Appendix A

Phase Diagrams of types of attractors of the governing eqs.

In this work, numerical computations have been reported for various values of $k_2$ and $r_e$ for $\omega = J$. Below we present a detailed investigation of the $k_2$ vs. $r_e$ space for values of $0.5 \leq k_2 \leq 6.0$ in steps of 0.5 and $r_e$ varying between 0 and 2 in steps of 0.2.

We ran the program for 2500 points of the Poincaré section (stroboscopic plot) and deleted the first 2250 points to remove the transients. All runs were started with the initial conditions $\theta = \phi = 45^\circ$. We obtained similar results when $\phi$ was replaced by $-\phi$. As a test case when we ran the program for $\theta = 90^\circ$ the trajectory plot reduced to a continuous curve, indicating regular behaviour for all the values of $k_2$ and $r_e$ considered. At $k_2 = 0.03$ for $r_e = 1.6$, the attractor slowly begins to broaden from a continuous curve and the Lyapunov exponent first becomes positive. There are a number of regular regimes in between the chaotic regimes. In our system, chaos usually appears as a broadening of the attractor as can be seen from the example given in Fig. 3.1. In certain regimes as in Ramamohan et al., (1994) the attractor broadens to such an extent that a subset of the phase space is completely filled.
For a comprehensive study of the chaotic behaviour of the system considered in this work we have analysed the problem for two different initial conditions. In both cases we varied the parameter \( r_e \) ranging from 0 to 2 in steps of 0.2 and \( k_2 \) ranging from 0 to 5 in steps of 0.5. We obtained different types of attractors for these ranges of the parameters and we have classified them into 6 types. These attractors are given in Fig. A.1 of this appendix. The corresponding phase diagrams in \( k_2 - r_e \) space are given in Tables A.1 and A.2.
Figure A.1: Attractors of different types for different values of $k_2$: (1) $\theta = 30^\circ, \phi = 45^\circ, r_e = 0.6, k_2 = 4.0$ (2) $\theta = 30^\circ, \phi = 45^\circ, r_e = 1.8, k_2 = 5.0$ (a regular attractor with two fixed points in the Poincaré section) (3) $\theta = 30^\circ, \phi = 45^\circ, r_e = 1.8, k_2 = 6.0$ (4) $\theta = 30^\circ, \phi = 45^\circ, r_e = 1.4, k_2 = 4.5$ (5) $\theta = 30^\circ, \phi = 45^\circ, r_e = 1.4, k_2 = 5.0$ (6) $\theta = 75^\circ, \phi = 30^\circ, r_e = 2.0, k_2 = 6.0$ (a regular attractor with more than two fixed points in the Poincaré section)
### Table A.2: A Phase diagram of $k_2$ vs. $r_e$ for $\omega = J$, initial conditions $\theta = 75^\circ, \phi = 30^\circ$

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<th>2.5</th>
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Appendix B

Similarities and Differences with the thesis of K. Satheesh Kumar

In this thesis, we have reviewed chaotic dynamics and suspension rheology briefly. Since the problem considered in this work and the problem handled by K. Satheesh Kumar lie in these two areas, he has also reviewed these two areas briefly. In these reviews there are some similarities. In what follows we have included in this appendix the similarities and differences of this thesis with the thesis entitled studies on the chaotic rheological parameters of periodically forced suspensions of weak Brownian slender bodies in simple shear flow submitted to the Cochin University of Science and Technology by K. Satheesh Kumar (1997)1

The main thrust of this thesis is the demonstration of chaos and a study of its properties in a physically realizable fluid dynamic system. That is, the chaotic evolution of the orientations of individual particles of finite aspect ratio ranging from 0 to 2 is analysed in detail. Similarly, K. Satheesh Kumar has analyzed, in his thesis, the effect of evolution of the orientations of individual particles of infinite aspect ratio. In short, the dynamics of periodically forced particles of finite aspect ratio is analyzed in this thesis whereas K. Satheesh

1studies on the chaotic rheological parameters of periodically forced suspensions of weak Brownian slender bodies in simple shear flow, Kumar, KS (1997)
Kumar has studied the rheology of a suspension of long slender rods (of infinite aspect ratio).

The analysis of dynamics in this thesis is of dilute suspensions of periodically forced spheroids in the presence of weak or negligible Brownian motion in simple shear flow. K. Satheesh Kumar has considered slender rods instead of spheroids keeping the other assumptions on the suspensions the same. In fact there is no similarity in the main contributions of this thesis with that of K. Satheesh Kumar. To get overall idea of the differences and similarities we list out the features of this thesis which differ from that of K. Satheesh Kumar.

We have numerically analysed a fluid dynamic system, namely, the dynamics of periodically forced spheroids in simple shear flow. This system is physically realizable and technologically important. During the computations we observed several practically and fundamentally important phenomena like strong dependence of the results on aspect ratio, a new Class I intermittency, etc. We have also proposed a new control algorithm.

