CHAPTER-3

MODELING OF INTERCONNECTED AC-DC POWER SYSTEMS

3.1 INTRODUCTION

The modeling of power system is essential before entering into the design of Load Frequency Control (LFC). In this chapter, a model of two-area and three area reheat thermal power system are developed. The load frequency control is concerned with the evaluation of response of the system for small changes (Allen 1984). In order to derive the transfer function of the interconnected power system, it is necessary to develop a linear model. A linear model can be developed by making the suitable approximations and assumptions. The two most common mathematical models are transfer function model and state space model. The mathematical equations of the interconnected power system are described below.

3.2 MODELING OF POWER GENERATING SYSTEM

3.2.1 Governor Model

Governors are the units that are used in power systems to sense the frequency bias caused by the load change and cancel it by varying the inputs of the turbines. The schematic diagram of a speed governing unit is shown in Figure 3.1. If without load reference, when the load change occurs, part of the change will be compensated by the valve/gate adjustment while the rest of the change is represented in the form of frequency deviation (Prabha Kundur
The goal of LFC is to regulate frequency deviation in the presence of varying active power load. The measured rotor speed $\omega_r$ is compared with reference speed $\omega_0$. The error signal $\Delta \omega$ or $\Delta f$ is amplified and integrated to produce a control signal $\Delta X_e$ which actuates the main steam supply valve. Where $R$ is the speed regulation, $T_g$ is the time constant of the governor and $K_g$ is the governor gain.

Thus, the load reference set point can be used to adjust the valve/gate positions so that the load change is cancelled by the power generation rather than resulting in a frequency deviation.

Figure 3.1 Schematic diagram of speed governing unit

The reduced form of Figure 3.1 is shown in Figure 3.2. The Laplace transform representation of the block diagram in Figure 3.2 is given by (Prabha Kundur 2008)

$$\Delta X_e(s) = \frac{K_g}{1 + sT_g} \left[ U(s) - \frac{1}{R} \Delta F(s) \right]$$  \hspace{1cm} (3.1)
A turbine unit in power systems is used to transform the natural energy, such as the energy from steam or water, into mechanical power ($\Delta P_m$) that is supplied to the generator. In LFC model, there are two kinds of commonly used turbines: non-reheat and reheat turbines, all of which can be modelled by transfer functions (Elgerd 2005).

The first stage of turbine is named as non-reheat turbine. From a response point of view this turbine is quite simple. Upon opening the control valve the steam flow will not reach the turbine cylinder instantaneously. A time delay (denoted by $T_t$) occurs between switching the valve and producing the turbine torque. The transfer function of the non-reheat turbine is represented as

$$G_{NR}(s) = \frac{\Delta P_t(s)}{\Delta X_e(s)} = \frac{K_t}{1 + sT_t} \quad (3.2)$$

where $\Delta X_e$ is the incremental change in valve/gate position.
$\Delta P_t$ is the incremental change turbine power.
$K_t$ is the gain of turbine.
$T_t$ is the turbine time constant.
The representation of non-reheat turbine model is shown in Figure 3.3. The turbine time constant $T_t$ assumes values in the typical range 0.1–0.5 seconds (Elgerd 2005).

$$
\Delta X_e(s) \xrightarrow{K_t \frac{1}{1+sT_t}} \Delta P_t(s)
$$

**Figure 3.3 Block diagram representation of Non-reheat turbine model**

Consider the two stage turbine shown in Figure 3.4. Assume that the High Pressure (HP) and Low Pressure (LP) stages have the per-unit megawatt ratings $K_r$ and 1- $K_r$ respectively.

Upon changing the control valve with the increment $\Delta X_e$ the HP turbine contributes the power component

$$
\Delta P_{t,HP} = K_r \Delta X_e \frac{K_t}{1+sT_t} \text{ MW}
$$

(3.3)

The reheater represents a delay $T_r$ and the LP stage thus contributes

$$
\Delta P_{t,LP} = (1-K_r) \Delta X_e \frac{K_t}{(1+sT_r)} \left( \frac{1}{1+sT_r} \right) \text{ MW}
$$

(3.4)
The total power is obtained by adding the Equation 3.3 and 3.4. One thus obtain the overall turbine transfer function.

\[
G_T = \frac{\Delta P_{T,HP} + \Delta P_{T,LP}}{\Delta X_e} = \frac{K_t}{1+sT_t} \frac{1+sK_rT_r}{1+sT_r} \quad \text{(3.5)}
\]

The time constant \(T_r\) assumes values in the typical range 4 – 10 seconds making the reheat turbine dynamically very slow (Elgerd 2005). The representation of reheat turbine model is shown in Figure 3.5.

