CHAPTER -2

NOTCH FILTER DESIGN TECHNIQUES
Digital Signal Processing (DSP) techniques are integral parts of almost all electronic systems. These techniques are rapidly developing day by day due to tremendous technological developments in high speed computers, integrated circuit fabrication and field programmable arrays (FPGA). With these, digital signal processing has now become more reliable and speed processing is almost near infinity [1-3]. DSP techniques find applications in variety of areas such as in speech processing, data transmission on telephone channels, image processing, instrumentation, bio-medical engineering, seismology, oil exploration, detection of nuclear explosion etc. There are many commonly used DSP operations such as Differentiation, Integration, Hilbert transformation and Filtering. Among these, filtering is the operation that is invariably used in most of the applications. There are various types of filters such as High pass (HP), Low pass (LP), Band stop (BS), Band pass (BP), and Notch filters. Digital notch filters are of two types, IIR and FIR. In this research work, we have focused our attention on the design techniques of application based digital FIR notch filters.

2.1 NOTCH FILTER CHARACTERISTICS AND APPLICATIONS

Detection, estimation and filtering of narrow band signals in the presence of noise represent some of the important applications of signal processing techniques. In most of these applications it is desired to remove the narrow band signal while leaving the broad band energy unchanged. This can be achieved by
passing the signals through a notch filter where the notches are centered on the narrow band signals.

2.1.1 Notch Filter Characteristics

A notch filter is essentially a band stop filter with a *very narrow stop band* and two pass bands. The amplitude response, $H_1(\omega)$, of a typical notch filter (designated as NF1) shown in Figure 2.1 is characterized by a notch frequency $\omega_d$ (radians) and -3 dB rejection band width (RBW). For an ideal notch filter, this RBW should be zero, the pass band magnitude should be unity and the attenuation at the notch frequency should be infinite. We may alternatively have the amplitude response $H_2(\omega)$ of a notch filter (designated as NF2) as shown in Figure 2.2. $H_2(\omega)$ has 180 degrees phase shift at the notch frequency $\omega_d$ i.e. it has opposite signs in the two pass bands. However, the magnitude response $|H_2(\omega)|$ is of the same type as that shown in Figure 2.1. In this thesis, we propose the design methodologies for both these type of filters. Before considering the design aspects of the notch filters, it is relevant to highlight some of the important applications of the notch filters.

2.1.2 Applications of Notch Filters

Some of the applications of the notch filters are as follows:
Notch filter characteristics of NF1 type

Figure 2.1 The normalized amplitude response $H_1(\omega)$ of the notch filter NF1
Notch filter characteristics of NF2 type

Figure 2.2 The normalized amplitude response $H_2(\omega)$ and $|H_2(\omega)|$ of the notch filter NF2
(i) In the area of communications, control, instrumentation and bio medical engineering, notch filters are generally used to eliminate 50/60 Hz power line interference.

(ii) Spatial notch filters are used in transmitting antenna arrays, which are omni-directional except for one null. This feature is used when it is desired to null out one listener in a known direction.

(iii) Notch filters are used to remove spectral lines from a broad band spectrum, so as to eliminate noise due to electromagnetic interference (EMI), as encountered in radars and direction finders [4].

(iv) Switching type of AC & DC motor drives, converters and inverters cause sinusoidal disturbances at certain harmonics of the line frequency. This can be a problem when measuring the velocity of an elevator using an analog tacho generator [14]. This is so because, the primary velocity signal now contains an additive ripple component consisting of line frequency and its low order (2\textsuperscript{nd}, 3\textsuperscript{rd} and 4\textsuperscript{th} order) harmonics. Use of notch filter eliminates such unwanted signals and enables accurate measurements.

(v) Besides the line frequency disturbances, mechanical resonance also cause narrow band interference in many industrial and automatic control
applications. A servomotor is commonly coupled to its load by inherently resilient drive shafts, and the combination possesses selective resonance frequencies. The overall system may not operate smoothly if the resonance frequency signals are not eliminated. Such interference is easily removed by using notch filters [12,13].

