CHAPTER I

INTRODUCTION
Section 1

In this section, certain concepts, terminologies and symbols of acceptance sampling relevant to this thesis are provided.

American National Standards Institute / American Society for Quality Control (ANSI /ASQC) Standard A2 (1987) defines acceptance sampling as the methodology that deals with procedures by which decision to accept or not to accept a lot is based on the results of the inspection of samples. According to Dodge (1969), the major areas of acceptance sampling are:

(1) lot-by-lot sampling by the method of attributes, in which each unit in a sample is inspected on a go-not-go basis for one or more characteristics;

(2) lot-by-lot sampling by the method of variables, in which each unit in a sample is measured for a single characteristic, such as weight or strength;

(3) continuous sampling of a flow of units by the method of attributes; and

(4) special purpose plans, including chain sampling, skip-lot-sampling, small-sampling plans, etc.
This thesis mainly relates to the study of certain sampling plans, and sampling schemes classified under the categories (1), (2) and (4) given above.

The concepts and terminologies relevant to this thesis are given below:

**Sampling Plan, Sampling Scheme and Sampling System**

ANSI /ASQC Standard A2 (1987) defines an acceptance sampling plan as a specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria. It defines an acceptance sampling scheme as ‘a specific set of procedures which usually consists of acceptance sampling plans in which lot size, sample size and acceptance criteria or the amount of 100% inspection and sampling are related. The well known MIL-STD-105D (1963) is a sampling scheme. Also, Tightened-normal-tightened (TNT) sampling inspection of Calvin (1977) is a sampling scheme. Stephens and Larson (1967) define a sampling system as an assigned grouping of two or three sampling plans and the rules for using (that is, switching between) these plans for sentencing lots or batches of articles to achieve blending of the advantageous features of the sampling plans. Quick Switching System (QSS) of Romboski (1969) is an example for a sampling system. In this thesis only sampling plans and sampling schemes are considered.
Inspection

ANSI/ASQC Standard A2 (1987) defines the term ‘inspection’ as ‘activities’, such as measuring, examining, testing, gauging one or more characteristics of a product and/or service and comparing these with specified requirements to determine conformity. A sampling scheme or a sampling system may contain three types of inspections namely normal, tightened and reduced inspection. ANSI/ASQC Standard A2 (1987) defines these as follows.

Normal Inspection

Inspection that is used in accordance with an acceptance sampling scheme when a process is considered to be operating at or slightly better than its acceptable quality level.

Tightened Inspection

A feature of a sampling scheme using stricter acceptance criteria than those used in normal inspection.

Reduced Inspection

A feature of a sampling scheme permitting smaller sample sizes than used in normal inspection.

Operating Characteristic (OC) Curve

Associated with each sampling plan there is an OC curve which portrays the performance of the sampling plan against good and poor quality. This curve
when referred to the two axes, the axis $p$, the proportion defective of the material offered for inspection and the axis $P_a(p)$, the probability of acceptance of a lot or process, is the locus of $[p, P_a(p)]$. The OC Curves are generally classified under Type A and Type B situations. ANSI / ASQC Standard A2 (1987) defines them as follows.

**OC Curve for Isolated or Unique Lots or a Lot from an Isolated Sequence**

‘A curve showing, for a given sampling plan, the probability of accepting a lot as a function of the lot quality’ (Type A).

**OC Curve for Continuous Stream of Lots**

‘A curve showing, for a given sampling plan, the probability of accepting a lot as a function of the process average’ (Type B).

**OC Curve for Continuous Sampling Plan**

‘A curve showing the long-run percentage of product accepted during the sampling phase(s) as a function of the quality level of the processes.

**OC Curve for Special Purpose Plans**

‘A curve showing, for a given sampling plan, the probability of continuing to permit the process to continue without adjustment as a function of the process quality.’

In sampling systems or schemes, according to Romboski (1969) one will have a ‘Composite OC curve’ which gives the steady state probability of
acceptance under the switching rules of the system or scheme as a function of process quality.

Hypergeometric model is exact to evaluate $P_a(p)$ under type A situation (when sampling is done from a lot with a finite number, $N$, of items and non replacement of sampling units). Under type B situation (when sampling from the conceptually infinite output of units that the process turns out under the same essential conditions) binomial model will be exact for the case of nonconforming (defective) units to calculate $P_a(p)$. Binomial model is also exact in the case of sampling from a finite lot with replacement. Poisson model is exact in calculating $P_a(p)$ which specifies a given number of nonconformities (defects) per unit (or nonconformities per hundred units). Variable sampling plans use normal distribution for calculating $P_a(p)$.

