CHAPTER - 5
PORTFOLIO ANALYSIS

- Traditional Portfolio Analysis
- Why Portfolios?
- Diversification
- Expected Return of a Portfolio
- Risk of a Portfolio
- Reduction of Portfolio Risk through diversification:
  - Security returns perfectly positively correlated
  - Security returns perfectly negatively correlated
  - Security returns uncorrelated
- Portfolios with more than two securities
- Risk-Return Calculations of portfolios with more than two securities
PORTFOLIO ANALYSIS:

“Portfolio analysis considers the determination of future risk and return in holding various blends of individual securities”.

Traditional Portfolio Analysis:

Traditional security analysis recognizes the key importance of risk and return to the investor. Most traditional methods recognize return as some dividend receipt and price appreciation over a forward period. But the return for individual securities is not always over the same common holding period, nor are the rates of return necessarily time-adjusted. An analyst may well estimate future earnings and a P/E to derive future prices. He will surely estimate the dividend. But he may not discount the values to determine the acceptability of the return in relation to the investor’s requirements.

In any case, given an estimate of return, the analyst is likely to think of and express risk as the probable downside price expectation (either by itself or relative to upside appreciation possibilities). Each security ends up with some rough measure of likely return and potential downside risk of the future.

Portfolios, or combinations of securities, are thought of as helping to spread risk over many securities. However, the interrelationship between securities may be specified only broadly or nebulously.

This is not to say that traditional portfolio analysis is unsuccessful. It is to say that much of it might be more objectively specified in explicit terms.

Why Portfolios?

The simple fact that securities carry different degrees of expected risk leads most investors to the notion of holding more than one security at a time, in an attempt to spread risks by not putting all their eggs into one basket. Diversification of one’s holdings is intended to reduce risk in an economy in which every asset’s returns are subject to some degree of uncertainty. Even the value of cash suffers
from the inroads of inflation. Most investors hope that if they hold several assets, even if one goes bad, the others will provide some protection from an extreme loss.

**Diversification:**

Efforts to spread and minimize risk take the form of diversification. The more traditional forms of diversification have concentrated upon holding a large number of security types (stock, bonds) across industry lines (utility, mining, manufacturing groups). The reasons are related to inherent differences in bond and equity contracts, coupled with the notion that an investment in firms in dissimilar industries would most likely do better than in firms within the same industry. Holding one stock each from mining, utility, and manufacturing groups is superior to holding three mining stocks. Carried to its extreme, this approach leads to the conclusion that the best diversification comes through holding large number of securities across industries.

Most people would agree that a portfolio consisting of two stocks is probably less risky than one holding either stock alone. However, experts disagree with regard to the ‘right’ kind of diversification and the “right” reason.

According to Harry Markowitz, investor attitudes toward portfolios depends exclusively upon

- Expected return and risk, and
- Quantification of risk.

**Expected Return of a portfolio:**

As a first step in portfolio analysis, an investor needs to specify the list of securities eligible for selection or inclusion in the portfolio. Next he has to generate the risk-return expectations for these securities namely, the expected rate of return (mean) and the variance or standard deviation of the return.

The expected return of a portfolio of assets is the weighted average of the return of the individual securities held in the portfolio. The weight applied to each return is the fraction of the portfolio invested in that security.
Example:

Let us consider a portfolio of two equity shares P and Q with expected returns of 15 per cent and 20 per cent respectively.

If 40 per cent of the total funds are invested in share P and the remaining 60 per cent, in share Q, then the expected portfolio return will be:

\[ (0.40 \times 15) + (0.60 \times 20) = 18 \text{ per cent} \]

Thus, expected portfolio return may be expressed as under:

\[ \bar{r}_p = \sum_{i=1}^{n} x_i \bar{r}_i \]

Where,

- \( \bar{r}_p \) = Expected return of the portfolio.
- \( x_i \) = Proportion of funds invested in security i.
- \( \bar{r}_i \) = Expected return of security i.
- \( n \) = Number of securities in the portfolio.

**Risk of a portfolio:**

The variance of return and standard deviation of return are alternative statistical measures that are used for measuring risk in investment. These statistics measure the extent to which returns are expected to vary around an average over time. The variance or standard deviation of an individual security measures the riskiness of a security in absolute sense. For calculating the risk of a portfolio of securities, the riskiness of each security within the context of the overall portfolio has to be considered. This depends on their interactive risk, i.e. how the returns of a security move with the returns of other securities in the portfolio and contribute to the overall risk of the portfolio.

