Preface

By a graph $G = (V, E)$, we mean a finite undirected graph with neither loops nor multiple edges. For graph theoretic terminology, we refer to Chartrand and Lesniak [3].

In Chapter 1, we collect some basic definitions and theorems on graphs which are needed for the subsequent chapters.

The concept of domination in graphs was first introduced by Ore [22]. At present the study of domination in graphs and related subset problems such as independence, irredundance and covering and more than 75 models of domination in graphs form one of the well developed areas in graph theory. For a nice treatment of the fundamentals of domination in graphs, one may refer to the book by Haynes et al. [16]. Surveys of several advanced topics in domination are given in the book edited by Haynes et al. [17].
Let \( G = (V,E) \) be a graph. A subset \( S \subseteq V \) is called a *dominating set* of \( G \) if every vertex in \( V \setminus S \) is adjacent to some vertex in \( S \). A dominating set \( S \) is called a *minimal dominating set* if no proper subset of \( S \) is a dominating set of \( G \).

The *domination number* of \( G \) is the minimum cardinality taken over all dominating sets in \( G \) and is denoted by \( \gamma(G) \).

Several models of domination have been investigated by several authors over the past 30 years either by imposing conditions on the subgraph induced by a dominating set \( S \) or \( V - S \), or by imposing other restrictions on degrees etc. In fact more than 75 models of domination have been listed in the appendix of the book by Haynes et al. [16].

Sampathkumar [24] introduced the concept of global domination.

A dominating set \( S \) of a graph \( G \) is called a *global dominating set* if \( S \) is also a dominating set of the complement \( \overline{G} \) of \( G \). The minimum cardinality of a global dominating set of \( G \) is called the *global domination number* of \( G \) and is denoted by \( \gamma_g(G) \) or simply \( \gamma_g \).
Cockayne et al. [4] introduced the concept of total domination in graphs. Let $G$ be a graph without isolated vertices. A dominating set of $G = (V, E)$ is called a total dominating set of $G$ if every vertex of $V$ is adjacent to some vertex in $S$, or equivalently, if the induced subgraph $\langle S \rangle$ has no isolated vertices. The minimum cardinality of a total dominating set of $G$ is called the total domination number of $G$ and is denoted by $\gamma_t(G)$ or simply $\gamma_c$.

Let $G$ be a graph such that both $G$ and $\overline{G}$ have no isolated vertices. A dominating set $S$ of $G$ is called a total global dominating set $S$ is a total dominating set of $G$ and also a total dominating set of $\overline{G}$. The minimum cardinality of a total global dominating set of $G$ is called the total global domination number of $G$ and is denoted by $\gamma_{tg}(G)$ or simply $\gamma_{tg}$.

Fractionalization of integer valued graph theoretic parameters is another fascinating research area in graph theory. The values of fractional parameters are useful not only for their own applications but also in providing information about the related integer-valued parameters.

Hedetniemi et al. [18] introduced the fractional version of domination in graphs.
Let $G = (V, E)$ be a graph. Let $g : V \to \mathbb{R}$ be any function. For any subset $S$ of $V$, let $g(S) = \sum_{v \in S} g(v)$. The weight of $g$ is defined by $|g| = g(V) = \sum_{v \in V} g(v)$.

A function $g : V \to [0, 1]$ is called a dominating function (DF) of the graph $G = (V, E)$ if $g(N[v]) = \sum_{u \in N[v]} g(u) \geq 1$ for all $v \in V$.

A dominating function $g$ of a graph $G$ is minimal (MDF) if for all functions $f : V \to [0, 1]$ such that $f \leq g$ and $f(v) \neq g(v)$ for at least one $v \in V$, $f$ is not a dominating function of $G$.

