CHAPTER 4

A NEW CARRIER BASED PULSE WIDTH MODULATION STRATEGY FOR VSI

4.1 INTRODUCTION

The present day applications demand ac power with adjustable amplitude and frequency. A well defined mode of operation of pulse width modulated voltage source inverters (PWM-VSI) facilitates to offer ac voltage at the desired frequency and amplitude according to the system requirements (Mohan et al 2002). Though a number of options are available to accomplish voltage linearity, still the trend is to continue to avail the role modulation strategies. The earnest choice is the so-called carrier-based PWM technique, which compares a control signal with a carrier waveform (Boost et al 1988, Batarseh 2004, Boundaja et al 2007).

The carrier based PWM methods are preferred due to their fixed switching frequency, low ripple current and well-defined harmonic spectrum characteristics. These strategies incorporate a “per carrier cycle average output voltage” equal to the reference voltage, employing either the triangle-intersection technique or a direct digital pulse programming approach (Grahame Holmes et al 2003). The characteristic advantage is that they serve to generate a linear relationship between the reference and output voltages within a limited range.
It is evident that utilization of the effective dc bus voltage is essential from cost and power density perspectives, which in turn necessitates the control signal to exceed the carrier level. However, for the portion of the modulating wave having a larger magnitude than the triangular wave, the peak value remains un-modulated and the gate signals are forced to remain on or off for a full carrier cycle leading to a nonlinear reference-output voltage relationship (Rashid 2004). There is therefore a definite need to construct new carrier functions which enable in extending the range of operation and serve a wide variety of applications (Jeevananthan et al 2005).

4.2 PROBLEM FORMULATION

The objective of this chapter is to suggest a new method to extend the linearity of the SPWM strategy to its full range of operation (Koura et al 1996, Holtz et al 1993, Rashid 2004). The proposed AMTCPWM envisages increasing the range of the SPWM control and eliminates the need of nonlinear modulation in an attempt to reach the square wave boundary. It is basically a high frequency triangular wave, which is amplitude modulated by a sinusoidal modulating signal of reference frequency. The approach is characterised linearly and its effectiveness in terms of lower THD and enhanced fundamental demonstrated using MATLAB based simulation for the basic single phase H-bridge inverter circuit. The hardware results of the designed prototype are included to validate the simulated performance.

4.3 PROPOSED STRATEGY

The characteristics of the SPWM and its three modes of operation are well known as seen in the Figure 4.1, which is that the normalized peak amplitude of fundamental frequency component can be expressed as a function of the amplitude modulation index ($M_a$). In the linear mode, where the value of $M_a$ is less than unity, the amplitude of the fundamental frequency voltage varies linearly with $M_a$ and pushes the harmonics into a high frequency range around the switching frequency and its multiples. The non-
linear or pulse dropping mode starts when the fundamental component is increased by allowing the reference command peak to exceed the peak of the carrier (Kerkman et al 1995, Rowan et al 1987, Hava et al 1998).

The fundamental component versus $M_a$ characteristics obtained using conventional SPWM is shown in Figure 4.1.

![Figure 4.1 Output fundamental Vs $M_a$ characteristics](image)

Figure 4.1 Output fundamental Vs $M_a$ characteristics

Figure 4.2 explains the relationship between the fundamental component gain ($G_f$) and $M_a$. For $M_a \leq 1$, $G_f$ remains constant. When the modulation index is greater than unity, the gain reduces sharply in a non-linear manner. However if $M_a$ approaches very large values, the gain approaches zero. It is essential to retain the linearity until the square wave output limit is reached as shown in Figure 4.3.

![Figure 4.2 Variation of fundamental component gain with $M_a$](image)

Figure 4.2 Variation of fundamental component gain with $M_a$
The PWM control signal in SPWM is obtained by comparing a high frequency triangular carrier of frequency, $f_c$ and amplitude 1 (per unit) and a low frequency sine wave of frequency, $f_m$ and amplitude $M_a$ (per unit) as shown in Figure 4.4. The relative amplitudes of the harmonics change with $M_a$. It is generally accepted that the performance of an inverter with any switching strategy can be related to the harmonic content of its output voltage. The precise value of the switching angles ($p_1$, $p_2$, $p_3$, …$p_i$) and hence duty cycle can be obtained through the triangular (carrier) and the sinusoidal (reference) equations.

