CHAPTER 7

FUZZY QUEUE MODEL TO INTERDEPENDENT COMMUNICATION SYSTEM WITH BULK ARRIVALS
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7.1 INTRODUCTION:

Queue models have led to accurate analysis of modern communication system. These models are playing significant role in modeling voice calls. With the advent of faxes and the internet, the nature of traffic has changed dramatically, As a result, packet switched networks data traffic (where modems do the talking instead of humans) are very much used which are different from voice traffic in their statistical characteristic.

Circuit switching was developed to handle voice traffic but it can also handle digital data; communication etc. Circuit switching implies that there is dedicated communication path between two stations. The path is connected sequence of links between network nodes. Packet switching involves dividing data messages into small bundles of information and transmitting them through communication networks to their indented destination using computer-controlled switches. With packet switching, data are divided into smaller segments called ‘Packets’ prior to transmission through the network. Hence, each message is divided packets, and each packet can take a different path through the network. Consequently, all packets do not necessarily arrive at the receiver and with the same time or in the same order in which they transmitted. Because packets are small, the hold time is generally quite short, message transfer is near real time and blocking cannot occur.

In packet switching operation single node to node link can be dynamically shared by many packets over time so line efficiency is greater, the packets
are queued up and transmitted as rapidly as possible over the link. It can perform data rate conversion because two stations of different data rates can exchange packets because each connects to its node at its proper data rate. In packet switching network, packets are accepted but the delivery delay increases. Properties of priorities can be used i.e if a node has a number of packets queued for transmission, it can transmit the higher priority packets first.

In all packet node communication systems, network resources are managed by statistical multiplexing or dynamic memory allocation in which a communication channel is effectively divided into an arbitrary number of logical bit rate channels or data streams. The delay in packet switching can be recommended by utilizing the statistical multiplexing in communication systems. Mathematical analysis of queue models provide the basic framework in communication network since a close resemblance can be seen between queue models and communication system. The messages can be treated as waiting lines and all the concerned activities, capacity of buffer can be regarded as capacity of queue system while service discipline may be taken as FIFO depending upon the situation. In fact, buffer is a region of physically memory storage used to temporarily hold data while it is being moved from one place to another, buffer are typically used when there is a difference between the rate at which data is received and the rate at which it can be processed or in case that these rates are variable (eg. Online video streaming). Most of the researchers working in this field studied the communication network as an interconnected queues with the assumption that arrival and service patterns are interdependent Jenq,(1984), Hosida (1993), Srinivasa Rao etal.(2000).
Few attempts have been made in developing and analyzing an interdependent communication networks which most often are found in packet radio systems, store and forward data or voice transmission, teleprocessing etc. through these dependencies can have marked effect on system performance and must be taken into account for any realistic analysis.

Srinivasa Rao etal.(2003) developed an interdependent communication networks with the assumption that arrival and transmission process at node of the network are correlated and follows a bi-variate Poisson process having joint probability mass function (pmf) of the form given by Milne (1974). Arrivals of packets are assumed to be single i.e each packet arrives of its own. But in packet switching the messages is divided into small packets of random length. Therefore, message arrivals to the buffer in bulk of packets. Srinivas Rao etal. (2006) further made an attempt to develop an interdependent communication system assuming that arrivals are in bulk.

Recently, Singh T.P (2011) extended the study made by Srinivas Rao (2006) in ordered to develop a queue model to the communication network. Using Chapmann Kolmogrov equations Singh T.P further calculated the average delay transmission and variance of number of packets in buffer through tables. This chapter is an extended work of Srinivasa Rao (2006) and Singh T.P (2011) in the sense that analysis of performance of multiplexing is measured by approximating the arrival packets and transmission bi-variate under fuzzy environment. Communication has been studied due to unpredictable and uncertain nature of demand at transmission line, congestion occurs in communication system. Therefore, the arrival or messages as well as the concerned activity in transmission of messages, as service patterns etc.
have been assumed fuzzy in nature. The Co- variances between the composite arrival and transmission completion have also been assumed fuzzy in nature. Our model is more realistic than the work done of Srinivasa & Singh T.P. This interdependence network can reduce the mean buffer length and the variability of buffer contents. The delay in packet switching can be reduced by utilizing the statistical multiplexing in communication network i.e reduces the average delay of transmission.