![Figure 3.5 Block diagram representation of reheat turbine model](image)

### 3.2.3 Generator Load Model

A generator unit in power systems converts the mechanical power received from the turbine into electrical power. But for LFC, one focus on the rotor speed output (frequency of the power systems) of the generator instead of the energy transformation. Since electrical power is hard to store in large amounts, the balance has to be maintained between the generated power and the load demand (Nagrath 2005). The representation of the generator load model is shown in Figure 3.6.

The system is originally running in its normal state with complete power balance is \(P_G = P_D\). The frequency is at normal value ‘f’.

By connecting additional load to the system, the load demand is increased by \(\Delta P_D\). The generator increases its output \(\Delta P_G\) to match the new load that is given by
\[ \Delta P_G = \Delta P_D \]  

(3.6)

where \( \Delta P_G = \Delta P_T \), incremental turbine power output (assuming generator incremental loss to be negligible) and \( \Delta P_D \) is the load increment.

From the power balance equation (Nagrath 2005), one have

\[ \Delta P_G - \Delta P_D = \frac{2H P_r}{f} \frac{d}{dt} \Delta f + D \Delta f \]  

(3.7)

Dividing throughout by \( P_r \) and rearranging, one get

\[ \Delta P_G (pu) - \Delta P_D (pu) = \frac{2H}{f} \frac{d}{dt} \Delta f + D(pu) \Delta f \]  

(3.8)

where \( D = \frac{\partial P_D}{\partial f} = \text{Damping coefficient in MW/Hz.} \)

Taking the Laplace transformation, one can write \( \Delta F(S) \) as

\[ \Delta F(s) = \frac{\Delta P_G(s) - \Delta P_D(s)}{D + \frac{2H}{f}s} \]  

(3.9)

\[ \Delta F(s) \left[ \frac{\Delta P_G(s) - \Delta P_D(s)}{1 + \frac{K_{PS}}{1 + sT_{PS}}} \right] \]  

(3.10)

\[ T_{PS} = \frac{2H}{Df} = \text{Power system time constant} \]

\[ K_{PS} = \frac{1}{D} = \text{Power system gain} \]
3.2.4 Tie-line Model

An extended power system can be divided into a number of load frequency control areas interconnected by means of tie lines. Without loss of generality we shall consider a two area case connected by a single tie line as shown in Figure 3.7.

![Figure 3.7 Tie line interconnected control area](image)

The objective of an interconnected power system is to regulate the frequency deviation between the control areas. Based on the demand, the power interchange between the areas takes place. The power out of area 1 is given by

\[
P_{\text{tie,1}} = \frac{|V_1|}{X_{12}} \frac{|V_2|}{X_{12}} \sin (\delta_1 - \delta_2) \tag{3.11}
\]

where \(\delta_1, \delta_2\) = Power angles of equivalent machines of the two areas.
For incremental changes in $\delta_1$ and $\delta_2$, the incremental tie line power can be expressed as

$$\Delta P_{\text{tie},1} = T_{12} (\Delta \delta_1 - \Delta \delta_2) \quad (3.12)$$

where

$$T_{12} = \frac{|V_1| |V_2|}{P_{r1} X_{12}} \cos (\delta_1 - \delta_2) = \text{Synchronizing power coefficient}$$

Since the incremental frequency $\Delta f$ is related to the phase angle of deviation,

$$\omega = 2\pi f \quad (3.13)$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \frac{d\delta}{dt} \quad (3.14)$$

i.e., $\Delta f = \frac{1}{2\pi} \frac{d}{dt} \Delta \delta$ \quad (3.15)

$$\Delta \delta = 2\pi \int \Delta f \, dt \quad (3.16)$$

Substitute the Equation 3.16 in Equation 3.12, one gets

$$\Delta P_{\text{tie},1} = 2\pi T_{12} \left( \int \Delta f_1 \, dt - \int \Delta f_2 \, dt \right) \quad (3.17)$$

where $\Delta f_1$ and $\Delta f_2$ are incremental frequency changes of areas 1 and 2 respectively.

