

CHAPTER III
PRODUCTIVITY AND PRODUCTION FUNCTION:
THEORETICAL ASPECT

Industrial technology can be broadly defined as the knowledge regarding the industrial arts existing at a point of time. The ruling technology sets the conditions for the optimum use of resources, i.e, it sets the limit on how much can be produced with a given amount of input. Given the levels of technology , there are various techniques of producing goods and services. Technical progress is the improvement in the knowledge about the industrial arts and implies that either greater output can be got with the same volume of inputs or the same output with lesser inputs.

Technological changes can be measured through the simple ratios and through production approach¹. The ratio analysis can be in terms of i) simple traditional partial productivity indices of labour and capital; ii) total or multi factor productivity indices which can be measured a) arithmetically² and b) geometrically³; and iii) decomposition of technical change into components, where changes in productivity and technology are analysed in terms of prices and costs⁴. By these methods, some of the parameters of the production function can be inferred.

In the production function approach , the various components of technological change can be estimated directly. A production function shows the relationship between the maximum output obtainable from a given set of inputs and the relation between the inputs themselves, in the existing state of technological knowledge. The technology is ~~in~~built here. The efficiency of technology, the degree of economies of scale, the degree of capital intensity of technology and ease with which factors can be substituted for each other, all can be measured by estimating the parameters of the underlying production function. Two specifications of production functions are the Cobb-Douglas and Constant Elasticity of Substitution.

The said methodology assumes that the objective of each industrial unit operating in a competitive market is to maximise its profit. The production factors are paid equal to their marginal product. This implies that the basic frame work is neoclassical, in which the demand of a specific factor is determined by the production function, the prices of output it produces and of the various factor services it uses. The equilibrium of the system determines the quantities and prices of the various factors used and products hired out.

III.1 Productivity Concepts

Productivity deserves the attention that it has received, for it is a measure of efficiency with which resources are converted into commodities and services that man want⁵.

The term 'Productivity' is used to denote the ratio of output to any or all associated inputs in real terms. Ratios of output to particular inputs may be termed partial productivity measures. Output per man-hour or output per unit of capital can be termed as partial productivity ratios. These ratios are useful for measuring the saving, in particular, achieved over time. But they don't measure over-all changes in productive efficiency. They are affected by the changes in the composition of input. i.e., by factor substitution⁶.

Generally, measurement of productivity is reckoned in labour. Economists and businessmen have used the term productivity in relation to the output secured for a given amount of labour. Productivity therefore means physical volume of output attained per worker or per man-hour. Change in output-labour ratio represents a change in the efficiency of labour as a factor input only in combination with changes in the magnitude and direction of other inputs like capital and technology. Output-labour ratio would be influenced by among others, the skill and dexterity of

the work force, capital labour substitution, technical improvements, managerial efficiency , etc. Changes in this ratio signify only whether labour as a factor input has been utilised better or otherwise in conjunctions with other factors of production.

According to professor R. Balakrishna, the indices thus derived are based on labour, they measure the industrial efficiency in general reckoned in terms of a specific factor. Any factor affecting output or labour may have an influence on labour productivity. Therefore, changes in the output of labour are measurement of general efficiency. So what is measured is the combined effect of the diverse influence at work in a productive function⁷.

Soloman Fabricant argues that change in output per man hour shows the combined effect on the product obtained from an hour of labour of two groups of factor. First, those causing increase in efficiency; and second, those causing changes in the volume of tangible and intangible capital available per man hour. If the relationship between value added (V) and labour (L) and capital (K) inputs are given by

$$V(t) = A(t) f [K(t) \cdot L(t)]$$

then

$$\frac{V(t)}{L(t)} = \frac{A(t) f [k(t) \cdot L(t)]}{L(t)}$$

Output per worker depends on $A(t)$, an index of technical change, amount of labour and capital⁸. Therefore a better measure is one that compares output with combined use of all resources.

The ratio of output to an input is known as partial productivity ratio. There are as many partial productivity indices as there are factors of production. The most important and most often used are the partial productivity indices of labour, capital and raw materials.

These productivity ratios can be represented by

$$APL = \frac{V}{L} = \frac{\text{output}}{\text{labour input}}$$

Which is the average product of labour or labour productivity;

by

$$APK = \frac{V}{K} = \frac{\text{output}}{\text{capital input}}$$

which is the average product of capital or capital productivity;

by

$$APM = \frac{V}{M} = \frac{\text{output}}{\text{material input}}$$

Which is the average product of material or raw material productivity.