7.2 Fuzzy Set:

In the universe of discourse $X$, a fuzzy subset $\tilde{A}$ on $X$ is defined by the membership function $\mu_{\tilde{A}}(X)$ which maps each element $x$ into $X$ to a real number in the interval $[0, 1]$. $\mu_{\tilde{A}}(X)$ Denotes the grade or degree of membership and it is usually denoted as

$$\mu_{\tilde{A}}(X): X \rightarrow [0,1].$$

If a fuzzy set $A$ is defined on $X$, for any $\alpha \in [0, 1]$, the $\alpha$-cuts $^\alpha A$ is represented by the following crisp set,

Strong $\alpha$-cuts: $^\alpha_+ A = \{ x \in X \mid \mu_{A}(x) > \alpha \}; \alpha \in [0,1]$

Weak $\alpha$-cuts: $^\alpha A = \{ x \in X \mid \mu_{A}(x) \geq \alpha \}; \alpha \in [0,1]$

Hence, the fuzzy set $A$ can be treated as crisp set $^\alpha A$ in which all the members have their membership values greater than or at least equal to $\alpha$. It is one of the most important concepts in fuzzy set theory.

Notations:

$\tilde{\lambda} : \text{Fuzzy arrival rate}$

$\tilde{\mu} : \text{Fuzzy service rate}$
7.3 TRIANGULAR FUZZY NUMBER:

A fuzzy number is simply an ordinary number whose precise value is somewhat uncertain. Taking \([a_1, a_2]\) supporting interval and the point \([a_m, 1]\) as the peak, we define a triangular fuzzy number \(\tilde{A}\) with membership function \(\mu_A(x)\) defined on \(R\) by confidence,

\[
\tilde{A} = \begin{cases} 
\frac{x-a_1}{a_m-a_1}, & a_1 \leq x \leq a_m, \\
\frac{a_2-x}{a_2-a_m}, & a_m \leq x \leq a_2, \\
0, & \text{otherwise}
\end{cases}
\]

In general practice, the points \(a_m \in (a_1, a_2)\) is located at the mid of the supporting interval i.e \(a_m = \frac{a_1 + a_2}{2}\).

Putting these values, we get

\[
\tilde{A} = \begin{cases} 
\frac{2(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq \frac{a_1+a_2}{2}, \\
\frac{2(a_2-x)}{a_2-a_1}, & \frac{a_1+a_2}{2} \leq x \leq a_2, \\
0, & \text{otherwise}
\end{cases}
\]

i.e three values \(a_1, a_m,\) and \(a_2\) construct a triangular number denoted by \(\tilde{A} = (a_1, a_m, a_2)\).

7.4 TRIANGULAR FUZZY NUMBER OPERATION:

Let \(\tilde{A} = (a_1, a_2, a_3)\) and \(\tilde{B} = (b_1, b_2, b_3)\) be two TFN, then the arithmetic operation on \(\tilde{A}\) and \(\tilde{B}\) are given as follows:

**Addition** \(\tilde{A} + \tilde{B} = [a_1+b_1, a_2+b_2, a_3+b_3]\)
Subtraction $\mathbf{A} - \mathbf{B} = [a_1-b_3, a_2-b_2, a_3-b_1]$

Multiplication $\mathbf{A} \times \mathbf{B} = [a_1b_2+a_2b_1-a_2b_2, a_2b_2, a_2b_3+a_3b_2-a_2b_2]$

Division $\mathbf{A} / \mathbf{B} = [2a_1/b_1+b_3, a_2/b_2, 2a_3/b_1+b_3]$

Provided $\mathbf{A}$ and $\mathbf{B}$ are all non-zero positive numbers.