Covariance is the statistical measure that indicates the interactive risk of a security relative to others in a portfolio of securities. The way security returns vary with each other affects the overall risk of the portfolio.
The covariance between two securities X and Y may be calculated as under:

\[
\text{Cov}_{xy} = \frac{\sum_{i=1}^{N} (R_x - \bar{R}_x) (R_y - \bar{R}_y)}{N}
\]

Where

- \(\text{Cov}_{xy}\) = Covariance between x and y.
- \(R_x\) = Return of security x.
- \(R_y\) = Return of security y.
- \(\bar{R}_x\) = Expected or mean return of security x.
- \(\bar{R}_y\) = Expected or mean return of security y.
- \(N\) = Number of observations.

### Calculation of Covariance

<table>
<thead>
<tr>
<th>Year</th>
<th>(R_x)</th>
<th>Deviation (R_x - \bar{R}_x)</th>
<th>(R_y)</th>
<th>Deviation (R_y - \bar{R}_y)</th>
<th>Product of deviations ((R_x - \bar{R}_x) (R_y - \bar{R}_y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>-4</td>
<td>17</td>
<td>5</td>
<td>-20</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>-2</td>
<td>13</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>2</td>
<td>10</td>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>4</td>
<td>8</td>
<td>-4</td>
<td>-16</td>
</tr>
</tbody>
</table>

\[
\bar{R}_x = \frac{56}{4} = 14
\]

\[
\bar{R}_y = \frac{48}{4} = 12
\]

\[
\text{Cov}_{xy} = \frac{\sum_{i=1}^{N} [R_x - \bar{R}_x] [R_y - \bar{R}_y]}{N} = \frac{-42}{4} = -10.5
\]

The covariance is a measure of how returns of two securities move together. If the returns of the two securities move in the same direction consistently the covariance would be positive. If the returns of the two securities move in opposite direction consistently the covariance would be negative. If the movements of returns are independent of each other, covariance would be close to zero.

Covariance is an absolute measure of interactive risk between two securities. To facilitate comparison, covariance can be standardised. Dividing the covariance between two securities by product of the standard deviation of each security gives such a standardised measure. This measure is called the coefficient of correlation. This may be expressed as:
\[ r_{xy} = \frac{\text{Cov}_{xy}}{\sigma_x \sigma_y} \]

Where
\[ r_{xy} = \text{Coefficient of correlation between } x \text{ and } y. \]
\[ \text{Cov}_{xy} = \text{Covariance between } x \text{ and } y. \]
\[ \sigma_x = \text{Standard deviation of } x. \]
\[ \sigma_y = \text{Standard deviation of } y. \]

Thus, covariance may be expressed as the product of correlation between the securities and the standard deviation of each of the securities. Thus,
\[ \text{Cov}_{xy} = r_{xy} \sigma_x \sigma_y \]

The correlation coefficients may range from -1 to 1. A value of -1 indicates perfect negative correlation between security returns, while a value of +1 indicates a perfect positive correlation. A value close to zero would indicate that the returns are independent.

The variance (or risk) of a portfolio is not simply a weighted average of the variances of the individual securities in the portfolio. The relationship between each security in the portfolio with every other security as measured by the covariance of return has also to be considered. The variance of a portfolio with only two securities in it may be calculated as under:

\[ \sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 (r_{12} \sigma_1 \sigma_2) \]

Where
\[ \sigma_p^2 = \text{Portfolio variance.} \]
\[ x_1 = \text{Proportion of funds invested in the first security.} \]
\[ x_2 = \text{Proportion of funds invested in the second security.} \]
\[ \sigma_1^2 = \text{Variance of first security.} \]
\[ \sigma_2^2 = \text{Variance of second security.} \]
\[ \sigma_1 = \text{Standard deviation of first security.} \]
\[ \sigma_2 = \text{Standard deviation of second security.} \]
\[ r_{12} = \text{Correlation coefficient between the returns of first and second security.} \]
Portfolio standard deviation can be obtained by taking the square root of portfolio variance.