Figure 4.3 Ideal SPWM characteristics

Figure 4.4 Generation of PWM signals using SPWM
The sinusoidal reference and triangular carrier are given by Equations (4.1) and (4.2) respectively.

\[ y = M \sin x \quad (4.1) \]

\[ x \pm \left( \frac{\pi}{2M_f} \right) y = \frac{r \pi}{M_f} \quad (4.2) \]

where ‘r’ is equal to one for first pair of triangular sections (straight lines), three for second pair, five for third pair and so on. The plus sign (+) sign should be taken for odd numbered line sections and minus sign (-) for even numbered line sections. The equations describing the natural sampled switching angles are transcendental and have the general distinct solutions for odd and even switching angles. The condition for any \( i^{th} \) switching angle, \( p_i \) is given in Equations (4.3) and (4.4) respectively for odd and even switching angles.

\[ M_\alpha \sin x + \frac{2M_f x}{\pi} - 2i = i \quad 1, 3, 5, \ldots \quad (4.3) \]

\[ M_\alpha \sin x - \frac{2M_f x}{\pi} - 2i = i \quad 2, 4, 6, \ldots \quad (4.4) \]

The duty cycle is calculated by simply adding the width of the individual pulses. The pulse width, total on time and the duty cycles are represented by Equations (4.5) to (4.7) respectively. It is realized that the inverter output, irrespective of control methods exhibits equal positive and negative half cycles, which results in zero dc component \( (a_0=0) \), and also does not posses any even harmonics due to half wave symmetry.
The width of the \( j^{th} \) pulse,

\[
\delta_j = P_{i+1} - P_i
\]  

(4.5)

\[
T_{on} = \sum_{j=1}^{n} \left[ x - \frac{\pi}{2M_f} M_a \sin x - \frac{2j}{2M_f} \right] - \left[ x + \frac{\pi}{2M_f} M_a \sin x - \frac{2j}{2M_f} \right]
\]  

(4.6)

The duty cycle,

\[
D = \frac{M_a}{M_f} \sum_{i=1}^{M-1} \sin \frac{i\pi}{M_f}
\]  

(4.7)

The Equations (4.8) and (4.9) give the generalized Fourier coefficients for the problem considered.

\[
a_n = \frac{V_{dc}}{n\pi\sqrt{2}} \sum_{k=1}^{i-1} \left( \sin np_{k+1} - \sin np_k \right) - \left( \sin np_{k+1} - \sin np_k \right)
\]  

(4.8)

\[
b_n = \frac{V_{dc}}{n\pi\sqrt{2}} \sum_{k=1}^{i-1} \left( \cos np_{k+1} - \cos np_k \right) - \left( \cos np_{k+1} - \sin np_k \right)
\]  

(4.9)

\[
C_n = \sqrt{a_n^2 + b_n^2}
\]  

(4.10)