These ratios show the amount of output per unit of labour, capital and material and if they rise, then there is an increase in the productivity of that factor. The inverse of these productivity ratios implies unit factor requirements per unit of output. Increase in any of the above partial productivity ratios means that over a period of time more output is possible with decreasing amount of inputs and there is saving in the use of a particular input over time.

If the underlying production function is CES then the changes in labour productivity index can be broken into -

- i) neutral technological changes;
- ii) changes in inputs and economies resulting from changes in the scale of operation ; and
- iii) non - neutral technological changes⁹.

Changes in average productivity of labour and capital could be due to changes in any one or several of the forces mentioned above and just the observation of the movements in labour

productivity index or capital productivity does not tell which force or set of forces generated that movement. These partial productivity ratios just denote the resources foregone in the production of an additional unit of output. Therefore, the average product of any single factor cannot be used as an index of overall efficiency.

Sometimes different partial productivity indices have opposite trends and in that case no judgment is possible about overall industrial efficiency. However, if all the partial productivity indices have similar trends, then it will be possible to draw inferences about the overall efficiency.¹⁰.

In spite of the fact that the partial productivity indices of labour and capital assume a one factor world, they are important and in a particular context the average productivity of labour alone or capital alone may be important depending on the question in mind. The partial productivity indices have been used to find out implied production with a view to approaching the question of the sources of growth of output.

III 2. Total Factor Productivity

The total factor productivity ratio is the most comprehensive one. This index measures the output per unit of

labour and capital combined. This can be calculated arithmetically and geometrically.

Kendrick's arithmetic measure is based on a linear production function of the form

$$V = aL + bK$$

where V is output, L and K denote labour and capital inputs and 'a' and 'b' are coefficients of labour and capital. Measured arithmetically, total factor productivity is given by

$$P = \frac{V}{a_0 L + b_0 K}$$

where V is an index of output; K and L are index of capital and labour respectively; a_0 and b_0 are the base year weights. The weights are either prices of labour and capital services or the percentage shares of labour and capital in a base year. The weighted inputs of labour and capital in each year are added to get the total input. Then an index of output as also of total input is prepared. The ratio of output index to that of total input will yield the arithmetical total factor productivity index.

Solow's geometric measure is based on the following forms of production function with constant returns to scale and neutral technological change. The functional form is

$$\frac{V}{L} = A(t) \left[\frac{K}{L} \right]^b$$

where $\frac{V}{L}$ is output per person, $\frac{K}{L}$ is capital per person and A and b are constants, expressing the above relation on logarithmic form it becomes,

$$\text{Log } \frac{V}{L} = \text{Log } A(t) + b \log \left[\frac{K}{L} \right]$$

putting this relation in incremental form

$$\frac{d \left[\frac{V}{L} \right]}{\frac{V}{L}} = \frac{d A(t)}{A(t)} + b \frac{d (K/L)}{(K/L)} \quad (\text{or})$$

$$\frac{dA(t)}{A(t)} = \frac{d(V/L)}{V/L} - b \frac{d(K/L)}{K/L}$$

$d(V/L)/(V/L)$ is the rate of change of output and $d(K/L)/K/L$ is the rate of change of capital per person and b is the capital's share of output. Therefore, the rate of change of total factor productivity is the difference between the rate of change of output and the rate of change of capital per person multiplied by capital's share of output. This yields $dA(t)/A(t)$ series, from which A(t) series can be derived by assuming the initial value of

A(t) as one. Thus, the rate of change of total factor productivity is the difference between the rate of change of labour and capital. The weights are the percentage shares of labour and capital.

In these models, the effects of technical progress (as represented by a time term) and capital accumulation are separated. The basic procedure is to estimate the contributions made to the growth in output by the increases in inputs of labour and capital over a period by multiplying the observed increases in inputs by the observed factor prices (taken as a measure of marginal products) and deducting the results from the overall growth in output, the residual is attributed technical progress.