The main contributions of this thesis can be summarised as follows. Firstly, we have analysed the results of our computations in detail and projected the aspect ratio dependence as a potential tool to segregate particles of a given shape from a suspension of particles having different shapes but similar sizes. We also have observed that the approach used by Strand(1989) for the strong Brownian limit is inappropriate in the chaotic regimes corresponding to the weak Brownian limit. Our results indicate a strong dependence of the solutions obtained on the aspect ratio of the spheroids. This strong dependence on the aspect ratio can be utilized to separate particles from a suspension of particles having different shapes but similar sizes.
Secondly, we have reported certain aspects of the problem that are of interest to the nonlinear dynamics community also. In this work we have reported a physically realisable system in which the possibility of an interesting and novel type of Class I intermittency has been demonstrated, namely it is an example of one of the very few physically realisable chaotic systems showing the phenomenon of a non hysteretic form of Class I intermittency with nearly regular reinjection period. Price and Mullin (1991) have observed experimentally a similar type of phenomenon in which a hysteretic form of intermittency with extreme regularity of the bursting is observed. The system described in this work appears to be one of the very few ODE systems describing a physically realisable system showing a non hysteretic form of Class I intermittency with nearly regular reinjection period. The regularity of the bursting is unaffected by variation in the control parameter. The maximum Lyapunov exponents of the bursts and laminar phase are estimated separately and indicate existence of chaos. The length of the laminar phase shows scaling behaviour typical of Class I intermittency near the tangent bifurcation and also shows new scaling behaviour. Return maps of the dynamical system are presented to explain the behaviour observed. The system also demonstrates some interesting features such as new scaling behaviour away from the onset of intermittency and the number of the bursts during a particular realization varying smoothly with the control parameter. These results are presented in terms of the evolution of the orientation of a spheroid subjected to an external periodic force immersed in a simple shear flow.

Thirdly, we have demonstrated that controlling the chaotic dynamics of periodically forced particles by a suitably engineered novel control technique which needs little information about the system and is easy to implement leads
to the possibility of better separation than otherwise possible. In this work we have demonstrated that controlling the chaotic dynamics of periodically forced particles leads to the possibility of better separation. Utilizing the flexibility of controlling chaotic dynamics in a desired orbit irrespective of initial state, it is demonstrated that it is theoretically possible to separate particles much more efficiently than otherwise possible from a suspension of particles having different shapes but similar sizes especially for particles of aspect ratio $r_e > 1.0$. The strong dependence of the controlled orbit on the aspect ratio of the particles demonstrated in this work may have many applications such as the development of computer controlled intelligent rheology. The results of this work also suggest that control of chaos in this problem may have many applications.

Finally, we have proposed a novel algorithm based on parametric perturbation for control of chaos in this work. The method proposed is comparatively easy to implement and needs almost no information about the system. One of the main advantages of this control algorithm is the possibility of pre-targeting the length of the controlled period obtained by suitably engineering the control technique. In addition we demonstrate certain advantages of this novel technique over two well-known algorithms, namely control by periodic parametric perturbation and control by addition of a second weak periodic force, such as the possibility of switching behaviour, pre-targeting the period, stabilising high period orbits etc. We have also demonstrated the applicability of the technique in certain numerical models of physical systems. We have demonstrated the successful application of the new algorithm in a rather difficult problem, namely, the control of the dynamics and the rheological parameters of periodically forced suspensions of slender rods in simple shear flow and also in the Bonhoeffer-Van der Pol (BVP) oscillator.
Appendix B Similarities and Differences

As part of considering different systems to test the applicability and suitability of the new control of chaos algorithm developed during this work, we have considered the case of chaotic rheological parameters of periodically forced slender rods only to compare the efficiency of the new control algorithm with some other control algorithms. The control of chaos algorithm developed in this work is used to control chaotic rheological parameters in the work of K. Satheesh Kumar, but the thrust in his work is on the possibility of obtaining novel rheological parameters and not on the control algorithm. In short, we have just reproduced the expressions of the rheological parameters from the thesis of K. Satheesh Kumar, as chaotic rheological parameters happen to be one of the different systems we considered to test the novel control of chaos algorithm for efficiency.

The other similarity is the assumptions of the model of the suspension considered in these two thesis are similar with deviation from the thesis of K. Satheesh Kumar in the particle shape. We developed the orientation evolution equations based on Brenner (1974) and his scaling. At the same time, the development of the orientation evolution equations in the work of K. Satheesh Kumar is based on the approach of Strand and Kim (1992) and the scaling in his thesis is also different from that of ours.

Except for these two similarities all other results are the original contributions of this thesis which has been published in international referred journals.

Literature Cited

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