If a given lot consists of $N$ units, a sample of $n$ units is drawn from the lot and the lot quality, expressed as the fraction defective is $p$, then the criteria (conditions) for the selection of the appropriate distribution are given by Schilling, Sheesley and Nelson (1978) as:

- **Hypergeometric**: when $n / N \geq 0.1$
- **Binomial**: when $n / N < 0.1$ and $np \geq 5$
- **Poisson**: when $n / N < 0.1$ and $np < 5$
- **Normal**: when $n / N < 0.1$ and $np(1-p) \geq 25$
Associated with an acceptance sampling plan, there are two types of risks, viz., producer’s and consumer’s risk which are defined as follows:

**Producer’s risk (α)**

For a given sampling plan, the probability of not accepting a lot, the quality of which has a designated numerical value representing a level which it is generally desired to accept.

**Consumer’s risk (β)**

For a given sampling plan, the probability of acceptance of a lot, the quality of which has a designated numerical value representing a level which it is seldom desired to accept.

**Average Sample Number (ASN)**

ANSI/ASQC Standard A2 (1987) defines ASN as ‘the average number of sample units per lot used for making decisions (acceptance or non-acceptance)’.

A plot of ASN against \( p \) is called the ASN curve.

**Average Outgoing Quality (AOQ) and Average Outgoing Quality Limit (AOQL)**

ANSI/ASQC Standard A2 (1987) defines AOQ as ‘the expected quality of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality’. Similarly, for a given sampling plan, AOQL is defined as “the maximum AOQ over all possible levels of incoming product
Quality’. Policies adopted for replacing or removing nonconforming units in sampling and screening phases cause the expressions for AOQ to vary. Beainy and Case (1981) have given expressions for AOQ for different policies adopted for single and double sampling attribute plans. In this thesis, AOQ is approximated as $pP_a(p)$. The assumption underlying this expression is that for all the accepted lots the average proportion defective is assumed to be $p$ and for all rejected lots the entire units are being screened and nonconforming units are replaced. It is further assumed that the sampling fraction $n/N$ is very small and can be ignored. A plot of AOQ against $p$ is called the AOQ curve.

**Acceptable Quality Level (AQL)**

ANSI / ASQC Standard A2 (1987) defines AQL as ‘the maximum percentage or proportion of variant units in a lot or batch that, for the purpose of acceptance sampling, can be considered satisfactory as a process average’.

**Limiting Quality Level (LQL)**

ANSI / ASQC Standard A2 (1987) defines LQL as ‘the percentage or proportion of variant units in a batch or lot for which, for the purpose of acceptance sampling, the consumer wishes the probability of acceptance to be restricted to a specified low value’.
**Indifference Quality Level (IQL)**

IQL is the percentage or proportion of variant units in a batch or lot for which, for purpose of acceptance sampling, both the producer and consumer wish the probability of acceptance to be restricted to a specified value namely 0.50 [see Hamaker (1950a)]. The point (IQL, 0.5) on the OC curve is also called as “Point of Control”.

**Maximum Allowable Percent Defective (MAPD)**

According to Mayer (1967), the proportion of nonconforming units corresponding to the point of inflection on the OC curve is interpreted as the maximum allowable percent defective. The point of the OC curve at which the steepest descent is realized is called the point of inflection.

**Designing Sampling Plans**

In designing a sampling plan, one has to accomplish a number of different purposes. According to Hamaker (1960), the most important of which are:

1. to strike a proper balance between the consumer’s requirement, the producer’s capabilities and the inspector’s capacity;
2. to separate bad lots from good;
3. simplicity of procedures and administration;
4. economy in the number of observations;
5. to reduce the risk of wrong decisions with increasing lot size;
6. to use accumulated sample data as a valuable source of information;
7. to exert pressure on the producer or supplier when the quality of the lot received is unreliable or not up to standard; and
8. to reduce sampling when the quality is reliable and satisfactory.

Hamaker (1960) also states that these aims are partly conflicting and all of them cannot be simultaneously realized. According to Case and Keats (1982), the design methodologies of acceptance sampling by attribute plans may be categorized as given in Table 1.1.

Table 1.1: Sampling Plan Design Methodologies.

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<th>Risk Based</th>
<th>Economically Based</th>
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<td>Non-Bayesian</td>
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<td>Bayesian</td>
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Risk based sampling plans are traditional in nature, drawing upon producer and consumer type of risks as depicted by the OC curve. Economically based sampling plans explicitly consider such factor as cost of inspection, accepting a defective unit and rejecting a non-defective unit in an attempt to design cost effective plan. Bayesian plan design takes into account
the past history of similar lots submitted previously for inspection purposes. Non-Bayesian plan design is not explicitly based upon the past history.