Example:

Two securities P and Q generate the following sets of expected returns, standard deviations and correlation coefficient:

\[ \bar{r} = 15 \text{ per cent} \quad 20 \text{ per cent} \]
\[ \sigma = 50 \text{ per cent} \quad 30 \text{ per cent} \]
\[ r_{pq} = 0.60 \]

A portfolio is constructed with 40 per cent of funds invested in P and the remaining 60 per cent of funds in Q.

The expected return of the portfolio is given by:

\[ \bar{r}_p = \sum_{i=1}^{n} x_i \bar{r}_i \]
\[ = (0.40 \times 15) + (0.60 \times 20) = 18 \text{ per cent} \]

The variance of the portfolio is given by:

\[ \sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1x_2 (r_{12} \sigma_1 \sigma_2) \]
\[ = (0.40)^2 (50)^2 + (0.60)^2 (30)^2 + 2 (0.40) (0.60) (-0.60) (50) (30) \]
\[ = 400 + 324 - 432 = 292 \]

The standard deviation of the portfolio is:

\[ \sigma_2 = \sqrt{292} = 17.09 \text{ per cent.} \]

The return and risk of a portfolio depends on two sets of factors:

- The returns and risks of individual securities and the covariance between securities in the portfolio, and
- The proportion of investment in each security.
The first set of factors is parametric to the investor i.e. he has no control over the returns, risks and covariance of individual securities. The second set of factors are choice variables i.e. the investor can choose the proportions of each security in the portfolio.

**Reduction of portfolio Risk through diversification:**

The process of combining securities in a portfolio is known as diversification. The aim of diversification is to reduce total risk without sacrificing portfolio return. In the example considered above, diversification has helped to reduce risk. The portfolio standard deviation of 17.09 is lower than the standard deviation of either of the two securities taken separately, which were 50 and 30 respectively.

To understand diversification mechanism, following three cases can be considered:

**Security returns perfectly positively correlated:**

When security returns are perfectly positively correlated the correlation coefficient between the two securities will be +1. The returns of the two securities then move up or down together.

The portfolio variance is calculated as under:

\[
\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1x_2 (r_{12} \sigma_1 \sigma_2)
\]

Since \(r_{12} = 1\), this variance may be rewritten as:

\[
\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1x_2 (\sigma_1 \sigma_2)
\]

The right hand side of the equation has the same form as the expansion of the identity \((a + b)^2\), namely \(a^2 + 2ab + b^2\). Hence, it may be reduced as

The standard deviation then becomes

\[
\sigma_p = \sqrt{(x_1 \sigma_1 + x_2 \sigma_2)^2}
\]

This is simply the weighted average of the standard deviations of the individual securities.
Taking the same example that we considered earlier for calculating portfolio variance, we shall calculate the portfolio standard deviation when correlation coefficients is +1.

Standard deviation of security P = 50
Standard deviation of security Q = 30
Proportion of investment in P = 0.4
Proportion of investment in Q = 0.6
Correlation coefficient = +1.0

Portfolio standard deviation may be calculated as:
\[
\sigma_p = (x_1\sigma_1 + x_2\sigma_2)^2
\]
\[
= (0.4) (50) + (0.6) (30)
\]
\[
= 38
\]

Being the weighted average of the standard deviations of individual securities, the portfolio standard deviation will lie between the standard deviations of the two individual securities. It will vary between 50 and 30 as the proportion of investment in each security changes.

Example:

If the proportion of investment in P and Q are 0.75 and 0.25 respectively, portfolio standard deviation becomes:

\[
\sigma_p = (0.75) (50) + (0.25) (30) = 45
\]

Thus, when the security returns are perfectly positively correlated, diversification provides only risk averaging and no risk reduction because the portfolio risk cannot be reduced below the individual security risk. Hence, diversification is not a productive activity when security returns are perfectly positively correlated.
**Security returns perfectly negatively correlated:**

When security returns are perfectly negatively correlated, the correlation coefficient between them becomes \(-1\). The two returns always move in exactly opposite directions.