Both these measures of total factor productivity implicitly assume a homogeneous production. Under a competitive equilibrium the Kendrick and Solow measures are equivalent.¹¹ The magnitude of technological change or the residual derived through the total factor productivity approach will depend upon the form of production function. If the latter is "misspecified", then errors will spill over into the residual.¹² This error can be eliminated by estimating the parameters of a correctly specified production function.

III.3. Production Function

The components of technological change can be estimated directly by finding out empirically the magnitudes of the parameters of the underlying production function. A production function is the relationship between the quantities of inputs and outputs for efficient production by all possible processes set up as a functional form. A production function may then be specified as follows :

$$V=f(K,L) \text{ for } K \geq 0 \text{ and } L \geq 0 \dots\dots(1)$$

where V is output, K and L are the capital and labour inputs respectively. The production function is assumed to be twice differentiable and for which K and L are specified to take on non-negative magnitudes. The production function portrays the level of output, the marginal and average productivities of factors, and marginal rates of substitution between pairs of factors, for all relevant patterns of factor inputs.

The production function can be represented by iso-quants each representing various combinations of inputs which produce a given output. A well-behaved production function must possess certain basic properties. i.e., a minimum set of neo-classical criteria.

The various partial derivatives of (1) will be as follows :

$$f_L = \delta f / \delta L \quad (1a)$$

$$f_K = \delta f / \delta K \quad (1b)$$

$$f_{LL} = \delta^2 f / \delta L^2 \quad (1c)$$

$$f_{KK} = \delta^2 f / \delta K^2 \quad (1d)$$

$$f_{KL} = \delta^2 f / \delta K \delta L = \delta^2 f / \delta L \delta K \quad (1e)$$

(1a) and (1b) are the marginal productivity of labour and capital, respectively. (1c) and (1d) are variation of marginal productivity of labour and capital with respect to labour and capital itself. (1e) is the variation of marginal productivity of capital (or labour) with respect to labour (or capital).

The first neo-classical criterion of a well-behaved production is that any increase in inputs should have a positive effect on output - marginal product of labour and capital should be positive, i.e., (1a) and (1b) should be positive.

$$\delta f / \delta L > 0 \text{ and } \delta f / \delta K > 0$$

This implies that the iso-quants generated by a well-behaved production function should be downward sloping or $dL = -g(dK)$, where g is positive function of labour and capital. If both labour and capital inputs add a positive amount to the output and in order to maintain the same level of output if one factor is increased, the other factor can be reduced. The second criterion for a



well-behaved production function is that the rate of change of each marginal product should be negative. Symbolically it means:

$$\delta^2 f / \delta L^2 < 0 \text{ and } \delta^2 f / \delta K^2 < 0$$

This criterion will ensure equilibrium and this implies that iso-quants generated by a well-behaved production function are not merely downward sloping but also convex to the origin. This implies that if only one factor is increased, keeping other factors constant then the total product increases but at a decreasing rate. This is the familiar law of diminishing returns. Each iso-quant, which is convex downwards, expresses the law of diminishing marginal rate of technical substitution (MRTS) between factors.

$$\text{MRTS} = -dL/dK = (df/dK)/(df/dL)$$

(i.e.), it means that the marginal product of one factor will increase when more of the second factor is added. The marginal product of labour should increase when the capital increases.

A third criterion is that a well-behaved production function should be able to show any degree of economies or diseconomies of scale. Usually a production function specifies a priori constant returns to scale. But it should be permitted to assume any degree of homogeneity that is dictated empirically. Non-constant returns to scale have important implication for growth and therefore, the

basis of degree of economies of scale should be empirical rather than a priori, symbolically this means:

$$f(\lambda K, \lambda L) = \lambda^{\alpha} f(K, L) \text{ where } \lambda^{\alpha} = \begin{matrix} > \\ = \\ < \end{matrix} 1$$

The production function is the relation between the quantities of factors and the quantity of product and is naturally given by technical consideration. Therefore, the technology is embedded in the production function and can be expressed in terms of it. Therefore, a production function can be represented through its parameters the efficiency of technology, technologically determined economies of scale, the capital intensity of technology and ease with which factors can be substituted for each other. All these four components of technological change can be expressed in terms of a production function.

The specific form of production function may be Cobb-Douglas and / or Constant Elasticity of Substitution. The essential difference between the two is that Cobb-Douglas assumes that elasticity of substitution is always equal to one. In Constant Elasticity of Substitution, the elasticity of substitution can take any value from zero to infinity.