Taking the Laplace transformation of the above equations, the signal $\Delta P_{\text{tie}}(s)$ is obtained as

$$\Delta P_{\text{tie},1}(s) = \frac{2\pi T_{12}}{s} \left[ \Delta F_1(s) - \Delta F_2(s) \right] \quad (3.18)$$
In an interconnected power system, different areas are connected with each other via tie-lines. The tie-line connections can be modelled as shown in Figure 3.8. Considering $i^{th}$ control area, the change in tie line power can be expressed as

$$
\Delta P_{\text{tie},ij}(s) = \frac{2\pi T_{ij}}{s} \left[ \Delta F_i(s) - \Delta F_j(s) \right]
$$

(3.19)

**Figure 3.8 Block diagram of the tie line model between the two areas**

### 3.3 MODELING OF TWO AREA AC-DC REHEAT THERMAL POWER SYSTEM

#### 3.3.1 Transfer Function Model

The transfer function model of a two-area thermal reheat power system using simplified model is shown in Figure 3.9. A two-area system may be represented for the load frequency control in terms of its components like governor system, turbine, generator, load and tie line between two areas. It is convenient to obtain the dynamic model in state variable form from the transfer function model (Nagrath 2005, Issarachai 2002).
Figure 3.9  Modelling of HVDC power modulator in the two area thermal reheat with parallel AC-DC power system

3.3.2 State Space Model

The state space model can be developed from the transfer function model shown in Figure 3.9. The following equations can be written by inspection.

\[
\Delta P_f(s) = \frac{K_{PSI}}{1 + sT_{PSI}} [\Delta P_{g1}(s) - \Delta P_{d1}(s) - \Delta P_{uel}(s)] \quad (3.20)
\]

\[
\Delta P_{g1}(s) = \frac{1 + K_{rl} T_{rl} s}{1 + sT_{rl}} [\Delta P_{r1}(s)] \quad (3.21)
\]
\[
\Delta P_{t1}(s) = \frac{K_{t1}}{1 + sT_{t1}} \left[ \Delta X_{e1}(s) \right] 
\] (3.22)

\[
\Delta X_{t}(s) = \frac{K_{g1}}{1 + sT_{g1}} \left[ u_{1}(s) - \frac{1}{R_{1}} \Delta F_{1}(s) \right] 
\] (3.23)

\[
\beta_{1} \Delta F_{1}(s) + \Delta P_{tie1}(s) \right] 
\] (3.24)

\[
\Delta P_{AC}(s) = \frac{2\pi T_{12}}{s} \left[ \Delta F_{1}(s) - \Delta F_{2}(s) \right] 
\] (3.25)

\[
\Delta P_{DC}(s) = \frac{K_{DC}}{1 + sT_{DC}} \left[ K_{\Delta F1}(s) + K_{\Delta PAC}(s) \right] 
\] (3.26)

\[
\Delta F_{2}(s) = \frac{K_{ps2}}{1 + sT_{ps2}} \left[ \Delta P_{g2}(s) - \Delta P_{d2}(s) - \Delta P_{tie2}(s) \right] 
\] (3.27)

\[
\Delta P_{g2}(s) = \frac{1 + K_{ps2}T_{12}s}{1 + sT_{12}} \left[ \Delta P_{t2}(s) \right] 
\] (3.28)

\[
\Delta P_{t2}(s) = \frac{K_{t2}}{1 + sT_{t2}} \left[ \Delta X_{e2}(s) \right] 
\] (3.29)

\[
\Delta X_{2}(s) = \frac{K_{g2}}{1 + sT_{g2}} \left[ u_{2}(s) - \frac{1}{R_{2}} \Delta F_{2}(s) \right] 
\] (3.30)

\[
u_{2}(s) = \frac{1}{s} \left[ \beta_{2} \Delta F_{2}(s) + \Delta P_{tie2}(s) \right] 
\] (3.31)
Taking inverse Laplace transforms for the above equations

\[
\Delta F_1 = \frac{1}{T_{PS1}} \left[ K_{PS1} \Delta P_{g1} - \Delta F_i - K_{PS1} \Delta P_{d1} - K_{PS1} \Delta P_{ue1} \right] \tag{3.32}
\]

