INTRODUCTION

Fuzzy set theory after its introduction by L.A. Zadeh [39] has become important with application in almost all areas of mathematics, of which one is the area of topology. Zadeh took the closed unit interval \([0,1]\) as the membership set. J.A. Goguen [12] considered different ordered structures for the membership set. He considered a fuzzy subset as a generalized characteristic function. Thus the ordinary set theory is a special case of the fuzzy set theory where the membership set is \([0,1]\). Goguen suggested that a complete and distributive lattice would be a minimum structure for the membership set.

The theory of general topology is based on the set operations of union, intersection and complementation. Fuzzy sets do have the same kind of operations. It is therefore natural to extend the concept of point set topology to fuzzy sets resulting in a theory of fuzzy topology. The study of general topology can be regarded as a special case of fuzzy topology, where all fuzzy sets in question take values 0 and 1 only. C.L. Chang [7] was the first to define a fuzzy topology. Since then an
extensive study of fuzzy topological spaces has been carried out by many researchers. Many mathematicians, while developing fuzzy topology have used different lattice structures for the membership set like (1) completely distributive lattice with 0 and 1 by T.E. Gantner, R.C. Steinlage and R.H. Warren [11], (2) complete and completely distributive lattice equipped with order reversing involution by Bruce Hutton and Ivan Reilly [21], (3) complete and completely distributive non-atomic Boolean algebra by Mira Sarkar [32], (4) complete chain by Robert Badarad [2] and F. Conard [8], (5) complete Brouwerian lattice with its dual also Brouwerian by Ulrich Höhle [18], (6) complete Boolean algebra by Ulrich Höhle [19], (7) complete and distributive lattice by S.E. Rodabaugh [31]. R. Lowen [24, 25] modified the definition of fuzzy topology given by C.L. Chang and obtained a fuzzy version of Tychonoff theorem, but he lost the concept that fuzzy topology generalizes topology. We take the definition of fuzzy topology in the line of Chang with closed interval [0,1] as membership lattice in the first three chapters. In the last chapter we replace [0,1] by an arbitrary complete and distributive lattice.

In topology, the concept group of homeomorphisms of a space has been studied by various authors. Many problems
relating the topological properties of a space and the algebraic properties of its group of homeomorphisms were investigated. In 1959 J. de Groot [14] proved that any group is isomorphic to the group of homeomorphisms of a topological space. A related problem is to determine the subgroup of the group of permutations of a fixed set $X$, which can be represented as the group of homeomorphisms of a topological space $(X,T)$ for some topology $T$ on $X$. The Ph.D. thesis of P.T. Ramachandran [29] is a study on these and related problems. Ramachandran proved that no non-trivial proper normal subgroup of the group $S(X)$ of all permutations of a fixed set $X$ can be represented as the group of homeomorphisms of a topological space $(X,T)$ for any topology $T$ on $X$. (see also [30]) Also he has proved that the group generated by cycles cannot be represented as group of homeomorphisms.

In this thesis, a major part is an attempt to have an analogous study in fuzzy topological spaces. In the first chapter we investigate some relations between group of fuzzy homeomorphisms of a fuzzy topological space $X$ and the group of permutations of the ground set $X$. It is observed that non-homeomorphic fuzzy topological spaces can have isomorphic group of fuzzy homeomorphisms. In contrast
to the results in the topological context, we prove in this chapter that the subgroups generated by cycles and proper normal subgroups of $S(X)$ can be represented as groups of fuzzy homeomorphisms for some fuzzy topology on $X$. The relation between group of fuzzy homeomorphisms and group of homeomorphisms of the associated topology are discussed and it is proved that for topologically generated fuzzy topological spaces both the groups are isomorphic.

In 1970 R.E. Larson [23] found a characterization for completely homogeneous topological spaces. A method to construct completely homogeneous fuzzy topological spaces is also given in the first chapter.

Čech closure spaces is a generalization of the concept of topological spaces. Eduard Čech, J. Novak, R. Fric, Ramachandran and many others have studied this concept and many topological concepts were extended to the Čech closure spaces. In 1985 A.S. Mashhour and M.H. Ghanim [27] defined Čech fuzzy closure spaces and they extended Čech proximity to fuzzy topology. In the second chapter we investigate the group of fuzzy closure isomorphisms and extend some results discussed for the fuzzy topological spaces in the first chapter to Čech fuzzy closure spaces. Also in this
chapter an attempt is made to study the lattice structure of the set of all Čech fuzzy closure operators on a fixed set X.

In 1936, G. Birkhoff [4] described the comparison of two topologies on a set and proved that the collection of all topologies on a set forms a complete lattice. In 1947, R. Vaidyanathaswamy [36] proved that this lattice is atomic and determined a class of dual atoms. In 1964, O. Fröhlich [9] determined all the dual atoms and proved that the lattice is dually atomic. In 1958, Juris Hartmanis [16] proved that the lattice of topologies on a finite set is complemented and raised the question about the complementation in the lattice of topologies on an arbitrary set. H. Gaifman [10] proved that the lattice of topologies on a countable set is complemented. Finally in 1966, A.K. Steiner [34] proved that the lattice of topologies on an arbitrary set is complemented. Van Rooij [37] gave a simpler proof independently in 1968. Hartmanis noted that even in the lattice of topologies on a set with three elements, only the least and the greatest elements have unique complements. Paul S. Schnare [33] proved that every element in the lattice of topologies on a set X, except the least and the greatest elements have at least n−1 complements when X is finite such that \(|X| = n > 2\) and have infinitely many complements when X is infinite.
In 1989, S. Babusundar [1] proved that the collection of all fuzzy topologies on a fixed set forms a complete lattice with the natural order of set inclusion. He introduced $t$-irreducible subsets in the membership lattice and proved that the existence of minimal $t$-irreducible subsets in the membership lattice is a necessary and sufficient condition for the lattice of fuzzy topologies to have ultra fuzzy topologies and solved complementation problem in the negative.

Lattice structure of the set of all fuzzy topologies on a given set $X$ is further explored in the third chapter. For a given topology $t$ on $X$, we have studied properties of the lattice $\mathcal{F}_t$ of fuzzy topologies defined by families of lower semicontinuous function with reference to $t$ on $X$. We obtain from the lattice $\mathcal{F}_t$ that the lattice of fuzzy topologies forms a complete atomic lattice. This lattice is not complemented and it has no dual atoms. Also it is proved that topologically generated fuzzy topologies and crisp topologies have complements.

Some of the results in the previous chapters raise the question of how far the results do hold good when the membership lattice is not $[0,1]$ but an arbitrary complete and distributive lattice. The results of chapter 4 are in this direction.