It is significant to note from the MATLAB simulation of the conventional SPWM scheme that the square wave operation gives 381.97V (peak) of fundamental voltage (when \( V_{dc}=300V \)), which is 1.27 times the maximum voltage obtained in SPWM linear range \( (M_a=1) \). The Figure 4.5 (a) and (b) show the output voltages of SPWM for \( M_f=15 \) and \( V_{dc}=300V \) at \( M_a \) values of 0.8 and 1 respectively while Figure 4.6 (a) and (b) show their respective spectra. It is understood the linear modulation region depicts, the dominant inner sideband frequencies with \( M_a=0.8 \) and \( M_a=1 \) exhibits outer side band frequencies as seen in Figures 4.6 (a) and (b) respectively.
However, if the reference signal exceeds 100% or falls below 0%, the resulting PWM signal would be always on or always off, respectively. This is called over modulation. This region may be avoided by proper conditioning of the control signal. In addition, for control signals resulting in PWM signals with duty cycle values as high as 99% or as low as 1%, the switch may never fully reach the opposite state and spend an undue amount of time in transitions. The over modulation range may pose problems such as (i) large amounts of sub-carrier frequency harmonic currents are generated (ii) the fundamental component voltage gain significantly decreases and (iii) the switching device gate pulses are unexpectedly dropped.
Though the literatures define it to begin from 1, its actual commencement is delayed as the linearity goes up to the modulation index of 1.15. The inverter voltage waveform degenerates for sufficiently large values of $M_a$ from a pulse width modulated waveform into a square wave. Over modulation causes the output voltage to contain many more low order harmonics (3, 5, 7…etc.,) with magnitudes in the range of 10 to 20% of the fundamental unlike linear range. As $M_a$ increases, the on-time becomes proportionally larger and improves the rms value of the fundamental component in a non-linear manner with reduced gain. The simulated spectral outputs are available in Figure 4.7 for $M_a$=1.8. It is inferred that through less number of chopping and allowing the centre pulses to be widened, a higher fundamental output is obtained.

![Figure 4.7 Output voltage and spectrum](image)

The existing solutions to over modulation suffer due to the nonlinearities in the transition region. They are either very complex or demand nonlinear inverse gain calculation which results considerable amount of error and also suitable only for three-phase system. There is no simple PWM algorithm which retains voltage gain linearity until the full utilization of dc input for single-phase inverter system in the literature. Therefore, it is typically recommended to limit the control signal to a range, which avoids over modulation as well as extremely narrow pulses. It is therefore suggested to design a modified SPWM control scheme, which offers linear gain characteristics and is devoid of complex computations.
The amplitude modulated triangular carrier PWM (AMTCPWM) is proposed to extract single mode operation of SPWM inverter by linearly hopping to the square wave region. The conventional sine wave is chosen as the reference signal and the carrier is amplitude modulated triangular signal as shown in Figure 4.8.

![Figure 4.8 Proposed AMTCPWM](image)

The switching angles may be computed as the same way as SPWM scheme through analytical relations. The conditions for odd and even switching angles are given in Equations (4.11) and (4.12) and the switching angles \((q_1, q_2, q_3, \ldots q_i)\) are indicated in the Figure 4.8. It is worth while to note that both in SPWM and AMTCPWM schemes, the number of pulses will be equal to \((M_f-1)\). The modulation index for the AMTCPWM \((M_t)\) is defined as the ratio between the amplitudes of the reference sine wave and modulating sine of triangular carrier. The positions of any \(i^{th}\) switching instant in odd and even triangles are presented using Equations (4.11) and (4.12) respectively.

\[
M_t \sin x_i + 2\sin \left( \frac{i\pi}{2M_f} \right) \left[ M_f - \left( \frac{i+1}{2} \right) \right] = 0 \quad \text{for} \quad i = 1, 3, 5, \ldots \quad (4.11)
\]

\[
M_t \sin x_i + 2\sin \left( \frac{(i+1)\pi}{2M_f} \right) \left[ \frac{i}{2} - \left( \frac{M_f x_i}{\pi} \right) \right] = 0 \quad \text{for} \quad i = 2, 4, 6, \ldots \quad (4.12)
\]
The time varying nature of triangular carrier amplitude introduces considerable amount of complexity in the proposed AMTCPWM in particular when attempted to implement in digital platform. The cycle to cycle variation of triangular carrier amplitude as guided by its modulating (sine) envelope is shown in Figure 4.9. This issue may be solved by having a relation to determine the amplitude for any $t^{th}$ triangle.