\[
\Delta P_{g1} = \frac{1}{T_{rl}} \Delta P_{g1} + \frac{1}{T_{rl}} \Delta P_{rl} - \frac{K_{rl}}{T_{rl}} \Delta P_{rl} - \frac{K_{rl} \Delta X_{e1}}{T_{rl}} \tag{3.33}
\]

\[
\Delta P_{rl} = \frac{K_{rl}}{T_{rl}} \Delta X_{e1} - \frac{1}{T_{rl}} \Delta P_{rl} \tag{3.34}
\]

\[
\Delta X_{e1} = \frac{K_{g1}}{T_{g1}} u_1 - \frac{K_{g1}}{R_{g1} T_{g1}} \Delta F_i - \frac{1}{T_{g1}} \Delta X_{e1} \tag{3.35}
\]

\[
u_1 = \beta_i \Delta F_i + \Delta P_{ue1} \tag{3.36}
\]

\[
\Delta P_{AC} = 2\pi T_{l2} \Delta F_i - 2\pi T_{l2} \Delta F_2 \tag{3.37}
\]

\[
\Delta P_{DC} = \frac{K_{API}}{T_{DC}} K_{DC} + K_{API} \frac{1}{T_{DC}} \Delta P_{DC} \tag{3.38}
\]

\[
\Delta F_2 = \frac{1}{T_{PS2}} \left[ K_{PS2} \Delta P_{g2} - \Delta F_2 - K_{PS2} \Delta P_{d2} - K_{PS2} \Delta P_{ue2} \right] \tag{3.39}
\]

\[
\Delta P_{g2} = \frac{1}{T_{r2}} \Delta P_{g2} + \frac{1}{T_{r2}} \Delta P_{r2} - \frac{K_{r2}}{T_{r2}} \Delta P_{r2} - \frac{K_{r2} \Delta X_{e2}}{T_{r2}} \Delta P_{r2} \tag{3.40}
\]

\[
\Delta P_{r2} = \frac{K_{r2}}{T_{r2}} \Delta X_{e2} - \frac{1}{T_{r2}} \Delta P_{r2} \tag{3.41}
\]
The above equations are put in the matrix form

\[ \begin{align*}
\dot{x} &= A x + B u + E d \\
\text{(3.44)}
\end{align*} \]

where the system state vector \([x]\) is

\[ \begin{bmatrix}
\Delta F_1 \
\Delta P_{g1} \
\Delta P_{r1} \
\Delta X_{el} \
\Delta P_{AC} \
\Delta P_{DC} \
\Delta F_2 \
\Delta P_{g2} \
\Delta P_{r2} \
\Delta X_{e2} \
u_2
\end{bmatrix} \] \quad (3.45)

System control input \([u]\) is

\[ [u] = \begin{bmatrix}
 u_1 \\
u_2
\end{bmatrix} \] \quad (3.46)

System disturbance input vector is \([d]\)

\[ [d] = \begin{bmatrix}
 \Delta P_{d1} \\
\Delta P_{d2}
\end{bmatrix} \] \quad (3.47)
The system matrix $A$ is written as

$$
A = \begin{bmatrix}
-1 + \frac{K_{p1}}{T_{p1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{T_{l1}} & -\frac{1}{T_{r1}} & \frac{1}{T_{l1}} & \frac{1}{T_{r1}} & -\frac{K_{q2}}{T_{p1}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{T_{l1}} & \frac{K_{q2}}{T_{r1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{K_{g1}}{R_{1}T_{g1}} & 0 & 0 & -\frac{1}{T_{g1}} & \frac{K_{g1}}{T_{g1}} & 0 & 0 & 0 & 0 & 0 \\
\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2\pi T_{12} & 0 & 0 & 0 & 0 & 0 & -2\pi T_{12} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{DC}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{r2}} & \left(\frac{1}{T_{r2}} \quad \frac{-K_{q2}}{T_{r2}} \right) & -\frac{K_{q2}K_{r2}}{T_{r2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{i2}} & \frac{K_{q2}}{T_{r2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{g2}} & \frac{K_{g2}}{T_{g2}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-K_{g2}}{R_{2}T_{g2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 \\
\end{bmatrix}
$$
where,  
A - System matrix (n x n)  
B - Input matrix (n x m)  
E - Disturbance matrix (n x p)  
x - State vector (n)  
u - Control vector (m)  
d - Disturbance vector (p)
3.4 MODELING OF THREE AREA AC-DC REHEAT THERMAL POWER SYSTEMS

