CHAPTER 4

BEHAVIOURAL STUDY OF QUEUE SYSTEM UNDER SERVICE SURRENDER FACILITY
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4.1 INTRODUCTION

This chapter is an attempt to discuss a queue system containing two different types of queues namely primary and secondary queues. Keeping in the mind of server’s certain limitations and depending upon the kind of service given by server, the existence of secondary queue is being decided. The average waiting time in the system by a customer and service wastage rate has been explored in the chapter. The results have been analyzed through graphs. The service wastage rate throws some light about the amount of instability in the system when the secondary queue is in existence.

Queue models provide the predictions of behaviour of the system (e.g. expected waiting time, the average number of waiting customers and so far). These predictions help us to anticipate situation and to take appropriate measures to shorten the queue length. In the literature, right from Erlang (1909) to Kendall (1951), Johnson,N.L. etal (1969) ,Prabhu and Bhatt (1963), Maggu (1970), Singh T.P (1996, 2005, 2007) and also up to the recent years, the main focus is given on the analytical behavior of queues which form in front of a service channel. But in practical life sometimes we find some specific types of queue system where the services can be returned after sometime which can be named the surrendering facility of service to the customer. Bartholomew and forbes (1979) search out the statistical technique for manpower planning model and find the complete length of the service (CLS) before surrendering. Recently, Kane
and N.Kane (2004) made analysis of queues in which some customers who have already taken the service surrenders it, the reason best known to them. This chapter is an extended work of Kane and Neela kane (2004). The analytical study of the system has been made through graph and different tables. The service wastage rates are derived and a graphical analysis of the system has been studied in wider sense followed by some significant observations.


4.2 PRACTICAL SITUATION:

The practical situation can be observed on a railway reservation counter or an aeroplane service where the reserved tickets are provided to the travellers. Due to some unavoidable circumstances, the travellers return the tickets to the service channel which has surrendering facility. Another situation can also be observed on a service system of locker facility given by banks or in library for issuing books to the students, admission in engineering/ business schools/medical colleges etc where one has an option to surrender the allotted seat.

4.3 FORMULATION OF PRIMARY AND SECONDARY QUEUE:

Consider a service channel with available services to a finite number of customers (say N) after serving to N customers the server has nothing to provide the service to (N+1)th customer waiting in queue. The waiting customer has an option either to register themselves into a new list or to quit from the system. The waiting list which conditionally depends on
service surrender can be named as secondary queue. Hence, initially the
customer joins the primary queue and later has a choice whether to join the
secondary queue or quit the system. The duration for maximum waiting
time in secondary queue is uncertain.

4.4 MATHEMATICAL DESCRIPTION OF SECONDARY QUEUE:

Let \( p \) be the probability that the customer enjoys the service for a unit time.
The probability that he surrenders the service some time during a unit time
is \((1-p) = q \) (say). Therefore the probability that a customer completes ‘x’
units of time before he surrenders the service is a Geometric variable with
p.m.f.

\[
P(X=x) = p^{x-1}(1-p), \quad x=1, 2, \ldots
\]

As the probability \( p \) differs from person to person having value in the range
[0,1], it is worth to consider it a random variable. We assume the CLS (U)
follows the compound Geometric beta distribution, can be analyzed the
secondary queue model in the current situation.

If \( u \) units of time are utilized by a customer before he surrenders the
service, then the random variable \( U \) follows the geometric distribution with
parameter \( p \) and p.m.f is given by:

p.m.f is given by:

\[
P(U=u) = p^{u-1}(1-p), \quad u=1,2,\ldots
\]

Further, \( p \) is also a random variable with Beta distribution like

\[
F(p) = [1/B(r,s)] p^{r-1}(1-p)^{s-1}, \quad r, s>0
\]

\[
0<p<1
\]
Then the probability distribution of Compound Geometric Beta distribution of CLS becomes

\[ P[U=u] = f(u) = \frac{B(r+u-1,s+1)}{B(r,s)}, \quad u=1,2,\ldots \]  
\[ (1.1) \]

\[ r, s > 0 \]

Assume that the event of surrendering service by a customer and allotment of service to a customer from a secondary queue happens only at the discrete time points.