(vi) In practical electrocardiogram (ECG) measurements, the primary signal is often contaminated by strong disturbances, which must be removed before the signal is registered for further analysis. The varying ECG contact potentials and breathing artefacts (below 0.5 Hz) cause unwanted base line drift [15]. For stress ECG recordings, this drift may some times make the recording impossible. Such unwanted signals are effectively removed by using appropriate linear phase notch filters. Elimination of dc component of an ECG is another use of notch filters [16]. Notch filters are also used to suppress the secondary artefacts that arise due to electrical interference, as well as cross coupling from frontal EEG while analyzing electro-oculogram [17].

(vii) Adaptive and broad null width notch filters find application in electronic counter countermeasure (ECCM) systems. An important measure for reducing the effects of either electronic countermeasure (ECM) or mutual interference is to avoid saturating, or overloading of the receiver, with
large interfering signals. This can be achieved with proper tuning of radar, associated with adaptive notch filters to evade narrow band spot jamming.

(viii) A filter with broad null width can be used to suppress interference frequencies that may arise due to accidental spikes around 50 Hz in anti spike circuits as a protection measure.

The aforementioned list of application of notch filters is only indicative and not complete. Now, we shall discuss the techniques commonly used to design the notch filters.

2.2 DIGITAL NOTCH FILTER DESIGN TECHNIQUES

Digital notch filters are of two types.[1-3, 5-9].

- Infinite Impulse Response(IIR) filters
- Finite Impulse Response(FIR) filters

Now the salient features of these techniques with special emphasis on FIR digital notch filters are discussed.

2.2.1 IIR Design Techniques

IIR filters are preferred in situations where linearity of phase response is not that important. These filters require much lower order compared to the FIR
ones for the same set of magnitude response characteristics. This implies simplified structure with fewer multipliers and adders. The commonly used IIR filter design methods require transforming the given specification to an equivalent analog filter. Through the use of known analog filter design techniques, analog notch filter is designed first and then it is converted back to the digital domain through inverse transformation. This approach has the advantage that the standard results of analog filter design can be conveniently used. Based upon this approach, one can design Butterworth, Chebyshev (i.e. equiripple) or least-squares error (LSE) design or the design based on filter parameter optimization techniques. The most commonly used transformation is the bilinear transformation connecting ‘$s$’ and ‘$z^{-1}$’ by the relation: $s = \frac{2}{T} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right)$. This is a conformal transformation, and hence the most popular.

As an alternative, one can design digital IIR filters directly in the $z$-domain without reference to the analog domain. There are three methods for designing the IIR filters directly.

(i) Padé approximation method.
(ii) Least-Squares design in time domain.
(iii) Least-Squares design in frequency domain.

Among the above, we discuss the first method as this approach has influence on our approach.
Suppose that the desired impulse response \( h_d(n) \) is specified for \( n \geq 0 \). The filter to be designed has the system function

\[
H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=0}^{N} a_k z^{-k}}
\]

where \( h(k) \) is the unit sample response. The filter has \( L = M + N + 1 \) parameters, namely, the coefficients \( \{a_k\} \) and \( \{b_k\} \), which can be selected to minimize some error criterion. Suppose that we minimize the sum of the squared errors

\[
\varepsilon = \sum_{n=0}^{U} [h_d(n) - h(n)]^2
\]

with respect to the filter parameters \( \{a_k\} \) and \( \{b_k\} \), where \( U \) is some preselected upper limit in the summation. In general, \( h(n) \) is a nonlinear function of the filter parameters and, hence, the minimization of \( \varepsilon \) involves the solution of a set of nonlinear equations. However, if we select the upper limit as \( U = L - 1 \), it is possible to match \( h(n) \) perfectly to the desired response \( h_d(n) \) for \( 0 \leq n \leq M + N \). This can be achieved in the following manner.