3.4.1 Transfer Function Model

Modern control theory is applied to design an optimal load frequency controller for a three area interconnected AC-DC reheat thermal power systems. The transfer function model of a three area AC-DC interconnected thermal reheat power system using simplified model is shown in Figure 3.10. It is conveniently assumed that each control area can be represented by an equivalent governor system, turbine, generator, load and tie line between areas to obtain the dynamic model in state variable form from the transfer function model.

3.4.2 State Space Model

The state space model can be developed from the transfer function model shown in Figure 3.10. The following equations can be written by inspection.

\[
\Delta P_i(s) = \frac{K_{\text{Psi}}}{1 + sT_{\text{Psi}}} \left[ \Delta P_{g1}(s) - \Delta P_{d1}(s) - \Delta P_{\text{net}}(s) \right] \tag{3.48}
\]

\[
\Delta P_{g1}(s) = \frac{1 + K_{r1}T_{r1}s}{1 + sT_{r1}} \left[ \Delta P_{r1}(s) \right] \tag{3.49}
\]
Figure 3.10  Modelling of three area interconnected thermal reheat with parallel AC-DC power system

\[
\Delta P_{i1}(s) = \frac{K_{ii}}{1 + sT_{ii}} [\Delta X_{e1}(s)] \tag{3.50}
\]

\[
\Delta X_{i}(s) = \frac{K_{gg1}}{1 + sT_{g1}} [u_i(s) - \frac{1}{R_i} \Delta F_i(s)] \tag{3.51}
\]
\[ u_1(s) = \frac{1}{s} \left[ \beta_1 \Delta F_1(s) + \Delta P_{ue12}(s) \right] \tag{3.52} \]

\[ \Delta P_{AC1}(s) = \frac{2\pi T_{12}}{s} \left[ \Delta F_1(s) - \Delta F_2(s) \right] \tag{3.53} \]

\[ \Delta P_{DC}(s) = \frac{K_{DC}}{1 + sT_{DC}} \left[ K_{DF1}(s) - K_{DFAC}(s) \right] \tag{3.54} \]

\[ \Delta F_2(s) = \frac{K_{PS2}}{1 + sT_{PS2}} \left[ \Delta P_{e2}(s) - \Delta P_{d2}(s) - \Delta P_{ue21}(s) \right] \tag{3.55} \]

\[ \Delta P_{e2}(s) = \frac{1 + K_{e2}T_{r2}s}{1 + sT_{r2}} \left[ \Delta P_{r2}(s) \right] \tag{3.56} \]

\[ \Delta P_{r2}(s) = \frac{K_{r2}}{1 + sT_{r2}} \left[ \Delta X_{e2}(s) \right] \tag{3.57} \]

\[ \Delta X_{e2}(s) = \frac{K_{e2}}{1 + sT_{e2}} \left[ u_2(s) - \frac{1}{R_2} \Delta F_2(s) \right] \tag{3.58} \]

\[ u_2(s) = \frac{1}{s} \left[ \beta_2 \Delta F_2(s) + \Delta P_{ue21}(s) \right] \tag{3.59} \]

\[ \Delta P_{AC2}(s) = \frac{2\pi T_{33}}{s} \left[ \Delta F_2(s) - \Delta F_3(s) \right] \tag{3.60} \]

\[ \Delta F_3(s) = \frac{K_{PS3}}{1 + sT_{PS3}} \left[ \Delta P_{e3}(s) - \Delta P_{d3}(s) - \Delta P_{ue32}(s) \right] \tag{3.61} \]

\[ \Delta P_{e3}(s) = \frac{1 + K_{e3}T_{r3}s}{1 + sT_{r3}} \left[ \Delta P_{r3}(s) \right] \tag{3.62} \]
\[ \Delta P_{r3}(s) = \frac{K_{r3}}{1 + sT_{r3}} [\Delta X_{e3}(s)] \]  
\[ \Delta X_{3}(s) = \frac{K_{g3}}{1 + sT_{g3}} [u_3(s) - \frac{1}{R_3} \Delta F_3(s)] \]  
\[ u_3(s) = \frac{1}{s} [\beta_3 \Delta F_3(s) + \Delta P_{tie32}(s)] \]

