Functional analysis is an abstract branch of mathematics that originated from classical analysis. Its development started about a century ago, and nowadays functional analytic methods and results are important in various fields of mathematics and its applications. The impetus came from linear algebra, linear ordinary and partial differential equations, calculus of variations, approximation theory and, in particular, linear integral equations, whose theory had the greatest effect on the development and promotion of the modern ideas. At present functional analysis has become an important tool in the investigation of all kind of problems in pure mathematics, physics, biology, economics, etc.. In fact, it is hard to find a branch in science where functional analysis is not used.

Formally, functional analysis begins with the concept of normed linear space given by Stefan Banach in his Ph.D. thesis published in 1922. This notion of the norm of a vector is the generalization of the notion of length or in other words the notion of metric and it obeys some simple and natural geometric principles. A linear space together with the norm defined for its elements is called a normed space. The introduction of normed spaces shifted the entire focus from the function to the algebraic properties of sets of functions. This leads to the algebraization of analysis. The concept of linear 2-normed spaces has been investigated by S. Gahler [39] in 1964 and has been developed extensively in different subjects by many authors. Further, to generalize the notion of length, area and volume of n-dimensional parallelopipeds, Gahler [39] introduced the concept of n-norms in a real vector space. A systematic development of n-normed linear spaces is due to S. S Kim and Y. J. Cho [65], R. Malceski [77], A. Misiak [91-93], H. Gunawan and M. Mashadi [48-49] and H. Gunawan [46-47]. Recently, H. G.
Dales and M. S. Moslehian [27], define multi-normed spaces, and investigate some properties of multi-bounded mappings on multi-normed spaces.

In mathematics, and particularly in functional analysis, a functional is traditionally a map from a vector space to the field underlying the vector space, which is usually the real numbers. In other words, it is a function which takes for its input-argument a vector and returns a scalar. Commonly the vector space is a space of functions, thus the functional takes a function for its input-argument, and then it is sometimes considered a function of a function. Its use originates in the calculus of variations where one searches for a function which minimizes a certain functional. A particularly important application in physics is searching for a state of a system which minimizes the energy functional. The traditional usage also applies when one talks about a functional equation meaning an equation between functionals: an equation $F = G$ between functionals can be read as an 'equation to solve', with solutions being themselves functions. In such equations there may be several sets of variable unknowns, like when it is said that an additive function ‘$f$’ is one satisfying the functional equation $f(x+y)=f(x)+f(y)$.

The functional equations form a modern branch of mathematics. The origin of functional equations came about the same time as the modern definition of function. During 1747 to 1750, J. d’Alembert published three papers. These three papers were the first on functional equations. Many celebrated mathematicians including S. M. Ulam, Th. M. Rassias, A.L. Cauchy, T. Aoki, F. Skof, D. H. Hyers, J. Bolyai, J. d’Alembert, L. Euler, C. F. Gauss, M. Fréchet, J.L. Jensen A. N. Kolmogorov, N. I. Lobaceveskkii, J. V. Pexider, S.D. Poisson have studied functional equations because of their apparent simplicity and harmonic nature.

Although the modern study of functional equations originated more than 250 years ago, a significant growth of this discipline occurred during the last fifty years. The comprehensive books by L. Szekelyhidi [151], P. Sahoo and
P. Kannappan [139], D.H. Hyers, G. Isac, Th.M. Rassias [53], have contributed immensely to the further advancement of this discipline.

In 1940 S.M. Ulam posed the following problem: If we replace a given functional equation by a functional inequality, then under what conditions we can say that the solutions of the inequality are close to the solutions of the equation.

For example, Let $G_1$ be a group and let $G_2$ be a metric group with the metric $d(\ldots)$. Given $\varepsilon > 0$, does there exist a $\delta > 0$ such that if a mapping $h: G_1 \rightarrow G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$, for all $x, y \in G_1$, then there is a homomorphism $H: G_1 \rightarrow G_2$ with $d(h(x), H(x)) < \varepsilon$, for all $x \in G_1$? If the answer is affirmative, we would say that equation of homomorphism $H(xy) = H(x)H(y)$ is stable. The interested reader should refer to S. M. Ulam [154] for an account on Ulam’s problems. In the next year D. H. Hyers [51], gave an affirmative answer to the above question for additive groups under the assumptions that the groups are Banach spaces. The works of Hyers initiated much of the present day research in stability theory of functional equations. In 1978, Th. M. Rassias [130] proved a generalization of Hyers’ theorem for additive mappings in the following way:

“Let $f$ be an approximately additive mapping from a normed vector space $E$ into a Banach space $B$, i.e., $f$ satisfies the inequality

$$
\| f(x + y) - f(x) - f(y) \| \leq \varepsilon (\| x \| + \| y \|)
$$

for all $x, y \in E$, where $\varepsilon$ and $r$ are constants with $\varepsilon > 0$ and $0 \leq r < 1$. Then the mapping $L: E \rightarrow B$ defined by $L(x) = \lim_{n \rightarrow \infty} 2^{-n} f(2^n x)$ is the unique additive mapping which satisfies

$$
\| f(x) - L(x) \| \leq \frac{2\varepsilon}{2 - 2^r} \| x \|
$$

for all $x \in E$. 

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The result of Th. M. Rassias has influenced the development of what is now called the Hyers-Ulam-Rassias stability theory for functional equations. In 1994, a generalization of Rassias’ theorem was obtained by Gavruta [43] by replacing the bound $\varepsilon (\| x \| + \| y \|)$ by a general control function $\varphi(x,y)$. Since then the stability problems of various functional equations and mappings such as the Cauchy equation, the Jensen equation $2f\left(\frac{x+y}{2}\right) = f(x) + f(y)$, the quadratic equation, homomorphisms, derivations and there Pexiderized versions with more general domains and ranges have been investigated by a number of authors.