The difference equation for the desired filter is

\[
y(n) = -a_1 y(n-1) - a_2 y(n-2) - \ldots - a_N y(n-N) + b_0 x(n) - b_1 x(n-1) + \ldots + b_M x(n-M)
\]  

Suppose that the input to the filter is a unit sample [i.e. \( x(n) = \delta(n) \)]. Then the response of the filter is \( y(n) = h(n) \) and hence (2.3) becomes:
\[ h(n) = -a_1 h(n-1) - a_2 h(n-2) - \ldots - a_N h(n-N) + b_0 \delta(n) - b_1 \delta(n-1) + \ldots + b_M \delta(n-M) \]  

(2.4)

Since \( \delta(n-k) = 0 \) except for \( n = k \), (2.4) reduces to

\[ h(n) = -a_1 h(n-1) - a_2 h(n-2) - \ldots - a_N h(n-N) + b_n, \quad 0 \leq n \leq M \]  

(2.5)

For \( n > M \), since \( \{b_k\} \) are all zero, (2.4) becomes

\[ h(n) = -a_1 h(n-1) - a_2 h(n-2) - \ldots - a_N h(n-N), \quad 1 \leq n \leq N \]  

(2.6)

The set of \((M + N + 1)\) linear equations in (2.5) and (2.6) can be used to solve for the filter parameters \( \{a_k\} \) and \( \{b_k\} \). We set \( h(n) = h_d(n) \) for \( 0 \leq n \leq M + N \), and use the linear equations in (2.6) to solve for the filter parameters \( \{a_k\}, 1 \leq k \leq N \). Then we use these values of \( \{a_k\} \) in (2.5) and solve for the parameter \( \{b_k\}, 0 \leq k \leq M \). Thus we obtain a perfect match between \( h(n) \) and the desired response \( h_d(n) \) for the first \( L = M + N + 1 \) values of the impulse response. This design technique is usually called the Padé approximation procedure.

Some modified IIR notch filter designs have also been reported in the literature. Hirano et al. [18] have realized IIR notch filter function by applying bilinear transformation on second order analog transfer function. The design requires only two multipliers and offers independent variation of notch frequency \( (\omega_d) \) and the 3 dB rejection bandwidth (BW). Laakso et al. [13] have proposed
first and second order IIR notch filters with zeros strictly on the unit circle and poles close to the zeros to ensure a narrow notch width. The first order notch filter is useful in eliminating zero frequency (i.e. dc component) only. The second order notch filter given by

\[ H_0(z) = K_0 \frac{1 - 2 \cos \omega_0 z^{-1} + z^{-2}}{1 - 2r \cos \omega_0 z^{-1} + r^2 z^{-2}} \]  \hspace{1cm} (2.7)

can be designed for an arbitrary notch frequency \( \omega_0 \). In (2.7) \( \omega_0 \) is the normalized frequency in radians, \( r \) is the radius of the complex conjugate pole pair located at the frequency \( \omega_0 \) and \( K_0 \) is a scaling factor such that the maximum gain of the filter equals unity. In this design, the rejection bandwidth (RBW) can be controlled through ‘\( r \)’. RBW decreases as ‘\( r \)’ goes closer to unit circle [13]. However, the quantization error increases when \( \Delta = 1 - | r | \) is made small (since the variance of the quantization error is proportional to \( 1 / \Delta^2 \) [1].

Much research has also been carried out in the area of adaptive notch filtering. In certain applications of signal processing, where it is desired to eliminate unknown or time varying narrow band or sine wave components from the observed time series, it becomes necessary to use adaptive notch filters (ANF). ANFs also find use in retrieval of sinusoids in noise and in tracking and enhancing time varying narrow band / sinusoidal signals in wide band noise [53-56]. Adaptive notch filter designs have been proposed by Thompson [54], Rao and Kung [55], Friedlander and Smith [40], and Nehorai [41], amongst others.
The computational efficiency, stability, convergence and numerical robustness of these methods depend upon the algorithm used for adaptation.