Taking inverse Laplace transforms for the above equations

\[ \Delta F_1 = \frac{1}{T_{PSI}} [K_{PSI} \Delta P_{g1} - \Delta F_1 - K_{PSI} \Delta P_{el} - K_{PSI} \Delta P_{tie12}] \]  
\[ \Delta P_{g1} = -\frac{1}{T_{r1}} \Delta P_{g1} + \frac{1}{T_{r1}} \Delta P_{el} - \frac{K_{rl}}{T_{rl}} \Delta P_{rl} - \frac{K_{rl} K_{rl}}{T_{rl}} \Delta X_{e1} \]
\[ \Delta P_{r1} = \frac{K_{rl}}{T_{rl}} \Delta X_{e1} - \frac{1}{T_{rl}} \Delta P_{r1} \]
\[ \Delta X_{e1} = \frac{K_{g1}}{T_{g1}} u_1 - \frac{K_{g1}}{R_1 T_{g1}} \Delta F_1 - \frac{1}{T_{g1}} \Delta X_{e1} \]
\[ u_1 = \beta_1 \Delta F_1 + \Delta P_{tie12} \]
\[ \Delta P_{AC} = 2\pi T_{l2} \Delta F_1 - 2\pi T_{l2} \Delta F_2 \]
\[ \Delta P_{DC} = \frac{K_{AF1}}{T_{DC}} K_{DC} + \frac{K_{APAC}}{T_{DC}} K_{DC} - \frac{1}{T_{DC}} \Delta P_{DC} \]
\[
\Delta F_2 = \frac{1}{T_{PS2}} \left[ K_{PS2} \Delta P_{g2} - \Delta F_2 - K_{PS2} \Delta P_{d2} - K_{PS2} \Delta P_{tie23} \right] 
\]  
(3.73)

\[
\dot{\Delta P_{g2}} = \frac{-1}{T_{r2}} \Delta P_{g2} + \frac{1}{T_{r2}} \Delta P_{r2} - \frac{K_{r2}}{T_{r2}} \Delta P_{r2} - \frac{K_{r2} K_{r2}}{T_{r2}} \Delta X_{e2} 
\]  
(3.74)

\[
\Delta P_{r2} = \frac{K_{r2}}{T_{r2}} \Delta X_{e2} - \frac{1}{T_{r2}} \Delta P_{r2} 
\]  
(3.75)

\[
\Delta X_{e2} = \frac{K_{g2}}{T_{g2}} u_2 - \frac{K_{g2}}{R_2 T_{g2}} \Delta F_2 - \frac{1}{T_{g2}} \Delta X_{e2} 
\]  
(3.76)

\[
u_2 = \beta_2 \Delta F_2 + \Delta P_{tie21} 
\]  
(3.77)

\[
\Delta P_{AC2} = 2 \pi T_{23} \Delta F_2 - 2 \pi T_{23} \Delta F_3 
\]  
(3.78)

\[
\dot{\Delta F_3} = \frac{1}{T_{PS3}} \left[ K_{PS3} \Delta P_{g3} - \Delta F_3 - K_{PS3} \Delta P_{d3} - K_{PS3} \Delta P_{tie32} \right] 
\]  
(3.79)

\[
\dot{\Delta P_{g3}} = \frac{-1}{T_{r3}} \Delta P_{g3} + \frac{1}{T_{r3}} \Delta P_{r3} - \frac{K_{r3}}{T_{r3}} \Delta P_{r3} - \frac{K_{r3} K_{r3}}{T_{r3}} \Delta X_{e3} 
\]  
(3.80)

\[
\Delta P_{r3} = \frac{K_{r3}}{T_{r3}} \Delta X_{e3} - \frac{1}{T_{r3}} \Delta P_{r3} 
\]  
(3.81)

\[
\Delta X_{e3} = \frac{K_{g3}}{T_{g3}} u_3 - \frac{K_{g3}}{R_3 T_{g3}} \Delta F_3 - \frac{1}{T_{g3}} \Delta X_{e3} 
\]  
(3.82)

\[
u_3 = \beta_3 \Delta F_3 + \Delta P_{tie32} 
\]  
(3.83)
In the optimal control scheme the control inputs $u_1$, $u_2$ and $u_3$ are generated by means of feedbacks from all the states with feedback constants to be determined in accordance with an optimality criterion.