In this thesis sampling plan design of category 1 (that is risk based non-Bayesian approach) alone is considered. According to Case and Keats (1982), only the traditional category 1 design is applied by vast majority of quality control practitioners due to their wider availability and ease of application. Hamaker (1958) is also of the same opinion.

According to Peach (1947), the following are some of the major types of designing plans, based on the OC curves, classified according to types of protection:

1. The plan is specified by requiring the OC curve to pass through (or nearly through) two fixed points. In some cases it may be possible to impose certain additional conditions. The two points generally selected are \((p_1, 1-\alpha)\) and \((p_2, \beta)\) where

\[
p_1 \text{ or } p_\alpha = \text{The quality level that is considered to be good so that producer expects lots of } p_1 \text{ quality or better to be accepted most of the time;}
\]

\[
p_2 \text{ or } p_\beta = \text{The quality level that is considered to be poor so that the consumer expects lots of } p_2 \text{ quality or worse to be rejected most of the time;}
\]
\[ \alpha = \text{The producer's risk of rejecting } p_1 \text{ quality and} \]
\[ \beta = \text{The consumer's risk of accepting } p_2 \text{ quality.} \]

Tables of Cameron (1952) and Schilling and Johnson (1980) are examples of this type of designing. Schilling and Sommers (1981) term \( p_1 \) as the Producer’s Quality level (PQL) and \( p_2 \) as the Consumer’s Quality Level (CQL). Earlier literature calls \( p_1 \) as the Acceptable Quality Level (AQL) and \( p_2 \) as the Rejectable Quality Level (RQL) or Lot Tolerance Percent Defective (LTPD). Peach and Littauer (1946) have defined the ratio \( p_2 / p_1 \) associated with given values of \( \alpha \) and \( \beta \) as the 'operating ratio' (OR). Traditionally the values of \( \alpha \) and \( \beta \) are assumed to be 0.05 and 0.10 respectively.

2. The plan is specified by fixing one point only through which the OC curve is required to pass and by setting up one or more conditions, not explicitly in terms of the OC curves. The set of LTPD tables of Dodge and Romig (1959) is an example for this type of designing.

3. The plan is specified by imposing upon the OC curve two or more independent conditions none of which explicitly involves the OC curve. The set of AOQL tables of Dodge and Romig (1959) is an example for this type of designing.
The Quality Characteristic

When an item is inspected, the quality characteristic is one which is observed or measured on the item which determines the item’s quality.

Attributes and Variables Plans

Two broad categories of sampling plans, known as attributes plans and variables plans, have been developed to deal with both qualitative and quantitative data. If the items in a sample are simply classified as defective or acceptable on the basis of the qualitative characteristic, then an attributes sampling plan is used. Alternatively, if the quality characteristic is quantitative (for example, the length or weight of an item), then either an attributes or variables plan may be employed. The attributes plan is still applicable because quantitative data can be converted into qualitative data by judging an item to be acceptable if the measurement of interest is within the specification limit(s) and to be rejectable otherwise. This approach does not, however, make full use of information conveyed by the sample measurements. If the form of the underlying distribution of the quality characteristic is known, then statistics based on the measurements themselves can be used in a variables sampling plan. Variable plans are usually based on the assumption that the quality characteristic has a normal distribution.
If the population standard deviation is known then the sample mean contains the information required to estimate the proportion defective. The known-sigma sampling plans have been given by Liberman and Resnikoff (1955) and Owen (1967).

If the population standard deviation is not known, then both the sample mean and sample standard deviation can be used to draw inferences about the proportion defective in the lot.

The main advantage of variables plan over attributes plan is that the former requires a smaller sample size than the later for approximately the same degree of protection. In spite of the reduced sample size, however, the cost of obtaining the exact measurement of the quality characteristic needed for the variables procedure may make it more expensive than an equivalent attributes plan with its larger sample size. In the past, the use of variables has been restricted to cases where there is destructive testing of expensive item (Gascoigne and Hill (1976)).

**Single Sampling Plan by Attributes**

In lot by lot sampling inspection by attributes the product is divided into inspection lots, a sample or several samples are drawn at random from each lot, and the decision to accept or reject the lot is based on the number of non-conforming units found in the sample or samples. When the decision is
always made on the evidence of only one sample, the sampling plan is
described as a Single Sampling Plan.

The operating procedure for a single sampling plan is given below:

(i) From each lot of size N, select a random sample of size n.

(ii) If the number of non-conforming units in the sample is less
    than or equal to the specified acceptance number c, accept the
    lot; otherwise reject the lot.