Juan E. Cousseau, Stefan Werner and Pedro Dario Donate have introduced a new family of IIR adaptive notch filters [56] that forms multiple notches using a second-order factorization of an all-pass transfer function. They have proposed two new adaptive filtering algorithms that can achieve fast convergence at low computational cost. This all-pass based notch realization introduces a different compromise between bias and signal to noise ratio (SNR) when compared with realizations previously reported in the literature. It achieves lower bias than other approaches at low SNR. This property is particularly attractive for the estimation and tracking of multiple sinusoids.

One of the major problems in IIR filters is that these designs have non linear phase response and, therefore, introduce phase distortion in general. It is possible to reduce phase distortion by cascading an all-pass phase equalizer. However, in this case, the advantage of lower order IIR filter is lost as cascading an all-pass filter results in an increased order of the over all filter which may some times be comparable to that of an equivalent FIR filter. Moreover, the IIR filters tend to be unstable (due to quantization) when the pole is very near to the unit circle. We, now examine some of the design techniques used for FIR notch filters.
2.2.2 FIR Design Techniques

There are essentially three types of standard design techniques for linear phase FIR filters.
- Frequency Sampling method
- Window Method
- Optimal Filter Design Method

The frequency sampling method is often not used for notch filter design because, the desired frequency response changes rapidly across the notch point leading to large interpolation error. The window method has the advantages that it is easy to use and closed form expressions are available for the window coefficients. Several windows have been reported in the literature, such as Hamming, Hann, Blackman, Bartlett, Papoulis, Lanczos, Tukey, Kaiser and Dolph-Chebyshev [1, 19] etc.

Vaidyanathan and Nguyen [20] introduced FIR eigen filters which are optimal in the least squares sense. Here, the objective function is defined only as the sum of pass band and stop band errors; the error of approximation in the transition band is not included. One of the advantages of eigen filters over other FIR filters is that, they can be designed to incorporate a wide variety of time domain constraints such as the step response and Nyquist constraint in addition to the usual frequency domain characteristics. This method has also been extended to include flatness constraints in the pass band.
The filters designed by using Kaiser window and eigen filter approaches are such that the ripple size grows as we move closer to the band edge. Because of this, the filter performance exceeds (i.e. better than the specification) for most frequencies, except around ω_p and ω_s [10], where ω_p and ω_s are respectively the pass band and stop band edges. The filter is thus, over designed and it is possible to reduce the filter length ‘N’ to meet the given set of specifications. This is achieved by the mini-max design method where the error is uniformly distributed in the pass band and stop band. Using Alternation Theorem and Remez – exchange algorithm, Parks and McClellan [21,22] have proposed versatile algorithm [23] for designing various types of FIR filters including differentiators and Hilbert transformers.

Out of all the FIR designs, Parks and McClellan iterative design yields the best results, although, it too has some inherent limitations. Equiripple designs only consider the specified pass bands and stop bands but the transition bands are not considered in the numerical solution. In fact, the transition regions are considered as ‘don’t care’ regions in the design procedure. As a result, the numerical solution may fail, especially in the transition region. For the optimum design, the filter length is determined by the narrow transition band. If the transition band is too wide, the algorithm will fail in the transition region resulting in overshoot of the frequency response [9].
Besides the standard techniques, a number of other methods are proposed for the design of notch filters in the literature. Among them, a few are discussed here. Tian-Hu yu et al. [24] has proposed two methods for designing the notch filters by exploiting the aforementioned design techniques. In one of the methods, a notch filter $H(\omega)$ is derived from a low pass filter $H_{LP}(\omega)$ by using the relation

$$H(\omega) = 2H_{LP}(\omega) - 1$$

(2.8)

when $H_{LP}(\omega)$ has the passband 0 to $\omega_d$. This transformation provides a change of phase by 180 degrees at the notch frequency $\omega_d$. The frequency response $H(\omega)$ may further be sharpened by using the amplitude change function (ACF) [25]. An alternative method in [24] is based on complementing a narrow pass band (tone) filter $B(\omega)$ to obtain the desired notch filter by using

$$H(\omega) = 1 - B(\omega)$$

(2.9)

Obviously, a narrow band filter $B(\omega)$ will have a large filter order. A number of techniques are, however, available in the literature [26-28] for reducing the number of multiplications.