Cobb - Douglas Production Function

The unrestricted Cobb-Douglas production function for two factors may be specified as follows :

$$V = A L^a K^b \quad \text{for } A > 0 \quad \text{and}$$
$$0 < a < 1$$
$$0 < b < 1$$

where V is output and L and K are labour and capital inputs and A, a and b are constants to be determined empirically. The marginal products of labour and capital can be found by differentiating partially with respect to labour and capital.

$$\text{Marginal product of labour} = \frac{\delta V}{\delta L} = a A L^{a-1} K^b = a \frac{V}{L}$$

$$\text{Marginal product of capital} = \frac{\delta V}{\delta K} = a A L^a K^{b-1} = b \frac{V}{K}$$

Since V, L, K, a, and b are all positive, the marginal products of labour and capital are positive, which fulfils the first neo-classical criterion of a well-behaved production function. Differentiating the marginal products of labour and capital respectively, We get

$$\frac{\delta^2 V}{\delta L^2} = a(a-1) A L^{a-2} K^b = a(a-1) \frac{V}{L^2}$$

$$\frac{\delta^2 V}{\delta K^2} = b(b-1) A L^a K^{b-2} = b(b-1) \frac{V}{K^2}$$

These expressions will be negative only when the value of a and b are less than one. Therefore, Cobb-Douglas production function will satisfy the second criterion only when a and b are less than one. Normally a and b are less than unity; a and b are equal to partial elasticity of output with respect to labour and capital respectively:

$$a = \frac{L}{V} \cdot \frac{\delta V}{\delta L}$$

$$b = \frac{K}{V} \cdot \frac{\delta V}{\delta K}$$

Therefore a and b represent individually the percentage change in output for percentage changes in labour and capital. The two co-efficients taken together measure the total percentage change in output for a given percentage change in labour and capital. This implies that $(a+b)$ show the degree of homogeneity in the Cobb-Douglas Production function. Doubling the labour and capital, the right-hand side becomes

$$A(2L)^a (2K)^b = 2^{a+b} A L^a K^b$$

the output increase by 2^{a+b} . If $a+b < 1$, the output increase would be less than double; if $a+b > 1$, it would be more than double; if $a+b = 1$, the output would just double. Therefore, there will be diseconomies of scale, constant returns to scale and increasing returns to scale. Economies of scale depend on whether $a+b$ is less than one, equal to one or greater than one. This implies that as a Cobb-Douglas production function can represent any degree of returns to scale it satisfies the third criterion also.

In the Cobb-Douglas Production function V, L and K represent output, labour and capital variables and A, a and b are constants to be determined empirically. A here is the efficiency parameter. For every input combination, the greater is A , the greater is the output level.

Since $\frac{\delta V}{\delta A} = \frac{V}{A}$, a proportional change in A produces a

proportional change in output, ceteris paribus. The sum of the partial elasticities of Cobb-Douglas production function, $a+b$, indicates the degree of returns to scale. The returns to scale can change according to changes in the scale of operations as well as in technology. The two cannot be separated. However, assuming that if the variations in the degree of returns to scale are due to

technological change only, then the sum of the elasticities will change but the ratio of the elasticities will remain unaltered¹⁴. The changes in the capital-intensity of a technology will lead to a change in 'a' relative to 'b'. In the Cobb-Douglas production function, the elasticity of substitution is unity and thus unchanging. Therefore, the changes in the elasticity of substitution can't be represented in the Cobb-Douglas production function. Thus of the four measurable properties of technological change, only three can be measured by Cobb-Douglas. Actually in the Cobb-Douglas frame work only a neutral technical change can be measured.

The Cobb-Douglas production function can be used to find out proximate causes of the sources of output growth. The output growth may be due to increase in the labour force, capital stock and technical change. The intensity of these sources of growth may vary across industries. For measuring the technical change through Cobb-Douglas, an exponential time trend is incorporated. Then the Cobb-Douglas production function may be specified as

$$V = A L^a K^B e^{rt}$$

where an exponential e^{rt} has been introduced to take care of technical progress.