**Single Sampling Plan by Variables**

A variable sampling plan is generally used whenever the characteristic of
interest is measurable on a continuous scale and is normally distributed with
mean \( \mu \) and standard deviation \( \sigma \), where \( \sigma \) may be known or unknown.

When the standard deviation \( \sigma \) is known, the single sampling variables
plan is specified by its sample size \( n_\sigma \) and its acceptance constant, \( k_\sigma \). The
operating procedure of a single sampling variables plan is given below:

A sample of size \( n_\sigma \), drawn at random from the lot, is inspected and the
lot is sentenced by the decision rule:

\[
\text{Accept the lot when } \bar{X} + k_\sigma \sigma \leq U;
\]

and \( \text{Reject the lot when } \bar{X} + k_\sigma \sigma > U; \) \hspace{1cm} (1.1)

where \( \bar{X} \) is the average quality characteristic derived from the sample and \( U \) is
the upper specification limit.
When used with a lower specification limit L, the acceptance criterion is

$$\bar{X} - k_\sigma \sigma \geq L.$$  

When the standard deviation $\sigma$ is unknown, the acceptance criteria would be

$$\bar{X} + k_S S \leq U \quad \text{and} \quad \bar{X} - k_S S \geq L$$

for the cases of upper and lower specification limits respectively, where $S^2$ is the unbiased estimate of $\sigma^2$.

**Designing Approach followed in this Thesis**

**Search Procedure**

In this approach, the parameters of a sampling plan are chosen, by trial and error by varying the parameters in the uniform fashion depending upon the properties of the OC function. An example of this approach is the one followed by Guenther (1969, 1970) while determining the parameters of the single and double sampling plans under the conditions for application of Binomial, Poisson and Hypergeometric models for OC curve. The advantage of the search procedure is that the sample sizes need not be rounded. The disadvantage of this procedure is that it needs elaborate computing facilities.
The following are the list of symbols which are frequently used in this thesis.

\[ \begin{align*}
N & = \text{Lot Size} \\
n & = \text{Sample size} \\
c & = \text{Acceptance number of an attribute single sampling plan} \\
c_1, c_2 & = \text{Acceptance numbers under tightened and normal single sampling attributes plans respectively} \\
\phi(x) & = \text{Density function of the standard normal distribution} \\
p & = \text{Lot quality or process quality} \\
P_a(p) & = \text{Probability of acceptance for given } p \\
p_{0.95}, p_{0.50}, & = \text{The lot or process quality for which the } p_{0.10}, p_{0.05} \text{ etc., Probability of acceptance is } 0.95, 0.50, 0.10, 0.05, \text{ etc., for a given sampling plan} \\
p_0 & = \text{Indifference Quality Level (IQL)} \\
p_m & = \text{The proportion defective at which AOQL occurs} \\
p_\star & = \text{Maximum Allowable Proportion Defective (MAPD), i.e., the proportion defective corresponding to the inflection point of the OC curve} \\
h_0 & = \text{Relative slope of the OC curve at } p_0
\end{align*} \]
Relative slope of the OC curve at $p^*$

Acceptable Quality Level (AQL)

Limiting Quality Level (LQL)

Producer’s risk

Consumer’s risk

Sample sizes of single sampling variables plan under $\sigma$-method (known $\sigma$) and $s$-method (unknown $\sigma$) respectively

Acceptance constants of single sampling variables plan under $\sigma$-method and $s$-method respectively

Acceptance constant of a single sampling variables plan under both $\sigma$-method and $s$-method

Acceptance constants of normal and tightened single sampling variables plans respectively

Acceptance constants of normal and tightened single sampling variables plans respectively under $\sigma$-method

Acceptance constants of normal and tightened single sampling variables plans respectively under $s$-method

Quality characteristic which is distributed as $N(\mu, \sigma^2)$
\[ \mu \quad = \quad \text{Process mean} \]
\[ \bar{X} \quad = \quad \text{Sample mean} \]
\[ \sigma^2 \quad = \quad \text{Process variance} \]
\[ S^2 \quad = \quad \text{Unbiased estimate of } \sigma^2 \]
\[ L \quad = \quad \text{Lower specification limit} \]
\[ U \quad = \quad \text{Upper specification limit} \]
\[ m \quad = \quad \text{Multiplicity factor in tightened sample size} \]

(i.e. \( n_T = m n_N \))
Section 2

In this Section, a brief review on the concept of tightened-normal-tightened sampling scheme by attributes and its designing for given AQL and LQL, AQL and AOQL, Indifference Quality Level and Maximum Allowable Percent Defective are presented. The contribution of the author relating to tightened-normal-tightened variables sampling scheme (TNTVSS(n_0;k_1,k_N)) is mentioned at the end of this section.