The most famous functional equation is the Cauchy equation

$$f(x+y) = f(x) + f(y)$$

any solution of which is called additive. It is known that, for real valued functions defined on the real line, every Lebesgue measurable solution of (1) is of the form $f(x)=cx$, for some constant $c$. Nonmeasurable solutions also exist but they are “wild”, being discontinuous everywhere and unbounded on every interval. A discussion of these facts may be found in the book Functional Equations in Several Variables, Cambridge University Press (2008) by J. Aczél and J. Dhombres.

Beginning around the year 1980 the topic of approximate homomorphisms or the stability of the equation of homomorphism was taken up by a number of mathematicians. In general, most of the authors used a whole vector space, group or semigroup for the domain of the approximately additive mapping. However, F. Skof [145] studied the stability problem for $\delta$-additive mappings defined on restricted domains in the plane.
The functional equation

\[ f(x+y) + f(x-y) = 2f(x) + 2f(y) \]

is called the quadratic functional equation. Every solution of the quadratic functional equation is said to be a quadratic mapping. The first stability theorem for the quadratic functional equation was proved by F. Skof [145] for a mapping from a normed space \( X \) into a Banach space \( Y \). Cholewa [20] extended Skof’s theorem by replacing \( X \) by an abelian group \( G \). This result was later generalized by S. Czerwik [24] in the spirit of Hyers-Ulam-Rassias. He also proved the stability of quadratic equation of Pexider type. Recently, the stability problem of the quadratic equation has been investigated by a number of mathematicians.

A study of approximately multiplicative functions from a vector space \( V \) into the real numbers was made by J.A. Baker, J. Lawrence and F. Zorzitto. They proved that for a given \( \delta > 0 \) and a function \( f: V \rightarrow \mathbb{R} \) such that \( |f(x+y)-f(x)f(y)| < \delta \) for all \( x, y \) in \( V \), then either \( f(x) \) remains bounded, with a bound depending on \( \delta \) or else \( f(x+y) = f(x)f(y) \) for all \( x, y \) in \( V \). This was followed by another generalization of the same result by L. Szekelyhidi [150].

Now, we give chapter wise detail of the thesis.

Chapter one is pivot to the thesis. It contains all basic definitions and important results. The aim of this chapter is to deal with the basic concepts so as to make it convenient to understand the rest of research work. We start with the definition of normed linear spaces in Section 1.1, which is followed by several examples. Section 1.2, contains definition of Banach spaces and its examples. In Section 1.3, the concept of 2-normed spaces given by S. Gahler [39] has been discussed. Section 1.4 deals with n-normed spaces followed by concept of 2-Banach spaces in section 1.5, Section 1.6 deals with Multi-Banach spaces defined by H. G. dales and M. S. Moslehian [27]. In section 1.7, the concept of
orthogonality on an arbitrary real normed space in the sense of J. Ratz [133] has been discussed. Section 1.8 contains some detailed literature on functional equations followed by definitions and examples of different Functional Equations. The concepts of stability of Functional Equations has been discussed in section 1.9. Section 1.10 deals with the stability of Cauchy Functional equations followed by some important results proved by several mathematicians. The stability results of Jensen functional equations have been given in section 1.11. Section 1.12 contains important results on the stability of Quadratic functional equations. Section 1.13 deals with important results on the stability of Cubic functional equations. In section 1.14 related results on the stability of Quadratic functional equations have been given. Section 1.15 contains related literature on Fixed point theory. In section 1.16, definition of 2-metric space has been given. In section 1.17, G-metric space defined by Mustafa and Sims [105] along with its basic concepts and results has been given.

In Chapter 2, we deal with the stability of quadratic and generalized quadratic Functional equations. This chapter has been divided in three sections. Section 2.1 is of introductory nature. In section 2.2, we have proved theorems on the Hyers-Ulam-Rassias stability of quadratic functional equations on orthogonality spaces in the sense of Ratz [133]. In the last section, we have introduced a generalized quadratic functional equation and proved the Hyers-Ulam-Rassias stability of this equation on Banach spaces. The results of this chapter have been supported by well constructed examples.

Chapter 3, contains results on the stability of various functional equations in 2-Banach spaces. This chapter has been divided in five sections. Section 3.1 deals with some usual definitions, notations and other results which have been used in the sequel. In section 3.2 and 3.3, we proved the Hyers-Ulam-Rassias stability of the quadratic functional equations in 2-Banach spaces, which are motivated by the results of W. G. Park [123]. In the later half of this chapter, we
have proved several theorems on the Hyers-Ulam-Rassias stability of Cubic functional equations in 2-Banach spaces.

Chapter 4 is devoted to the stability of Jensen type quadratic functional equations in multi-Banach spaces. In section 4.1, some basic concepts concerning multi-Banach spaces has been given. In section 4.2 and 4.3, we use direct approach to prove some results on the stability of Jensen type quadratic functional equations on multi-Banach spaces, whereas in the second half of the chapter we use fixed point approach to establish the proofs of various theorems on the Hyers-Ulam-Rassias stability of Jensen type quadratic functional equations in multi-Banach spaces.

In Chapter 5, we prove some fixed point theorems in G-metric spaces. We start with basic definitions and basic results concerning G-metric spaces in first section. In second section we deal with a common fixed point theorem for weakly compatible maps in G-metric spaces. Section 5.3 contains some fixed point theorems using E.A. property for a pair of weakly compatible mappings satisfying a contractive condition of integral type in G-metric spaces. The results of this chapter have been supported through well constructed examples.