Figure 4.9 Calculation of modified carrier amplitude

The amplitude of $t^{th}$ modulated triangular carrier ($V_{mt}$) is computed using Equation (4.13)

$$V_{mt} = \sum_{t=0}^{M_f-1} \sin \left[ \frac{\pi}{M_f} \left( \frac{1}{2} + t \right) \right]$$

(4.13)

4.4 PERFORMANCE EVALUATION

The primary function of a single phase VSI is to convert a fixed dc voltage to a single-phase ac voltage with variable magnitude and frequency. A simplified circuit diagram of a three-level voltage source inverter suitable for high-power medium-voltage applications is shown in Figure 4.10. It is composed of four active switches, $S_1$ to $S_4$. 

$$V_{mt} = \sum_{t=0}^{M_f-1} \sin \left[ \frac{\pi}{M_f} \left( \frac{1}{2} + t \right) \right]$$
The output voltages for AMTCPWM at $M_t=0.8$ and $M_t=0.98$ are simulated in Figure 4.11 (a) and (b) respectively while their respective harmonic spectra are shown in Figure 4.12 (a) and (b). It is clear that the proposed scheme renders proportionate variation of harmonic profile unlike SPWM and obviates the need for a unique filter design.

![Figure 4.10 Power circuit of single phase inverter](image)

(a) $M_t=0.8$                                (b) $M_t=0.98$

**Figure 4.11 Output voltage - AMTCPWM**

![Figure 4.11 Output voltage - AMTCPWM](image)

(a) $M_t=0.8$                                (b) $M_t=0.98$

**Figure 4.12 Harmonic spectra of output voltage – AMTCPWM**

![Figure 4.12 Harmonic spectra of output voltage – AMTCPWM](image)
Figure 4.13 shows the variation of fundamental output voltage for various values of $M_t$ and assures the gain linearization property of the proposed scheme. The THD values for the entire output voltage range seen in Figure 4.14 exhibit slightly higher values than that of the conventional SPWM. The values of modulation index, THD and lower order (sub-carrier) harmonics at the output voltage (peak) of 240V in both SPWM and AMTCPWM methods with 300V input and $M_t = 15$ are tabulated in Table 4.1. It is understood that the lower order harmonics increase considerably in the case of AMTCPWM method. Table 4.2 compares the carrier frequency harmonics and point out that the AMTCPWM always suppresses the upper side band harmonics. Hence the AMTCPWM successfully extends the linearity of SPWM until square wave with marginally increased distortion. Though the increased THD and slightly higher lower order harmonics may increase the filter size and cost still the benefits gained by linear control out weighs this additional burden.

![Figure 4.13 Fundamental voltage Vs $M_t$ - AMTCPWM](image)
Figure 4.14 Fundamental voltage Vs THD

Table 4.1 Comparison of SPWM and AMTCPWM-sub-carrier harmonics and THD

<table>
<thead>
<tr>
<th>Method</th>
<th>Method</th>
<th>THD (%)</th>
<th>h₃ (%)</th>
<th>h₅ (%)</th>
<th>h₇ (%)</th>
<th>h₉ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPWM</td>
<td>Mₙ</td>
<td>0.800</td>
<td>60.84</td>
<td>0.70</td>
<td>0.17</td>
<td>0.27</td>
</tr>
<tr>
<td>AMTCPWM</td>
<td>Mₙ</td>
<td>0.628</td>
<td>86.68</td>
<td>32.46</td>
<td>18.63</td>
<td>12.28</td>
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Table 4.2 Comparison of SPWM and AMTCPWM-carrier frequency harmonics

<table>
<thead>
<tr>
<th>Method</th>
<th>2M₁⁻³</th>
<th>2M₁⁻¹</th>
<th>2M₁⁺¹</th>
<th>2M₁⁺³</th>
<th>4M₁⁻³</th>
<th>4M₁⁻¹</th>
<th>4M₁⁺¹</th>
<th>4M₁⁺³</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h₂₇ (%)</td>
<td>h₂₉ (%)</td>
<td>h₃₁ (%)</td>
<td>h₃₃ (%)</td>
<td>h₅₇ (%)</td>
<td>h₅₉ (%)</td>
<td>h₆₁ (%)</td>
<td>h₆₃ (%)</td>
</tr>
<tr>
<td>SPWM</td>
<td>17.50</td>
<td>39.14</td>
<td>39.08</td>
<td>17.76</td>
<td>14.12</td>
<td>13.43</td>
<td>13.21</td>
<td>13.97</td>
</tr>
<tr>
<td>AMTCPWM</td>
<td>18.88</td>
<td>55.50</td>
<td>36.42</td>
<td>9.63</td>
<td>6.18</td>
<td>22.92</td>
<td>13.21</td>
<td>4.02</td>
</tr>
</tbody>
</table>
4.5 HARDWARE IMPLEMENTATION

The experimental arrangement of the fabricated system shown in Figure 4.15 is constituted of an uncontrolled rectifier, dc link filter and VSI. Pulses are generated by means of FPGA based open loop controller.