TNT zero acceptance number sampling scheme was originally Proposed by Calvin (1977) and investigated further by Soundararajan and Vijayaraghavan (1990), Govindaraju and Subramani (1992), and Senthilkumar (2000).

In designing sampling inspection plans, according to Schilling (1982), particularly in the area of compliance testing and safety-related items, an acceptance number of zero nonconforming units is particularly desirable. Also, a single sampling plan having an acceptance number of zero with a small sample size is often employed in situations involving costly or destructive testing by attributes. The small sample size is warranted due to the costly nature of testing and the zero acceptance number arises out of the desire to maintain a steep OC
curve. But a single sampling plan having an acceptance number of zero has the following disadvantages:

1. A single nonconforming unit in the sample calls for rejection of the lot.
2. The OC curve of all such plans have a uniquely poor shape, in that the probability of acceptance starts to drop rapidly for the smallest values of $p$.

In contrast, single sampling plans having $c = 1$ or more, as well as double and multiple sampling plans, lack these undesirable characteristics, but require a larger sample size.

TNT Sampling scheme is applicable in compliance and safety-related testing. In the area of compliance testing and especially for safety-related items, the following features seem desirable in a sampling plan.

1. Rejection of the lot if any defective items are found in the sample.
2. A well-defined relationship between the sampling plan and the size of the lot being inspected.
3. A clear indication of the economic impact of the quality levels utilized in the plan.
4. Simplicity and clarity in use.

Calvin (1977) devised a tightened-normal-tightened (TNT) sampling inspection scheme involving switching between two sampling plans. MIL-STD-105D (1963) contains a scheme with a fixed sample size but two
different acceptance criteria. Maintaining the acceptance criteria and switching between two sample sizes can also accomplish TNT sampling. This approach is particularly appealing with zero acceptance number plans.

Switching between two sample sizes with zero acceptance number creates an inflection point in the OC curve similar to that achieved in chain sampling plan of Dodge (1955). The result is an improved probability of acceptance at low percent defective where the smaller sample size dominates. These sampling plans are useful in small lot situations and in situations where it is beneficial to accept lots based on samples containing no defects.

**TNT Zero Acceptance Number Sampling**

The usual zero acceptance number operating characteristic (OC) curve decreases exponentially, producing a very severe sampling plan. TNT plan utilizes two \( c = 0 \) sampling plans of different sample sizes together with switching rule to build up the shoulder of the OC curve in a manner similar to that of the switching rules of MIL-STD-105D (1963). Calvin (1977) points out that, while increasing producer protection, the switching rules have no real effect on LQL which remains essentially that of the tightened plan. The switching rules associated with using the sampling plans are essentially those proposed by Dodge (1959). The criterion for switching between normal and tightened plan for MIL-STD-105D (1963) is as follows:
**Normal to Tightened**

When normal inspection is in effect, tightened inspection shall be instituted when two out of five consecutive lots or batches have been rejected on original inspection.

**Tightened to Normal**

When tightened inspection in effect, normal inspection shall be instituted when five consecutive lots or batches have been considered acceptable on original inspection.

The operating procedure of TNT zero acceptance sampling scheme is as follows:

1. Inspect using tightened inspection with the larger sample size $n_1$ and $c = 0$.

2. Switch to normal inspection when $t$ lots in a row are accepted under tightened inspection.

3. Inspect using normal inspection with the smaller sample size $n_2$ and $c = 0$.

4. Switch to tightened inspection after a rejection if an additional lot is rejected in the next $s$ lots.
Thus, a TNT scheme is specified by four parameters namely, \( n_1, n_2, s, \) and \( t \), where \( n_1 \) is the tightened (larger) sample size, \( n_2 \) is normal (smaller) sample size, \( s \) is the criterion for switching to tightened inspection and \( t \) is the criterion for switching to normal inspection.

According to Calvin (1977), the OC function of the TNT scheme is given by

\[
P_a(p) = \frac{P_T(1-P_N')(1-P_T')(1-P_N)+P_NP_T'(1-P_T)(2-P_N')}{(1-P_N')(1-P_T')(1-P_N)+P_T'(1-P_T)(2-P_N')}
\]  

(1.2)

where \( P_T \) is the probability of acceptance under tightened inspection and \( P_N \) is the probability of acceptance under normal inspection.