The fractional domination number $\gamma_f(G)$ and the upper fractional domination number $\Gamma_f(g)$ are defined as follows:

$$\gamma_f(G) = min\{|g| : g \text{ is a dominating function of } G\}$$

and

$$\Gamma_f(G) = max\{|g| : g \text{ is a minimal dominating function of } G\}.$$ 

Arumugam et al. [1] introduced the concept of fractional global domination in graphs.

In Chapter 2 we introduce the concept of total global domination in graphs.

A function $g : V \to [0, 1]$ is called a total global dominating
function \((TGDF)\) of \(G\), if for every \(v \in V\),
\[
g(N(v)) = \sum_{u \in N(v)} g(u) \geq 1
\]
and \(g(N[v]) = \sum_{u \notin N[v]} g(u) \geq 1\). A \(TGDF\) \(g\) of a graph \(G\) is called
minimal \((MTGDF)\) if for all functions \(f : V \rightarrow [0, 1]\) such that \(f \leq g\) and \(f(v) \neq g(v)\) for at least one \(v \in V\), \(f\) is not a \(TGDF\).

The fractional total global domination number \(\gamma_{ftg}(G)\) is defined as follows:

\[
\gamma_{ftg}(G) = \min\{|g| : g \text{ is an MTGDF of } G\}
\]
where \(|g| = \sum_{v \in V} g(v)\).

The fractional total global domination number is the optimal solution of the following linear programming problem (LPP).

Minimize \(z = \sum_{i=1}^{n} g(v_i)\)

Subject to \(\sum_{u \in N(v)} g(u) \geq 1\) for all \(v \in V\),

\[
\sum_{u \notin N[v]} g(u) \geq 1 \text{ for all } v \in V \text{ and }
\]
\(0 \leq g(v) \leq 1\) for all \(v \in V\).

In Chapter 2, we present several basic results on fractional total global domination in graphs.

Meir and Moon [21] introduced the concept of a \(k\)-packing and distance \(k\)-domination in a graph as a natural generalisation of the
concept of domination. Let $G = (V, E)$ be a graph and $v \in V$. For any positive integer $k$, let $N_k(v) = \{u \in V : d(u, v) \leq k\}$ and $N_k[v] = N_k(v) \cup \{v\}$. A set $S \subseteq V$ is a distance $k$-dominating set of $G$ if $N_k[v] \cap S \neq \phi$ for every vertex $v \in V - S$. The minimum (maximum) cardinality among all minimal distance $k$-dominating sets of $G$ is called the distance $k$-domination number (upper distance $k$-domination number) of $G$ and is denoted by $\gamma_k(G)$ ($\Gamma_k(G)$).

Arumugam et al. [2] introduced the concept of fractional distance domination in graphs. In Chapter 3 of this thesis we initiate a study of fractional total distance domination in graphs.

Let $G = (V, E)$ be a graph without isolates. A function $f : V \to [0, 1]$ is called a total $k$-dominating function (TKDF) of $G$, if for every $u \in V$, $f(N_k(v)) = \sum_{u \in N_k(v)} f(v) \geq 1$.

A total $k$-dominating function (TKDF) $f$ of a graph $G$ without isolates is called a minimal total $k$-dominating function (MTKDF), if for all functions $g : V \to [0, 1]$, $g$ is not a total $k$-dominating function with $g < f$ and $f(v) \neq g(v)$ for at least one $v \in V$.

The fractional total $k$-domination number $\gamma^t_{k_f}(G)$ and the upper fractional total $k$-domination number $\Gamma^t_{k_f}(G)$ are defined as follows:
\[ \gamma_{kf}(G) = \min \{ |f| : f \text{ is an MTKDF of } G \} \]

\[ \Gamma_{kf}(G) = \max \{ |f| : f \text{ is an MTKDF of } G \}. \]

In Chapter 3, we present several basic results on fractional total \( k \)-domination and bounds for the fractional total \( k \)-domination number of a graph.

In Chapter 4, we introduce the concept of total global convexity graphs and total distance convexity graphs and determine the same for several classes of graphs.