Cobb-Douglas assumes a priori that elasticity of

substitution between labour and capital is equal, which may not be empirically true. Hence such an assumption may lead to some specification error¹⁵. Also the elasticity of substitution is a crucial economic parameter having important policy implications for economic growth, international trade, relative distribution of income and resource allocation. An industry which has relatively high elasticity of substitution will usually have a higher output rate as compared to an industry which has low elasticity of substitution¹⁶.

Constant Elasticity of Substitution

Constant elasticity of substitution (CES) production function permits elasticity of substitution to take on any value from zero to infinity. The form of this function is as follows:

$$V = A \left[\delta K^{-p} + (1 - \delta) L^{-p} \right]^{-1/p} \quad (\text{CES I})$$

where V, K and L refers to output, capital and labour input and A, δ and p are efficiency distribution and substitution parameters respectively and the elasticity of substitution is

$$es = \sigma = \frac{1}{1+p}$$

In this function elasticity of substitution can take any constant value from zero to infinity. This function will generate

iso-quants which will be downward sloping and convex to the origin. However, in this formulation, only constant returns to scale can be represented. This implies that this formulation is incapable of characterising any degree of returns to scale. Technical progress may be introduced into the ACMS production function in the following three ways¹⁷. Hicks neutral technical change can be introduced by putting $A = A_0 e^{rt}$ and hence the function will be as follows:

$$V = A_0 e^{rt} \left[\delta K^{-p} + (1 - \delta)L^{-p} \right]^{-1/p} \quad (\text{CES II})$$

This means that in Hicks neutral case the efficiency of both factors changes equally.

In Harrod neutral case only labour is gaining in efficiency, thus the functional form will be

$$V = A \left[\delta K^{-p} + (1 - \delta)(Le^{rt})^{-p} \right]^{-1/p} \quad (\text{CES III})$$

The Solow type of neutrality is that of capital gaining in efficiency and may be expressed as

$$V = A \left[\delta (e^{rt} K)^{-p} + (1 - \delta)L^{-p} \right]^{-1/p} \quad (\text{CES IV})$$

These formulations only test whether there is any neutral technical change. Therefore, the CES production function has also been modified to study the rate of factor augmentation and the

extent to which the technical progress is biased towards uneven factor saving¹⁸.

$$V = \left[(E_L L)^{-p} + (E_K K)^{-p} \right]^{-1/p} \quad (\text{CES V})$$

Where E_L L and E_K K are labour and capital inputs, respectively, in efficiency units, L and K being measured unconventionally.

These formulations of CES production function assume constant returns to scale. For giving up the assumption of constant returns to scale, Brown and De cani¹⁹ introduce one more parameter, m, which can characterise any degree of returns to scale. This has the same general form but can exhibit any degree of returns to scale. This is as follows:

$$V = A \left[\delta K^{-p} + (1 - \delta) L^{-p} \right]^{(-m/p)} \quad (\text{CES VI})$$

In this a new parameter m has been introduced: m will be greater, equal or less than one for increasing, constant and decreasing returns to scale, respectively.

In these formulations the neutral technological changes are represented by efficiency and scale parameters, A and m and non-neutral technical changes by capital-intensity and substitution parameters δ and p.

The more general form of CES function fulfils all three neo classical criteria of a well - behaved production function .

All these CES production function are non - linear in the parameters and therefore, difficult to estimate. The parameters of CES production function can be estimated either directly or indirectly by using marginal productivity conditions.

V.E.S. production function

Recently some attempts have been made to get a new production function to meet the criticism leveled against both Cobb-Douglas and CES production functions. The general approach of these studies have been the assumption that the elasticity of substitution is a linear function of the ratio of two inputs and then to integrate the resulting differential equation to arrive at the implied production function. The resulting production function is the generalisation of the CES which possesses the desirable properties of Variable elasticity of substitution.

III.4.Methodology for the Present Study

In order to test the objectives of the study, the important statistical and mathematical tools such as mean, coefficient of variation, regression models, etc are used. Growth of sugar

industry in India and the selected regions is analysed by computing percentage rate of changes and trend rate of growth. This study also examines whether the growth rate was accelerating or decelerating. The percentage rates of change in sugar production related variables have been measured by taking year to year changes which would reveal the period of higher rate of increase in production and factors responsible for the same. It is measured as follows:-

$$\frac{R - R^{-1}}{R^{-1}} \times 100$$

Where R refers to current year value and R^{-1} refers to previous year value. The trend rates of growth have been estimated in semi-log form. That is

$$Y = ae^{bt}$$

$$(OR) \quad \ln Y = a_1 + b_1 t + u_1 \quad (1) \text{ and}$$

$$Y = a e^{bt} + ct^2$$

$$(OR) \quad \ln Y = a_2 + b_2 t + b_3 t^2 + u_2 \quad (2)$$

Where Y is the variable for which trend rate of growth is to be estimated at t is the time, with 1973-74 = 1. The coefficient of t^2 in equation (2) will exhibit accelerating or decelerating trend in the dependent variable. U_1 and U_2 are error terms.