The type B OC curve would be resulted from (1.2) by plotting \( P_a(p) \) against \( p \) when sampling is done from an infinite universe or process or a large lot. For a given value of the process fraction defective, \( p \), the binomial distribution or the Poisson distribution as an approximation to the binomial can be utilized in the computation of probability. Under binomial conditions, \( P_T \) and \( P_N \) are defined as

\[
P_T = (1-p)^{n_1} \quad \text{and} \quad P_N = (1-p)^{n_2}
\]

In a special case when \( s = 4 \) and \( t = 5 \), the OC function of the TNT scheme becomes the scheme OC function of MIL-STD-105D (1963), involving only tightened and normal inspection. The scheme OC function of
MIL-STD-105D(1963) that involves tightened and normal inspections was derived by Dodge (1965), Hald and Thyregod (1965) and Stephens and Larson (1967).

Senthilkumar (2000) has studied the procedure for the selection of TNT plans under f-binomial approximation to the hyper geometric distribution for the OC curve.

Soundararajan and Vijayaraghavan (1990) have constructed tables under the conditions of Poisson distribution for selecting the parameters of the TNT plans based on different sets of criteria. A method for the conversion of a given set of conditions to the other equivalent set is also indicated. Under the Poisson assumption, when c= 0 both for tightened and normal inspection

\[ P_T = \exp(-n_1p) \quad \text{and} \quad P_N = \exp(-n_2p) \]

Soundararajan and Vijayaraghavan (1992) introduced the sampling inspection scheme designated as TNT \((n_1,n_2;c)\) which refers to a TNT scheme where the normal and tightened single sampling plans have the common acceptance number \(c\), but on tightened inspection the sample size is \(n_1\), and on normal inspection the sample size is \(n_2(<n_1)\).

The conditions for application and the operating procedure of TNT \((n_1,n_2;c)\) attributes scheme are given below.
1. Production is steady so that results on current, preceding and succeeding lots are broadly indicative of a continuing process and submitted lots are expected to be of essentially the same quality.

2. Successive lots are submitted for inspection in the order of their production.

The operating procedure of TNT \( (n_1,n_2;c) \) attribute scheme is as follows:

1. Inspect under tightened inspection using the single sampling plan with sample size \( n_1 \) and acceptance number \( c \).

2. If \( t \) lots in a row are accepted, switch to normal inspection (step3).

3. Inspect under normal inspection using the single sampling plan with sample size \( n_2 < n_1 \) and acceptance number \( c \).

4. If an additional lot is rejected in the next \( s \) lots after a rejection, switch to tightened inspection.

The OC function of the scheme is given by equation (1.2) where \( P_T \) is the proportion of lots expected to be accepted when using \( (n_1,c) \) plan and \( P_N \) is the proportion of lots expected to be accepted when using \( (n_2,c) \) plan. Thus, the tightened-normal-tightened sampling scheme is specified by the parameters \( n_1 \), \( n_2 \), and \( c \), and \( s \) and \( t \) constitute the criteria for switching between tightened and normal inspection respectively. Under binomial conditions, \( P_T \) and \( P_N \) are defined as
\[ P_T = \sum_{i=0}^{c} \binom{n_1}{i} p^i (1-p)^{n_1-i} \quad \text{and} \quad P_N = \sum_{i=0}^{c} \binom{n_2}{i} p^i (1-p)^{n_2-i} \]

Under Poisson conditions, \( P_T \) and \( P_N \) are defined as

\[ P_T = \sum_{r=0}^{c} \exp(-n_1p)(n_1p)^r/r! \quad \text{and} \quad P_N = \sum_{r=0}^{c} \exp(-n_2p)(n_2p)^r/r! \]

The behavior of OC curves of TNT \( (n_1,n_2;c) \) scheme is studied and it is given that for smaller values of fraction non-conforming, \( p \), the OC curve of TNT scheme coincides with the OC curve of normal plan and as quality deteriorates, the scheme OC curve moves towards that for tightened inspection and almost coincides with it beyond the indifference quality level, \( p_0 \) \([P_\alpha(p_0)=0.50]\).

Using the operating ratio of an OC curve as a basis for defining the equivalence of two sampling plans, the TNT \( (n_1,n_2;c) \) scheme is compared with conventional single and double sampling plans and it is shown that the TNT scheme is more efficient than single and double sampling plans.

Govindaraju and Subramani (1992) have studied the procedure for tightened-normal-tightened scheme involving the minimum sum of producer’s and consumer’s risks for a specified acceptable quality level and limiting quality level.
The proportion defective corresponding to the inflection point of an OC curve is denoted by \( p^* \), and it was interpreted by Mayer (1967) as MAPD. The desirability of developing a set of sampling plans indexed by \( p^* \) has been dealt by Mandelson (1962) and Soundararajan (1975). The MAPD for a single sampling plan under the Poisson model was obtained by Soundararajan (1975).