The portfolio variance may be calculated as:

\[
\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \rho_{12} \sigma_1 \sigma_2
\]

Since \(\rho_{12} = -1\) this may be rewritten as:

\[
\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 - 2x_1 x_2 \rho_{12} \sigma_1 \sigma_2
\]

The right hand side of the equation has the same form as the expansion of the identity \((a - b)^2\), namely \(a^2 - 2ab + b^2\). Hence, it may be reduced as:

\[
\sigma_p^2 = (x_1 \sigma_1 + x_2 \sigma_2)^2
\]

The standard deviation then becomes:

\[
\sigma_p = x_1 \sigma_1 + x_2 \sigma_2
\]

For the illustrative portfolio considered above, we can calculate the portfolio standard deviation when the correlation coefficient is \(-1\).

\[
\sigma_p = (0.4) (50) - (0.6) (30) = 2
\]

The portfolio risk is very low. It may even be reduced to zero.

\[
\sigma_p = (0.375) (50) - (0.625) (30) = 0
\]

Example:

If the proportion of investment in P and Q are 0.375 and 0.625 respectively, portfolio standard deviation becomes:

Here, although the portfolio contains two risky assets, the portfolio has no risk at all. Thus, the portfolio may become entirely riskfree when security returns are perfectly negatively correlated. Hence, diversification becomes a highly productive activity when securities are perfectly negatively correlated, because portfolio risk can be considerably reduced and sometimes even eliminated. But, in reality, it is rare to find securities that are perfectly negatively correlated.
Security returns uncorrelated:

When the returns of two securities are entirely uncorrelated, the correlation coefficient would be zero.

The formula for portfolio variance is:

\[ \sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 - 2x_1x_2 r_{12} \sigma_1 \sigma_2 \]

Since \( r_{12} = 0 \), the last term in the equation becomes zero; the formula may be rewritten as:

\[ \sigma_p = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 \]

The standard deviation then becomes:

\[ \sigma_p = \sqrt{x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2} \]

For the illustrative portfolio considered above the standard deviation can be calculated when the correction coefficient is zero.

\[ \sigma_p = (0.4)^2(50)^2 + (0.6)^2(30)^2 \]

\[ = \sqrt{400 + 325} \]

\[ = 26.91 \]

The portfolio standard deviation is less than the standard deviations of individual securities in the portfolio. Thus, when security returns are uncorrelated, diversification reduces risk and is a productive activity.

We may now tabulate the portfolio standard deviations of our illustrative portfolio having two securities P and Q, for different values of correlation coefficients between them. The proportion of investments in P and Q are 0.4 and 0.6 respectively. The individual standard deviations of P and Q are 50 and 30 respectively.
From the above analysis we may conclude that diversification reduces risk in all cases except when the security returns are perfectly positively correlated. As correlation coefficient declines from +1 to -1, the portfolio standard deviation also declines. But the risk reduction is greater when the security returns are negatively correlated.

**Portfolios with more than two securities:**

The benefits from diversification increase as more and more securities with less than perfectly positively correlated returns are included in the portfolio. As the number of securities added to a portfolio increases, the standard deviation of the portfolio becomes smaller and smaller. Hence, an investor can make the portfolio risk arbitrarily small by including a large number of securities with negative or zero correlation in the portfolio.

But, in reality, no securities show negative or even zero correlation. Typically, securities show some positive correlation, that is above zero but less than the perfectly positive value (+1). As a result, diversification results in some reduction in total portfolio risk but not in complete elimination of risk. Moreover, the effects of diversification are exhausted fairly rapidly i.e. most of the reduction in portfolio standard deviation occurs by the time the portfolio size increases to 25 or 30 securities. Adding securities beyond this size brings about only marginal reduction in portfolio standard deviation.

Adding securities to a portfolio reduces risk because securities are not perfectly positively correlated. But the effects of diversification are exhausted rapidly because the securities are still positively correlated to each other though not

<table>
<thead>
<tr>
<th>Correlation coefficients</th>
<th>Portfolio standard deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>38.00</td>
</tr>
<tr>
<td>0.6</td>
<td>34.00</td>
</tr>
<tr>
<td>0.0</td>
<td>26.91</td>
</tr>
<tr>
<td>-0.6</td>
<td>17.09</td>
</tr>
<tr>
<td>-1.0</td>
<td>2.00</td>
</tr>
</tbody>
</table>

**Portfolio Standard Deviations**
perfectly correlated. Had they been negatively correlated, the portfolio risk would have continued to decline as portfolio size increased. Thus, in practice, the benefits of diversification are limited.