Fluctuations in sugarcane production, sugarcane crushed, sugar production, recovery and duration of crushing days, etc. in India and selected regions are studied by estimating the coefficient of variation. (C.V) Coefficient of variation is calculated by using the following formula:-

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

Where σ = standard deviation and \bar{X} = arithmetic mean.

The various partial productivity ratios and total factor productivity index are measured as under:-

1. Labour Productivity

Here the labour productivity for All India and the selected regions are measured in the following ways.

$$i) \quad a) \quad LP = \frac{RGVO}{L}$$

$$b) \quad LP = \frac{RGVO}{MH}$$

$$ii) \quad a) \quad LP = \frac{RVA}{L}$$

$$b) \quad LP = \frac{RVA}{MH}$$

$$\text{iii) a) } LP = \frac{Q}{L}$$

$$\text{b) } LP = \frac{Q}{MH}$$

2. Capital productivity

In this study we measure the capital productivity in real terms. For capital productivity measurement of All India and the selected regions we use the following methods.

$$\text{a) } CP = \frac{RGVO}{RCS}$$

$$\text{b) } CP = \frac{RVA}{RCS}$$

$$\text{c) } CP = \frac{Q}{RCS}$$

3. Raw materials productivity

In this study we also measure the raw materials productivity. For productivity calculations the real value of raw material is used. The real value of raw material is obtained by dividing the gross value of raw material by price of sugarcane. Since sugarcane occupies the major portion of gross value of raw

materials the sugarcane price is used to calculate the real value of raw materials. Price index of sugarcane is calculated by taking 1973 - 74 as the base year. Material productivity is measured as,

$$MP = \frac{RGVO}{RRM}$$

4. Total factor productivity

The total factor productivity index represents the ratio between the actual output in constant prices and the output which the particular combination of labour, material, fuel and capital would have produced working at their base year efficiency. The Kendrick measure of total factor productivity is estimated. That is,

$$TFPQ_t = \frac{Q_t}{\sum S_i^0 (X_i^t / X_i^0)}$$

Where Q_t is the physical quantity of output in current year, S_i^0 is the share of each year cost in the base year and (X_i^t / X_i^0) is the i^{th} real factor input ratio between t^{th} and base year periods.

Alternatively the total factor productivity is measured by considering real gross value added as output assuming material and fuel productivities are constant. Thus

$$TFP_K = \frac{RVA_t}{w L_t + r K_t}$$

where RVA^t is the real value added in current year, L^t and K^t are current labour and real capital stock and \bar{w} and \bar{r} are wage rate and rate of return on capital in the base year.

The influence of output and technology on total factor productivities have been estimated by fitting the following multiple regression function²⁰:-

$$\text{Log TFP}_K = a + b_1 \log V + b_2 t + u \quad (1)$$

$$\text{Log LP} = a + b_1 \log V + b_2 t + u \quad (2)$$

Where TFP_K and LP are the total factor productivity index and labour productivity index respectively, V is the real value added, t is the time, with 1973-74 = 1 and u is the error term.

Production functions

In order to determine the relevant form of production function, the following functions have been estimated at the regional and national levels for the sugar industry during the period 1973-74 to 1990-91.

1. Variable Elasticity of Substitution Production Function

The variance or otherwise of elasticity of substitution to capital-labour ratio is tested by fitting variable elasticity substitution function. It is expressed as follows :-

$$\text{Log } V/L = a + b_1 \log w + b_2 \log K/L + u$$

Where V is value added, L is labour, K is capital, w is wage rate and a, b_1 and b_2 are constants. u is the error term.