![Experimental setup](image)

**Figure 4.15 Experimental setup**

System-level-integration (SLI) using reprogrammable FPGA technology is made possible by advances in IC wafer technology, especially in the area of deep submicron lithography. Today, state-of-the art waferfabs find FPGA as an excellent mechanism for testing new wafer technology because of their reprogrammable nature. Incidentally, this trend in the waferfabs means that FPGA companies have early access to the newest deep sub-micron technologies, dramatically increasing the number of gates available to designers, as well as reducing the average gate cost sooner in the technology life-cycle than before. This trend, together with innovative system level architecture features, is leading FPGA’s to become the preferred architecture for SLI (Jeevananthan et al 2007c, http://www.xilinx.com)
The proposed carrier function is implemented by static random access memory (SRAM) based FPGA architecture (Xilinx-Spartan-3 xc3s400-4-pq208) seen in Figure 4.16. The XC3s400-4 pq208 has 400K logic gates, 8064 logic cells, CLB 896, distributed RAM bits 56K and maximum user input/output pins of 264(http:// www.xilinx.com). The design is complied, simulated using Modelsim and finally downloaded to the device through Xilinx software. The joint text action group (JTAG) downloads the data between the Xilinx software and the Spartan 3 board.

A single-phase inverter is assembled and tested with a non-inductive resistor load of 110Ω. The input dc source voltage of 200V is obtained using a single phase diode rectifier and capacitive filter of 3mF. The output voltage waveform obtained when the inverter is controlled using AMTCPWM technique (Mt =0.8 and Mf =20) is depicted in Figure 4.17. Figure 4.18 (a) shows the frequency spectrum of SPWM (Ma=1) and Figure 4.18 (b) the same for AMPTCPWM (Mf=0.8). The results obtained over a range of modulation indices serve to closely correlate the simulation and hardware responses as observed from Table 4.3.
Figure 4.17 Output voltage –AMTCPWM

(a) SPWM     (b) AMTCPWM

Figure 4.18 Harmonic spectra

Table 4.3 Comparison of simulation and experimental results

<table>
<thead>
<tr>
<th>$M_t$</th>
<th>Simulation $V_1$ (V)</th>
<th>THD (%)</th>
<th>Experimental $V_1$ (V)</th>
<th>THD (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>72.44</td>
<td>130.30</td>
<td>75.21</td>
<td>133.32</td>
</tr>
<tr>
<td>0.6</td>
<td>107.5</td>
<td>91.62</td>
<td>109.11</td>
<td>94.16</td>
</tr>
<tr>
<td>0.8</td>
<td>142.40</td>
<td>62.57</td>
<td>145.00</td>
<td>64.10</td>
</tr>
</tbody>
</table>
4.6 SUMMARY

It has been acknowledged that the voltage linearity, harmonic distortion, and over modulation range performance characteristics of a modulator are mainly dependent on the voltage-utilization level. The linear voltage range of a PWM-VSI is mainly determined by its modulator characteristics. The main advantage of AMTCPWM is that it does not require any mode change and also has contributed to cause exactly the same number of switching per cycle.

The AMTCPWM technique has been developed to provide full utilization (up to square wave region) without any pulse dropping and mode changing. It has been generated with a view to retain voltage linearity up to the square range and extract higher output voltages. The close degree of comparison between the hardware and simulation results for a similar operating state has been brought out. The developed carrier functions have served to improve the fundamental for a given switching frequency in a VSI. In spite of the slightly inferior spectrum of AMTCPWM, the proportionate variation of harmonic frequencies with the depth of modulation will ease the filter design and will explore further innovation for PWM-VSIs.