**7.5 DEFEUZZIFICATION OF TFN:**

If $\mathbf{A} = (a_1, a_2, a_3)$ is a TFN then its associated crisp number is given by Yager’s formula as follows:

$$A = \frac{a_1+2a_2+a_3}{4}$$

**7.6 MODEL DESCRIPTION & NOTATION:**

In this communication network model, we consider the arrival of packets and number of transmission are correlated. Both follows a bi-variate Poisson process having joint probability mass function based on the line of Milne (1974) & Srinivasa Rao, etal.(2000). The capacity of buffer is assumed to be infinite and the number of packets arrival in any module is taken as a random variable $x$ in fuzzy environment.

$\lambda_x$ : Arrival rate of message of size having $x$ packets in fuzzy.

$\varepsilon_x$ : Covariance between arrival of packets and number of transmission completion in fuzzy.

$\mu$ : Average transmission rate in fuzzy.

$C_x$ : Probability that a batch of size $x$ packets will arrive to buffer in fuzzy environment.
The composite arrival rate of packets $\bar{\lambda} = \sum_x \bar{\lambda}_x$ and the covariance of the composite arrivals and transmission completions $\bar{\varepsilon} = \sum_x \bar{\varepsilon}_x$.

The covariance is generated through bit dropping of flow control mechanism inducing the dependence between arrival of messages and service transmissions. The network diagram is shown as:

\[
\begin{array}{c}
\text{Bit dropping or Flow control}
\end{array}
\]

7.7 MATHEMATICAL STUDY:

Connecting the probability consideration, the differential difference equation of the fuzzy system is in transient form can be depicted as:

\[
\begin{align*}
\bar{P}_n'(t) &= -(\bar{\lambda} + \bar{\mu} - 2\bar{\varepsilon})\bar{P}_n(t) + (\bar{\mu} - \bar{\varepsilon})\bar{P}_{n+1}(t) + (\bar{\lambda} - \bar{\varepsilon}) \sum_{r=1}^n \bar{P}_{n-r}(t) C_r, \\
&\quad n \geq 1 \quad (1)
\end{align*}
\]

\[
\bar{P}_0'(t) = -(\bar{\lambda} - \bar{\varepsilon})\bar{P}_0(t) + (\bar{\mu} - \bar{\varepsilon})\bar{P}_1(t)
\]

7.8 In steady state:

The steady state condition is reached when the behavior of the system becomes independent of the time.

The steady state equations are $t \to \infty$

\[
\begin{align*}
0 &= -(\bar{\lambda} + \bar{\mu} - 2\bar{\varepsilon})\bar{P}_n + (\bar{\mu} - \bar{\varepsilon})\bar{P}_{n+1} + (\bar{\lambda} - \bar{\varepsilon}) \sum_{r=1}^n \bar{P}_{n-r} C_r, \\
&\quad n \geq 1 \\
0 &= -(\bar{\lambda} - \bar{\varepsilon})\bar{P}_0 + (\bar{\mu} - \bar{\varepsilon})\bar{P}_1, \quad n \geq 1 \quad (2)
\end{align*}
\]

Assuming p.g.f of the number of packets in the buffer and the number of packets that the message is divided:
\[ P(\tilde{z}) = \sum_{n=0}^{\infty} P_n \tilde{z}^n, |\tilde{z}| \leq 1 \]

\[ C(\tilde{z}) = \sum_{n=0}^{\infty} C_n \tilde{z}^n, |\tilde{z}| \leq 1 \]

Multiplying (2) with \( \tilde{z}^n \) & summing over \( n = 0 \) to \( \infty \), after simplification, we get

\[ 0 = -(\lambda - \varepsilon) \tilde{P}(\tilde{z}) - (\mu - \varepsilon) [\tilde{P}(\tilde{z}) - \tilde{P}(0)] + \frac{(\mu - \varepsilon)}{\tilde{z}} [\tilde{P}(\tilde{z}) - \tilde{P}(0)] + (\lambda - \varepsilon) \tilde{C}(\tilde{z}) \tilde{P}(\tilde{z}) \]