Suresh and Balamurali (1993) have provided tables and procedures for the selection of TNT\((n_1,n_2;c = 0)\) scheme indexed by maximum allowable percent defective (MAPD).

Suresh and Balamurali (1994) have also provided the tables and procedures for designing TNT\((n;c_1,c_2)\) scheme when \( c_1=0 \) and \( c_2=1 \) indexed by maximum allowable percent defective (MAPD).

Vijayaraghavan and Soundararajan (1996) have studied the TNT scheme of type \((n;c_1,c_2)\), which uses the tightened plan with parameters \((n,c_1)\) and the normal plan with parameter \((n,c_2)\). The procedures for the selection of tightened-normal-tightened (TNT) sampling scheme of type TNT\((n;c_1,c_2)\) are given. It has been shown that the TNT schemes provide smaller sample size than the matched single sampling plan.

The conditions for application of this scheme are the same as that of TNT\((n_1,n_2;c)\). The operating procedure of TNT\((n;c_1,c_2)\) attribute scheme is as follows:
1. Start with tightened inspection, with the sample size n and acceptance number c_1.

2. When t lots in a row are accepted under tightened inspection, switch to normal inspection.

3. Inspect under normal inspection, with the sample size n and acceptance number c_2.

4. When an additional lot is rejected in next s lots after a rejection, switch to tightened inspection.

The OC function of the scheme is given by equation (1.2) where P_T is the proportion of lots expected to be accepted when using (n,c_1) plan and P_N is the proportion of lots expected to be accepted when using (n,c_2) plan. Thus, the tightened-normal-tightened sampling scheme is specified by the parameters n, c_1, and c_2, and s, and t constitute the criteria for switching between tightened and normal inspection respectively.

Under binomial conditions, P_T and P_N are defined as

\[ P_T = \sum_{i=0}^{c_1} \binom{n}{i} p^i (1-p)^{n-i} \quad \text{and} \quad P_N = \sum_{i=0}^{c_2} \binom{n}{i} p^i (1-p)^{n-i} \]

Under Poisson conditions, P_T and P_N are defined as

\[ P_T = \sum_{r=0}^{c_1} \exp(-np)(np)^r / r! \quad \text{and} \quad P_N = \sum_{r=0}^{c_2} \exp(-np)(np)^r / r! \]
In the second chapter, the author introduces a new scheme of sampling inspection plan namely tightened-normal-tightened variable sampling scheme of type TNTVSS\( (n_0;k_T,k_N) \). The behaviour of OC curves of the TNTVSS\( (n_0;k_T,k_N) \) is studied. The TNTVSS is compared with the conventional variables single sampling plan and also with the attributes TNT \( (n;c_T,c_N) \) scheme. The efficiency of TNTVSS with respect to smaller sample sizes has been established over the attribute scheme. The TNT scheme with an unknown \( \sigma \) variable plan as the reference plan is also introduced along with the procedure of selection of the parameters. The method of designing the scheme based on the given AQL, \( \alpha = 0.05 \), LQL and \( \beta = 0.10 \) is indicated.

A procedure and tables for the selection of TNTVSS\( (n_0;k_T,k_N) \) indexed by AQL and AOQL, whenever rejected lots are 100% inspected for replacement of defective units, are given.

TNTVSS\( (n_0;k_T,k_N) \) indexed by Indifference Quality Level is presented. A procedure for designing a TNTVSS\( (n_0;k_T,k_N) \), which makes use of the indifference quality level as quality index and the relative slope at this point as the measure of sharpness of inspection is given. A table is also constructed using which one can select a sampling scheme for given values of \( p_0 \) and \( h_0 \) under \( \sigma \)-method and S-method respectively.
The expressions for finding inflection point for the TNT variables sampling scheme, for both the cases of known \( \sigma \) and unknown \( \sigma \) have been derived. A table is also constructed using which one can select a sampling scheme for given values of \( p \), and \( h \), under \( \sigma \)-method and S-method.

A method of finding \( \text{TNTVSS}(n_0;k_T,k_N) \) that is matched with a given TNT attributes scheme \( \text{TNT}(n;c_T,c_N) \) is provided. TNT variables sampling scheme is also compared with single sampling attributes and variables plans.