The VES production function with inclusion of time trend is also considered for the analysis. The following log function has been used for fitting VES production function.

$$\text{Log } V/L = a + b_1 \log w + b_2 \log K/L + b_3 t + u$$

Where a, b_1 , b_2 and b_3 are constants.

2. Constant Elasticity of Substitution Production Function

The elasticity of substitution and the rate of technical change are estimated by using the models derived from the constant elasticity of substitution production function. The well known statistical models obtained from CES function for estimating elasticity of substitution and technical change are as follows :-

$$\text{Log } V/L = a + b_1 \log w + u \quad (1)$$

$$\text{Log } V/L = a + b_1 \log w + b_2 t + u \quad (2)$$

Where V is Value added, L is labour, w is wage rate and t is time. a, b₁ and b₂ are constants. U is the error term.

3. Cobb-Douglas production function

Cobb-Douglas production function is used to estimate the input elasticities, neutral technical progress and returns to scale.

$$V = A L^a K^b$$

When transformed into log form, we have

$$\text{Log } V = \text{Log } A + a \log L + b \log K + u \quad (1)$$

Where V is the real value added at constant prices, L is labour and K is adjusted fixed capital stock. A, a and b are constants. u is the error term.

a and b are determined by the method of least squares. The equation (1) doesn't measure the technical progress. Hence an exponential trend has been incorporated in the equation (1) in order to account for and measure neutral technological change.

$$\text{Log } V = \text{Log } A + a \log L + b \log K + c t + u \quad (2)$$

Where A, a, b and c are constants.

Marginal Productivity of Labour and Capital

Marginal Productivities of labour and capital have been computed for each individual year for the period from 1973-74 to

1990-91 for All India and the six selected regions by the following formulae.

$$M.P.L = a(V/L)$$

$$M.P.K = b(V/K)$$

On the other hand if the Cobb-Douglas production function is of the form $V = A L^a K^{1-a}$, then the marginal productivities of labour and capital are :-

$$M.P.L = a(V/L)$$

$$M.P.K = (1-a) V/K$$

Regional Efficiency in Sugar Industry

In order to determine the efficiency of the selected regions during 1973-74 to 1990-91, the model used here is Cobb-Douglas production function with state dummy variables. The logarithmically transformed Cobb-Douglas production function which includes five dummy variables and K and L as follows:-

$$\text{Log } V_{it} = a_1 + b_1 D_1 + b_2 D_2 + b_3 D_3 + b_4 D_4 + b_5 D_5 + C_1 \log L_{it} + C_2 \log K_{it} + u$$

(1)

Where D_1, D_2, D_3, D_4 and D_5 are dummy variables. The subscript 'i' refers to the region and 't' to time.

$D_1 = 1$ for the region Andhra Pradesh; 0 for others

$D_2 = 1$ for the region Bihar ; 0 for the others
 $D_3 = 1$ for the region Karnataka; 0 for the others
 $D_4 = 1$ for the region Maharashtra; 0 for the others
 $D_5 = 1$ for the region Tamil Nadu; 0 for the others
 $D_1=D_2=D_3=D_4=D_5=0$ for the region Uttar Pradesh.

The terms $a_1, (a_1 + b_1), (a_1 + b_2), (a_1 + b_3), (a_1 + b_4)$ and $(a_1 + b_5)$ are state intercepts corresponding to the regions Uttar Pradesh, Andra Pradesh, Bihar, Karnataka, Maharashtra and Tamil Nadu. These intercepts may be considered as efficiency indices.

To investigate the stability of regional efficiency the following 'F' test which has been suggested by Chow²¹ is used. The test is

$$F^* = \frac{\Sigma e^2 - (\Sigma e_1^2 + \Sigma e_2^2)/K}{(\Sigma e_1^2 + \Sigma e_2^2)/n_1 + n_2 - 2K}$$

Where

- Σe^2 = unexplained variation of the pooled sample corresponding to the period 1973-74 to 1990-91,
- Σe_1^2 = Unexplained variation of the sample corresponding to the period 1973-74 to 1981-82,
- Σe_2^2 = Unexplained variation of the sample corresponding to the period 1982-83 to 1990-91.

n = n_1+n_2 = Number of samples.

K = Number of coefficients including the intercept a_1 .

The over all fitness of the models are verified by the estimated values of R^2 , standard error and F-values. D.W. Statistic is presented to test the first - order auto correlation.

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