(3)

From (3)

\[ \tilde{P}(\tilde{z}) = \frac{(\mu - \varepsilon)(1 - \tilde{z})}{(\mu - \varepsilon)(1 - \tilde{z}) - (N - \tilde{z})(1 - \tilde{C}(\tilde{z}))} \tilde{P}_0 \]

(4)

Let the batch size \( \tilde{C}_x \) is geometrically distributed:

i.e \( \tilde{C}_x = (1 - \alpha)\alpha^n \quad 0 < \alpha < 1 \) then,

\[ \tilde{C}(\tilde{z}) = \frac{(1 - \alpha)\tilde{z}}{1 - \alpha \tilde{z}} \]

(5)

Use (5) in (4) we have

\[ \tilde{P}(\tilde{z}) = \frac{(\mu - \varepsilon)(1 - \alpha \tilde{z})\tilde{P}_0}{(\mu - \varepsilon)(1 - \alpha \tilde{z}) - (\lambda - \varepsilon)\tilde{z}} \]

(6)

For \((\lambda - \varepsilon) < (\mu - \varepsilon)(1 - \alpha)\)

### 7.9 PERFORMANCE MEASURE:

Using condition \( \tilde{P}(1) = 1 \)

Probability that the system is empty i.e. \( \tilde{P}(0) = 1 - \frac{(\lambda - \varepsilon)}{(\mu - \varepsilon)(1 - \alpha)} \)

\[ = 1 - \tilde{P}_0 \]

(3.1)

Expanding \( \tilde{P}(\tilde{z}) \) and collecting the coefficient of \( \tilde{z}^i \), the probability that the system size is \( i \)'as,
\[ \bar{p}_t = (1-\bar{\rho}_0)[\alpha + (1-\alpha)\bar{\rho}_0]^{i-1}(1-\alpha)\bar{\rho}_0 \quad \text{(3.2)} \]

Note: In (3.2) if we take limit \( \alpha \to 0 \) we get

\[ \bar{p}_t = (1-\bar{\rho}_0) (\bar{\rho}_0)^i \quad i > 0 \]

This gives the interdependent communication network without bulk arrivals in fuzzy environment.

3.1 The average fuzzy number of packets in the system is given by:

(i) \[ \bar{L} = \frac{\bar{\rho}_0}{(1-\alpha)(1-\bar{\rho}_0)} \quad \text{where} \quad \bar{\rho}_0 = \frac{(\lambda - \bar{\epsilon})}{(1-\alpha)(\bar{\mu} - \bar{\epsilon})} \]

(ii) The variance of the fuzzy number of packets in the system

\[ \text{Variance} = \frac{\alpha \bar{\rho}_0 (1-\bar{\rho}_0) + \bar{\rho}_0}{(1-\alpha)^2(1-\bar{\rho}_0)^2} \]

Value of Average number of packets i.e. \( \bar{L} \) is given in table (1) and (2)

Table-1

For \( \lambda = (3, 4, 5) \) \quad \( \bar{\mu} = (8, 7, 6) \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \bar{\epsilon} \to )</th>
<th>(.1,.2,.3)</th>
<th>(.26,.34,.36)</th>
<th>(.38,.40,.42)</th>
<th>(.5,.6,.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>(1.28,1.81,2.33)</td>
<td>(1.20,1.73,2.19)</td>
<td>(1.19,1.66,2.13)</td>
<td>(1.04,1.59,2.08)</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>(1.54,2.18,2.82)</td>
<td>(1.52,2.09,2.75)</td>
<td>(1.52,2.08,2.64)</td>
<td>(1.30,1.96,2.53)</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>(2.01,2.78,3.68)</td>
<td>(1.94,2.65,3.49)</td>
<td>(1.86,2.64,3.39)</td>
<td>(1.59,2.42,3.15)</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>(2.66,3.79,4.92)</td>
<td>(2.56,3.60,4.64)</td>
<td>(2.52,3.42,4.50)</td>
<td>(2.08,3.11,4.13)</td>
<td></td>
</tr>
</tbody>
</table>