Byrnes [29] considered spatial notch filters, and derived a polynomial whose magnitude on the unit circle is close to a constant in all the directions except one. Such polynomials are used to design transmitting antenna arrays, which are omni-directional except for one null. The designed filter, however, is a
nonlinear phase FIR filter. Also, in this design the weights are complex in nature and, therefore, the number of multiplications and additions required for computation increases. This in turn also increases the memory requirements.

Another method for designing FIR notch filter was proposed by M.H.Er [30] where the symmetry constraint for the coefficients $h(n)$ was relaxed and therefore the design yields nonlinear phase FIR filters. Two procedures have been proposed in [30] for varying the null width. In the first approach, the mean squared error between the desired unity response and the response of the filter over frequency band of interest is minimized subject to a null constraint and its zero derivative constraints at the frequency of interest. The null width can be increased in discrete steps by setting more derivatives to zero at the notch frequency. In the second approach, a null power constraint over a frequency band of interest is introduced. This approach is found to be more effective in controlling the null width as compared to the derivative constraint methodology. Both of these approaches adopt optimization techniques, which have been efficiently solved in [30]. The limitation of such a design, however, is that it yields nonlinear phase, and does not provide closed form mathematical formula for computation of design weights. Every time the notch frequency $\omega_d$ changes, the optimization procedure has to be re run to obtain the design weights.

Novel analytical designs for maximally flat (MF) and equiripple (ER) FIR notch filters have been proposed in [31-33, 57], based on Zolotarev polynomials
and Jacobs’s elliptical functions. In [31] P. Zahradnik and M. Vlcek have presented a fast analytical tuning procedure for FIR notch filters. This analytical tuning procedure is based on the differential equation of the transformed Chebyshev polynomial. Proposed tuning procedure adjusts a single frequency of the frequency response to the desired value while preserving the nature of the filter.

The same authors have presented a novel analytical design method [32] for highly selective digital optimal equiripple FIR comb filter. So designed comb filters are optimal in the Chebyshiv sense. In this method the number of notch bands, the width of the notch bands and the attenuation in the pass bands can be independently specified. Next with these specifications they have evolved the degree formula and differential equation for generating polynomial of the comb filter. Based on the differential equation, they have described an algebraic recursive procedure for the evolution of impulse response of the filter. This procedure is very robust but highly involved. By using this, highly selective equiripple FIR comb filters with thousands of coefficients can be designed.

P. Zahradnik and M. Vlcek have suggested recursive algorithms [33] for computation of impulse response of notch filters, using many results proposed by them in [35]. These algorithms also, require complex mathematical manipulations. These notch filter designs are optimal and very useful ones though the calculation of the impulse response $h(n)$ requires computations of a number of
parameters. In this method the recursive algorithms are derived which lead to the impulse response coefficients of the notch filter.

Recently in 2009, another design method proposed by P. Zahradnik et al. [57] is an extension to the analytical design of digital FIR comb filter [32]. This design runs from the filter specification through the degree formula to evaluate the impulse response coefficients, by an extremely efficient recursive algorithm. The proposed design method outperforms the standard procedures in terms of speed and robustness. Here also, the design approach requires highly involved mathematics.

To summarize, the aforementioned design approaches for FIR notch filters have the following drawbacks:

1. The window technique is a non optimal design [9].
2. The eigen filter approach results in excessive ripples near the band edges [20].
3. Parks-McClellan method [22-24] as well asEr(374,544),(421,566)’s method [30] are iterative and give no explicit or recursive formulas for computing the design weights.
5. The adaptive notch filters are algorithm dependent.
(6) Algorithms proposed by Zahradnik and Vlcek [31-33, 57] require highly involved mathematical manipulations. The calculation of the impulse response $h(n)$ requires computation of a number of parameters.

We, in this thesis, have proposed alternative design approaches for the design of application specific FIR notch filters. These approaches are free from most of the above listed undesired features.