4.5 Theorem: The average waiting time of a customer in the secondary queue is \((r+s-1)/(s-1)\)

Proof: From (1.1) and proceeding on the basis of line of Kane & Neela kane (2004), we have:

\[ E(u) = \left[ \frac{1}{B(r,s)} \right] \sum_{u=1}^{\infty} uB(r + u - 1, s + 1) \]

\[ = \frac{1}{B(r,s)} \left\{ \int_0^1 u^{r-1}(1-u)^s du + 2 \int_0^1 u^r (1-u)^s du + \cdots \right\} \]

\[ = \frac{1}{B(r,s)} \left\{ \int_0^1 (1-u)^s [u^{r-1} + 2u^r + 3u^{r+1} + \cdots] du \right\} \]

\[ = \frac{1}{B(r,s)} \left\{ \int_0^1 [u^{r-1}(1-u)^{-2}(1-u)^s] du \right\} \]

\[ = \left( \frac{(r+s-1)/(s-1)}{[B(s-1,r)]} \right) \]

\[ = (r+s-1)/(s-1) \]  
\[ (1.2) \]

Hence, the overall average waiting time in the system could be expressed as \( W + [(r+s-1)/(s-1)] \), where \( W \) is the average waiting time in primary queue.
4.6 SERVICE WASTAGE RATE:

The distribution function of the Compound Geometric Beta distribution which is the waiting time distribution in the secondary queue is given by

\[ F(U) = \sum_{u=1}^{U} f(u) \]

\[ = \sum_{u=1}^{U} \left[ \frac{B(r + u - 1, s + 1)}{B(r, s)} \right] \]

After simplification, we get

\[ F(U) = 1 - \left[ \frac{(r+s+U)}{s} \right] f(U+1) \quad (1.3) \]

If \( G(U) \) is the probability that a customer utilizes the service at least for a period of time \( U \),

\[ \therefore G(U) = 1 - F(U-1) \]

\[ = \left[ \frac{(r+s+U-1)}{s} \right] f(U) \quad (1.4) \]

\( G(U) \) can be considered the survival function of the customer in the system.

On the basis of an argument by Bartholomew, DJ and Forbes, A.F. (1979) leads to the rate of service wastage denoted by \( W_u \) as

\[ W_u = \frac{f(U)}{G(U)} \]

\[ = \left[ \frac{s}{(r+s+U-1)} \right] \quad (1.5) \]

Empirically the equation (1.5) can provide some conclusion based on the following tables 1 to 4 and graph 1(a), 1(b), 2(a), 2(b), 3(a), 3(b) & 4(a), 4(b) respectively. In graph 1(a), 1(b), 2(a), 2(b) 3(a), 3(b), 4(a), 4(b) horizontal lines shows the value of \( U \) and vertical lines shows the values of \( s \) and \( r \) resp.
Table 1

<table>
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4.7 GRAPHICALLY REPRESENTATION:

![Graphs](image1.png)

1 (a) ![Graphs](image2.png)
1(b) ![Graphs](image3.png)

2 (a) ![Graphs](image4.png)
2(b) ![Graphs](image5.png)

3(a) ![Graphs](image6.png)
3 (b) ![Graphs](image7.png)

4.8 ANALYSIS OF TABLES AND GRAPHS:

1) From table 1, 2, 3 clearly $W_u$ is a function of $U$, $r$ and $s$.

2) The parameters ‘r’ and ‘s’ assumes their own values as per a particular queuing system under our study.

3) For a known value of $E(u)$, $W_u$ gives the amount of instability in the system as clear from table and graph.
4) The different values of $W_u$ exhibit the monotonic behavior of the system for different sets of value of $U, r$ and $s$.

5) From graph, it is clear for smaller value of $r$, the service wastage rate decreases rapidly in beginning and then gradually tends to stability.

**4.9 CONCLUDING REMARKS:**

It has been observed that the monotonic behavior of queuing system with service surrender facility is dominated by secondary queue. In the above discussed system a customer faces one of the following situations while joining the system.

(a) When the customer joins the system there is a primary queue and service is available with the server. The two possibilities arise in this case:

Either (i) Customer get the service after reaching the service counter.

Or (ii) Service is not available with the server and then customer has to decide whether to join the secondary queue or not.

(b) At the time of joining the system, there is a primary queue and server has nothing to provide the customer. In this situation secondary queue is in progress for example when the candidates do not get seat in engineering or medical institute of repute in first counselling then either they get registered themselves for second counselling or they do not register themselves it’s all depends upon their situations.

(c) At the time of joining the system there is no primary queue customer directly approaches the server for registering himself in secondary queue for example in any store, some material being out of stock then the store keeper registered the needed customer’s demand as a secondary queue.

(d) From the graph it is clear that, for smaller value of parameter ‘$r$’ the service wastage decreases rapidly in the beginning and then gradually
stabilizes after some interval of time. Further $W_u$ will increase after a long time showing a monotonic behavior for different set of value $U$, $r$ and $s$ which is close to real world problem since in real situation due to certain factors such as death of customer, for better and alternate opportunities in waiting list of certain jobs, the wastage rate gives the amount of instability in the system.

The average time taken by a customer to reach the server i.e $W$ is different in all these cases but the average waiting time in the secondary queue remains the constant which is equal to $(r+s-1)/(s-1)$. In order to find the expected waiting time of a customer in the system it is necessary to take into account the status of the system at the time of joining it by a customer. The status of the system at that time will decide the value of $W$. 