma_{\text{intrinsic}}$), observational error ($\sigma_{\text{error}}$), internal extinction ($\sigma_{\text{internal extinction}}$) and depth ($\sigma_{\text{depth}}$). By deconvolving the effects of observational er-
ror, extinction and population effects from the dispersion in magnitude, an estimate of depth can be obtained. By applying an non linear least square fit, the observed colour and magnitude distributions (as explained in section 2.2.1 on the RC stars) are fitted with a Gaussian + quadratic polynomial term and parameters are obtained. The parameters required for the estimation of the line of sight depth are $\sigma_{\text{mag}}$ and $\sigma_{\text{colour}}$. The errors associated with both the parameters are also estimated.

The following relations are used to estimate the resultant dispersion due to depth.

\[
\sigma_{\text{col}}^2 = \sigma_{\text{internal reddening}}^2 + \sigma_{\text{intrinsic}}^2 + \sigma_{\text{error}}^2
\]

\[
\sigma_{\text{mag}}^2 = \sigma_{\text{depth}}^2 + \sigma_{\text{internal extinction}}^2 + \sigma_{\text{intrinsic}}^2 + \sigma_{\text{error}}^2
\]

The average photometric errors in I and V band magnitudes were calculated for each region and the error in I magnitude and $(V - I)$ colour were estimated. These were subtracted from the observed width of magnitude and colour distribution respectively, thus accounting for the photometric errors (last term in the above equations). The contribution from the heterogeneous population of the RC stars which we discussed in section 1.6.2, $\sigma_{\text{intrinsic}}$ for both colour and magnitude are also subtracted from the observed dispersions. The $\sigma_{\text{intrinsic}}$ values in colour and magnitude taken for the LMC are 0.025 mag and 0.01 mag respectively. In the case of the SMC, 0.03 mag and 0.075 mag are taken as the $\sigma_{\text{intrinsic}}$ in colour and magnitude respectively. These values are taken from Girardi & Salaris (2001). After correcting for the population effects and the observational error in colour, the remaining spread in colour distribution (first equation) is taken as due to the internal reddening, $E(V - I)$. This is converted into extinction in I band, $A_I$. This is used to deconvolve the effect of internal extinction from the spread in magnitude. The resultant width in magnitude is converted into depth in kpc using distance modulus formula.

The error in the estimation of the dispersion corresponding to depth is obtained from the errors involved in the estimation of width of colour and magnitude distribution. The random error associated with the width corresponding to depth is $\Delta \text{depth}^2 = \Delta \text{width}^2 + \Delta (V - I)^2 \text{width}$. Thus the associated error in the estimation of depth can be calculated for all the locations. This error will also translate as the minimum depth that can be estimated.
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- **RR Lyrae stars**: Using the dereddened $I_0$ magnitude of each RR Lyrae star, the dispersion in the surveyed region of the MCs can be found. The data are binned to estimate dispersion. For each bin the mean magnitude and the dispersion can be estimated. The dispersion after correcting for age and evolutionary effects is a measure of the depth of the MCs. The corrections applied for the metallicity and evolutionary effects in the dispersion while estimating the line of sight depth are explained in detail in the respective chapters.

2.2.3 Conversion to Cartesian coordinate system

The relative distance of each region of the MCs is obtained from the variation in the $I_0$ magnitude. Here, the variation in $I_0$ is considered only due to the line of sight distance variation within the galaxy. The error in magnitude is also converted into error in distance.

Then the $x$, $y$, and $z$ coordinates are obtained using the transformation equations given below (van der Marel & Cioni 2001, see also Appendix A of Weinberg & Nikolaev 2001).

$$x = -D\sin(\alpha - \alpha_0)\cos\delta,$$
$$y = D\sin\delta\cos\delta_0 - D\sin\delta_0\cos(\alpha - \alpha_0)\cos\delta,$$
$$z = D_0 - D\sin\delta\sin\delta_0 - D\cos\delta_0\cos(\alpha - \alpha_0)\cos\delta,$$

where $D_0$ is the distance to the center of the MCs and $D$, the distance to the each sub-region is given by $D = D_0 + \Delta D$. The $(\alpha, \delta)$ and $(\alpha_0, \delta_0)$ represents the R.A and Dec of the region and the center of the MCs respectively.

2.2.4 Plane fitting procedure

The disks of the MCs are assumed to have a planar geometry. The structural parameters of the disks of the MCs, like the inclination, $i$ and the position angle of the line of nodes, $\phi$ are obtained by applying a plane fitting procedure to the $x,y,z$ cartesian coordinates of regions in the MCs. The equation of the plane used is given by

$$Ax + By + Cz + D = 0$$
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From the coefficients of the plane $A,B \& C$, $i$ and $\phi$ can be calculated using the formula given below.