The above equations can be organized in the following vector matrix form

$$
\dot{x} = A_1 x + B_1 u + E_1 d
$$

(3.84)

where the system state vector $[x]$ is

$$
[x] = [\Delta F_1 \Delta P_{g1} \Delta P_n \Delta X_{e1} u_1 \Delta P_{AC1} \Delta P_{DC} \Delta F_2 \Delta P_{g2} \Delta P_{r2} \Delta X_{e2} u_2 \Delta P_{AC2} \Delta F_3 \Delta P_{g3} \Delta P_{r3} \Delta X_{e3} u_3]^T
$$

(3.85)

System control input is $[u] = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

(3.86)

System disturbance input vector is $[d] = \begin{bmatrix} \Delta P_{d1} \\ \Delta P_{d2} \\ \Delta P_{d3} \end{bmatrix}$

(3.87)
\[
A = \begin{bmatrix}
-1 & \frac{K_{b1}}{T_{a1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-1}{T_{a1}} \left( \frac{1}{T_{a1}} - \frac{K_{b1}}{T_{a1}} \right) & -\frac{K_{b1} K_{a1}}{T_{a1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{-1}{T_{a1}} & \frac{K_{b1}}{T_{a1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{K_{b1}}{R T_{a1}} & 0 & 0 & \frac{-1}{T_{a1}} & \frac{K_{b1}}{T_{a1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 \alpha T_0 & 0 & 0 & 0 & 0 & -2 \alpha T_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{a1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{a1}} & \frac{K_{b1}}{T_{a1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-K_{b1}}{R T_{a1}} & 0 & 0 & \frac{-1}{T_{a1}} & \frac{K_{b1}}{T_{a1}} & 0 & 0 & 0 & 0 \\
\beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 \alpha T_0 & 0 & 0 & 0 & 0 & -2 \alpha T_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{a1}} & \frac{K_{b1}}{T_{a1}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{a1}} \left( \frac{1}{T_{a1}} - \frac{K_{b1}}{T_{a1}} \right) & -\frac{K_{b1} K_{a1}}{T_{a1}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{a1}} & \frac{K_{b1}}{T_{a1}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-K_{b1}}{R T_{a1}} & 0 & 0 & \frac{-1}{T_{a1}} & \frac{K_{b1}}{T_{a1}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{K_{g1}}{T_{g1}} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & \frac{K_{g2}}{T_{g2}} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & \frac{K_{g3}}{T_{g3}} & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-\frac{K_{Ps1}}{T_{Ps1}} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & -\frac{K_{Ps2}}{T_{Ps2}} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

where,

- \( A_1 \) - System matrix (n x n)
- \( B_1 \) - Input matrix (n x m)
- \( E_1 \) - Disturbance matrix (n x p)
x - State vector (n)
u - Control vector (m)
d - Disturbance vector (p)

The prime objective is to minimize the ACE which stabilizes the system frequency for a sudden load disturbance. The objective function of the load frequency controller (Nagrath 2005, Vaibhav Donde 2001) is given by,

\[ \text{ACE}_i = \beta \Delta F_i + \Delta P_{\text{tie},i} \]  

(3.88)

where, i - Number of areas,
\(\Delta F\) - Change in frequency,
\(\Delta P_{\text{tie},i}\) - Change in tie-line power
\(\beta\) - Biasing factor (\(<= 1\))

3.5 SUMMARY

The problem of controlling the power output of the generators of a closely knit electric area is considered so as to maintain the scheduled frequency. The boundaries of a control area will generally coincide with that of an individual electricity board company. The load frequency control is one of the mechanisms to maintain or restore the frequency within the specified limit.

To understand the load frequency control problem, the modelling of governor, turbine, generator load and tie-line were designed. Based on the concept, the modelling of two area interconnected AC-DC thermal reheat power system was developed and it will be extended to the three area interconnected AC-DC thermal reheat power systems.
Load frequency control with integral controller achieves zero steady state frequency error and a fast dynamic response, but it exercises no control over the relative loadings of the AC-DC tie line interconnected multi area power systems.