The total risk of an individual security comprises two components; the market related risk called systematic risk and the unique risk of that particular security called unsystematic risk. By combining securities into a portfolio the unsystematic risk specific to different securities is cancelled out. Consequently, the risk of the portfolio as a whole is reduced as the size of the portfolio increases. Ultimately when the size of the portfolio reaches a certain limit, it will contain only the systematic risk of securities included in the portfolio. The systematic risk, however, cannot be eliminated. Thus, a fairly large portfolio has only systematic risk and has relatively little unsystematic risk. That is why there is no gain in adding securities to a portfolio beyond a certain portfolio size.

The following figure depicts the diversification of risk in a portfolio:
The above figure shows the portfolio risk declining as the number of securities in the portfolio increases, but the risk reduction ceases when the unsystematic risk is eliminated.

**Risk-return calculations of portfolios with more than two securities:**

The expected return of a portfolio is the weighted average of the returns of individual securities in the portfolio, the weights being the proportion of investment in each security. The formula for calculation of expected portfolio return is the same for a portfolio with two securities and for portfolios with more than two securities. The formula is:

\[
\bar{r}_p = \sum_{i=1}^{n} x_i \bar{r}_i
\]

Where

- \( \bar{r}_p \) = Expected return of portfolio.
- \( x_i \) = Proportion of funds invested in each security.
- \( \bar{r}_i \) = Expected return of each security.
- \( n \) = Number of securities in the portfolio.

Let us consider a portfolio with four securities having the following characteristics:

<table>
<thead>
<tr>
<th>Security</th>
<th>Returns (per cent)</th>
<th>Proportion of investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td>0.2</td>
</tr>
<tr>
<td>B</td>
<td>17</td>
<td>0.3</td>
</tr>
<tr>
<td>C</td>
<td>23</td>
<td>0.1</td>
</tr>
<tr>
<td>D</td>
<td>20</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The expected return of this portfolio may be calculated using the formula:

\[
\bar{r}_p = \sum_{i=1}^{n} x_i \bar{r}_i
\]

\[
\bar{r}_p = (0.2)(12) + (0.3)(17) + (0.1)(23) + (0.4)(20)
\]

\[
= 17.8 \text{ per cent}
\]

The portfolio variance and standard deviation depend on the proportion of investment in each security, as also the variance and covariance of each security included in the portfolio.
The formula for portfolio variance of a portfolio with more than two securities is as follows:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}$$

Where

- $\sigma_p^2$ = Portfolio variance.
- $x_i$ = Proportion of funds invested in security $i$ (the first of a pair of securities).
- $x_j$ = Proportion of funds invested in security $j$ (the second of a pair of securities).
- $\sigma_{ij}$ = The covariance between the pair of securities $i$ and $j$.
- $n$ = Total number of securities in the portfolio.

The double summation $\sum_{i=1}^{n} \sum_{j=1}^{n}$ indicates that $n^2$ number of values are to be summed up. These values are obtained by substituting the values of $x_i$, $x_j$ and $\sigma_{ij}$ for each possible pair of securities.

Example:
Let us consider a portfolio with three securities A, B and C. The proportion of investment in each of these securities are 0.20, 0.30 and 0.50 respectively. The variance of each security and the covariance of each possible pair of securities may be set up as a matrix as follows:

<table>
<thead>
<tr>
<th>Weight</th>
<th>Variance-Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>A</td>
</tr>
<tr>
<td>0.3</td>
<td>B</td>
</tr>
<tr>
<td>0.5</td>
<td>C</td>
</tr>
</tbody>
</table>

The entries along the diagonal of the matrix represent the variance of securities A, B and C. The other entries in the matrix represent the covariances of the respective pairs of securities such as A and B, A and C, B and C.
Once the variance-covariance matrix is set up, the computation of portfolio variance is a comparatively simple operation. Each cell in the matrix represents a pair of two securities.

For example, the first cell in the first row of the matrix represents A and A; the second cell in the first row represents securities A and B, and so on. The variance and covariance in each cell has to be multiplied by the weights of the respective securities represented by that cell. These weights are available in the matrix at the left side of the row and the top of the column containing the cell. This process may be started from the first cell in the first row and continued for all the cells till the last cell of the last row is reached. When all these products are summed up, the resulting figure is the portfolio variance. The square root of this figure gives the portfolio standard deviation.