Inclination, $i = \arccos(C/\sqrt{A^2 + B^2 + C^2})$

Position Angle, $\phi = \arctan(-A/B) \cdot \text{sign}(B) \pi/2$.

2.2.5 Inertia tensor analysis

The parameters of the spheroidal/ellipsoidal component of the SMC, like the axes ratio, inclination of the longest axis with the line of sight axis, $i$ and the position angle of the projection of the ellipsoid, $\phi$ are estimated using the inertia tensor analysis. A detailed description of the tensor analysis is given below.

Moment of inertia of a body characterizes the mass distribution within the body. For any rotating three dimensional system, we can compute the moment of inertia about the axis of rotation, which passes through the origin of a local reference (XYZ) frame, using the inertia tensor. The origin of the system is the center of mass of the body. Consider a system made up of $i$ number of particles, each particle with a mass $m$. For each particle the $(x, y \text{ and } z)$ coordinates with respect to the center of mass is known. Then, the moment of inertia tensor, $I$ of the system is given by

$$I = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

where

$I_{xx} = \Sigma_i m_i (y_i^2 + z_i^2)$
$I_{yy} = \Sigma_i m_i (x_i^2 + z_i^2)$
$I_{zz} = \Sigma_i m_i (x_i^2 + y_i^2)$
$I_{xy} = I_{yx} = \Sigma_i m_i (x_i + y_i)$
$I_{yz} = I_{zy} = \Sigma_i m_i (y_i + z_i)$
$I_{xz} = I_{zx} = \Sigma_i m_i (x_i + z_i)$

The components, $I_{xx}$, $I_{yy}$ and $I_{zz}$ are called the moments of inertia while $I_{xy}$, $I_{yx}$, $I_{xz}$, $I_{zx}$, $I_{yz}$ and $I_{zy}$ are the products of inertia. These components given above are basically specific to the local reference frame and reflect the mass distribution within the system in relation to the local reference frame. If we align the axes of the local reference frame in such a
way that the mass of the system is evenly distributed around the axes then the product of inertia terms vanish. This aligning the axes of local reference frame in such a way that the product of inertia terms vanish, would mean the transformation of the local reference frame (XYZ) to a system (X’Y’Z’) . In the new frame of reference, inertia tensor, $I'$ is given by

$$I' = \begin{pmatrix} I'_{xx} & 0 & 0 \\ 0 & I'_{yy} & 0 \\ 0 & 0 & I'_{zz} \end{pmatrix}$$

The terms $I'_{xx}$, $I'_{yy}$ and $I'_{zz}$ are the non zero diagonal terms of the inertia tensor in the new reference frame (X’Y’Z’) and are called the principal moments of inertia of the body. The three axes in the new reference frame are called the principal axes of the body.

To determine the principal axes of the system we have to diagonalize the inertia tensor $I$, which is obtained with respect to the local reference frame. The diagonalization of the inertia tensor provides three eigen values ($\lambda_1$, $\lambda_2$ and $\lambda_3$) which correspond to the moments of inertia ($I'_{xx}$, $I'_{yy}$ and $I'_{zz}$) about the principal axes. For each eigen value we can compute the corresponding eigen vector. The eigen vectors corresponding to each eigen values are given as $e_{j_i} = e_{11} i' + e_{12} j' + e_{13} k'$, $e_{j_i} = e_{21} i' + e_{22} j' + e_{23} k'$ & $e_{j_i} = e_{31} i' + e_{32} j' + e_{33} k'$ where $i'$, $j'$ and $k'$ are unit vectors along the X’, Y’ and Z’ axes respectively. The transformation of the XYZ system to X’Y’Z’ can be obtained using the transformation matrix, $T$ which is made up of the 9 components of the 3 eigen vectors. The transformation matrix, $T$ is given by

$$T = \begin{pmatrix} e_{11} & e_{21} & e_{31} \\ e_{12} & e_{22} & e_{32} \\ e_{13} & e_{23} & e_{33} \end{pmatrix}$$

From the eigen values and the transformation matrix, the axes ratio and the orientation of the characteristic ellipsoid that best describes the spatial distribution of the particles in the system can be obtained. As the moment of inertia of a system characterizes the resistance of the system to rotation, the component of moment of inertia along the major axis of the system will be least and maximum along the minor axis. Thus the three eigen values which represent the moments of inertia of the three axes (such that $I'_{xx} > I'_{yy} > I'_{zz}$) of the ellipsoid can be written as

- $I'_{xx} = M(a^2 + b^2)$
- $I'_{yy} = M(a^2 + c^2)$
- $I'_{zz} = M(b^2 + c^2)$
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where $a, b \& c$ are the semi-axes of the ellipsoid such that $a > b > c$ and $M$ the total mass of the system. Using the above relations we can estimate the axes ratio of the ellipsoid which best describes the spatial distribution of the particles in the system. The transformation matrix, $T$ describes the spatial directions or the orientation of the ellipsoid with respect to the local reference frame.