Table-2

After defuzzified the average number of packets we get

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \bar{\epsilon} \to )</th>
<th>(.1,.2,.3)</th>
<th>(.26,.34,.36)</th>
<th>(.38,.40,.42)</th>
<th>(.5,.6,.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.80</td>
<td>1.71</td>
<td>1.66</td>
<td>1.57</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>2.18</td>
<td>2.11</td>
<td>2.08</td>
<td>1.93</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>2.81</td>
<td>2.68</td>
<td>2.63</td>
<td>2.39</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>3.79</td>
<td>3.6</td>
<td>3.46</td>
<td>3.10</td>
<td></td>
</tr>
</tbody>
</table>
Table-3

Variance of number of packets in the system

<table>
<thead>
<tr>
<th>α</th>
<th>$\bar{\epsilon}$ →</th>
<th>(.1,.2,.3)</th>
<th>(.26,.34,.36)</th>
<th>(.38,.40,.42)</th>
<th>(.5,.6,.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>(4,5.49,7)</td>
<td>(3.62,5.14,6.33)</td>
<td>(3.52,4.81,6.10)</td>
<td>(2.99,4.42,5.74)</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>(5.67,7.72,9.89)</td>
<td>(5.51,7.20,9.40)</td>
<td>(4.73,7.20,8.14)</td>
<td>(4.43,6.49,8.12)</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>(9.20,11.92,15.56)</td>
<td>(8.61,11.03,14.74)</td>
<td>(8.08,11.03,13.58)</td>
<td>(6.22,9.52,12.56)</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>(15.63,20.72,25.81)</td>
<td>(14.47,19.0,23.53)</td>
<td>(13.93,17.46,22.5)</td>
<td>(10.77,14.86,18.95)</td>
<td></td>
</tr>
</tbody>
</table>

Table-4

After defuzzified the variance of number of packets in the system

<table>
<thead>
<tr>
<th>α</th>
<th>$\bar{\epsilon}$ →</th>
<th>(.1,.2,.3)</th>
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<th>(.38,.40,.42)</th>
<th>(.5,.6,.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>5.49</td>
<td>5.05</td>
<td>4.81</td>
<td>4.39</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>7.75</td>
<td>7.32</td>
<td>6.81</td>
<td>6.38</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>12.15</td>
<td>11.35</td>
<td>10.93</td>
<td>9.45</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>20.72</td>
<td>19</td>
<td>17.83</td>
<td>14.86</td>
<td></td>
</tr>
</tbody>
</table>

7.10 CONCLUSION:

We find, in general the average numbers of packets in network as well as in the buffer are decreasing as the independent parameters are increasing provided these are positive and other parameters are fixed. Also we find that the average number of packets in fuzzy network and in buffer are increasing as $\alpha$ increases for given value of $\hat{\lambda}$, $\bar{\mu}$, $\bar{\epsilon}$. The mean buffer length of this network is less than that of classical network with bulk arrival without interdependence. Similarly when the covariance of the composite arrivals and transmission completion are fuzzy in nature then result horizontally are also decreasing. The whole system parameters become fuzzy in nature and the system behavior shows same interdependence. From table (III) & (IV) we again observe same behavior of the variability of number of packets in buffer. In general, as
$\alpha$ increases variance also increases provided the other parameters fixed. On defuzzification the result also show the similar behavior.

If in this interdependence communication system arrivals are assumed in crisp set then the result of the model tally with the work made by the Srinivasa Rao (2006) and Singh T.P (2011).