The variance of the illustrative portfolio given above can now be calculated.

\[
\sigma_p^2 = (0.2 \times 0.2 \times 52) + (0.2 \times 0.3 \times 63) + (0.2 \times 0.5 \times 36) \\
+ (0.3 \times 0.2 \times 63) + (0.3 \times 0.3 \times 38) + (0.3 \times 0.5 \times 74) \\
+(0.5 \times 0.2 \times 36) + (0.5 \times 0.3 \times 74) + (0.5 \times 0.5 \times 45) \\
= 53.71
\]

The portfolio standard deviation is:

\[
\sigma_p = \sqrt{53.71} = 7.3287
\]

We have seen earlier that covariance between two securities may be expressed as the product of correlation coefficient between the two securities and standard deviations of the two securities.

Thus,

\[
\sigma_{ij} = r_{ij} \sigma_i \sigma_j
\]

Where

\(\sigma_{ij}\) = Covariance between security i and security j.

\(r_{ij}\) = Correlation coefficient between security i and security j.

\(\sigma_i\) = Standard deviation of security i.

\(\sigma_j\) = Standard deviation of security j.
Hence, the formula for computing portfolio variance may also be stated in the following form:

\[
\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j r_{ij} \sigma_i \sigma_j
\]

Let us calculate the portfolio variance and standard deviation for a portfolio with the following characteristics.

<table>
<thead>
<tr>
<th>Security</th>
<th>( x_i )</th>
<th>( \sigma_i )</th>
<th>Correlation coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.35</td>
<td>7</td>
<td>P and Q = 0.7</td>
</tr>
<tr>
<td>Q</td>
<td>0.25</td>
<td>16</td>
<td>P and R = 0.3</td>
</tr>
<tr>
<td>R</td>
<td>0.40</td>
<td>9</td>
<td>Q and R = 0.4</td>
</tr>
</tbody>
</table>

It may be noted that correlation coefficient between P and P, Q and Q, R and R is 1. The variance-covariance matrix may be set up as follows:

<table>
<thead>
<tr>
<th>Weight</th>
<th>0.35 P</th>
<th>0.25 Q</th>
<th>0.40 R</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>1 x 7 x 7</td>
<td>0.7 x 7 x 16</td>
<td>0.3 x 7 x 9</td>
</tr>
<tr>
<td>0.25</td>
<td>0.7 x 16 x 7</td>
<td>1 x 16 x 16</td>
<td>0.4 x 16 x 9</td>
</tr>
<tr>
<td>0.40</td>
<td>0.3 x 9 x 7</td>
<td>0.4 x 9 x 16</td>
<td>1 x 9 x 9</td>
</tr>
</tbody>
</table>

The portfolio variance can now be calculated using this variance-covariance matrix as shown below:

\[
\sigma_P^2 = (0.35 \times 0.35 \times 1 \times 7 \times 7) + (0.35 \times 0.25 \times 0.7 \times 7 \times 16) + (0.35 \times 0.40 \times 0.3 \times 7 \times 9) + (0.25 \times 0.35 \times 0.7 \times 16 \times 7) + (0.25 \times 0.25 \times 1 \times 16 \times 16) + (0.25 \times 0.40 \times 0.4 \times 16 \times 9) + (0.40 \times 0.35 \times 0.3 \times 9 \times 7) + (0.40 \times 0.25 \times 0.4 \times 9 \times 16) + (0.40 \times 0.40 \times 1 \times 9 \times 9)
\]

\[
= 65.4945
\]
The portfolio standard deviation is:

$$\sigma_p = \sqrt{65.4945} = 8.09$$

A portfolio is a combination of assets. From a given set of ‘n’ securities, any number of portfolios can be created. The portfolios may comprise of two securities, three securities, all the way up to ‘n’ securities. A portfolio may contain the same securities as another portfolio but with different weights. Thus, new portfolios can be created either by changing the securities in the portfolio or by changing the proportion of investment in the existing securities. Each portfolio is characterised by its expected return and risk. Determining the expected return and risk (variance or standard deviation) of each portfolio that can be created from a set of selected securities is the first step in portfolio management and is called portfolio analysis.