In the third chapter, the author introduces a new scheme namely tightened-normal-tightened variable sampling scheme \( \text{TNTVSS}(n_1,n_2;k) \) \([n_1,k)
and \((n_2,k) \) are respectively the tightened and normal variable sampling plans \( n_1=mn_2, \ m \geq 1 \]. This chapter also gives the procedure for construction and selection of \( \text{TNTVSS}(n_1,n_2;k) \) for various combinations of entry parameters. Under the conditions for application of Normal distribution for the OC curve of \( \text{TNTVSS}(n_1,n_2;k) \), the following designing methodologies are used for the selection of scheme parameters.

1. Designing the scheme for given AQL, LQL, \( \alpha \) and \( \beta \)
2. Designing the scheme for given AQL and AOQL.
3. Designing the scheme for given IQL (\( p_0 \)) and \( h_0 \).
4. Designing the scheme for given MAPD (\( p_\) ) and \( h_\) .
Section 3

This section deals with the review of Repetitive Group Sampling plan by attributes and its designing based on AQL and LQL, AQL and AOQL, indifference quality level and maximum allowable percent defective. The tables and some mathematical results relating to repetitive group sampling variables plan developed by the author and reported in chapter IV are given at the end.

Sherman (1965) has proposed a sampling plan, called repetitive group sampling (RGS) plan. The operation of the plan is similar to that of the sequential plan. According to Sherman, the RGS plan gives minimum sample size as well as desired protection. Although, the RGS plans are less efficient than sequential sampling plans, they are usually more efficient than single sampling plans.

The conditions for application and operating procedure of RGS attributes plan are given below.

Conditions for Application

The product to be inspected comprises a series of successive lots produced by an essentially continuing process.

1. The size of the lot is taken to be sufficiently large.
2. Under normal conditions the lots are expected to be of essentially the same quality.

3. The product comes from a source in which the consumer has confidence.

**Operating Procedure**

1. Take a random sample of size $n$.

2. Count the number of defectives, $d$, in the sample.

3. If $d \leq c_1$, accept the lot;
   
   if $d > c_2$, reject the lot;
   
   if $c_1 < d \leq c_2$, repeat steps 1, 2 and 3.

Thus, the RGS plan is specified by the parameters $n$, $c_1$, and $c_2$, where $n$ is the sample size, $c_1$ is the acceptance criterion of single sampling attributes plan, and $c_2$ is the rejection criterion of single sampling attributes plan.

**Operating Characteristic Function**

The OC function of the RGS plan, according to Sherman (1965), is

$$P_A(p) = \frac{P(d \leq c_1; n)}{1 - P(d \leq c_2; n) + P(d \leq c_1; n)}$$
Under the assumption of the Poisson model, Soundararajan and Ramasamy (1984, 1986) have provided tables and procedures for the selection of an RGS plan for different combinations of entry parameters, viz., (AQL, LQL), (AQL, AOQL), (p₀, h₀) and (p, h).

Govindaraju (1987) has shown that the dependent stage sampling (DSS) plan of Wortham and Mogg (1970) and quick switching system of type QSS-1(n;c_N,c_T) of Romboski (1969) possess equivalent OC functions as that of the RGS plan.

Hence, the tables constructed for designing RGS plans can be directly used for selecting the dependent stage sampling plan as well as quick switching system. Romboski (1969) developed tables for QSS-1 system and Soundarajan and Arumainayagam (1992) developed tables for the dependent stage sampling plan, quick switching system and the RGS plans. These tables provide sampling plans for stated conditions. However, in some cases the plans are not uniquely defined by the tables.

Subramani (1991) has studied the procedure for Repetitive Group Sampling plan by attributes involving the minimum sum of producer’s and consumer’s risks for a specified acceptable quality level and limiting quality level.
Jothikumar and Raju (1998) have provided a procedure and a table for designing and selecting the minimum risks repetitive group sampling plan RGS for given \((\text{AQL}, 1-\alpha)\) and \((\text{LQL}, \beta)\) under the conditions of the applications of binomial distribution for the OC-curve.

In the fourth chapter, the author introduces a new plan, called repetitive group sampling variable inspection plan designated by \((\text{RGSVP}(n; k_1, k_2))\). The RGS variables plan is specified by the parameters \(n\), \(k_1\), and \(k_2\), where \(n\) is the sample size, \(k_1\) is the acceptance criterion of single sampling variables plan, and \(k_2\) is the rejection criterion of single sampling variables plan. The method of designing the plan based on the AQL, (Acceptable Quality Level), \(\alpha\) (Producer's risk), LQL (Limiting Quality Level) and \(\beta\) (consumer's risk) is indicated. The RGS variables plan with an unknown \(\sigma\) variable plan as the reference plan is also introduced along with the procedure for selection of the parameters. It is shown that the RGS variables plan provides a smaller sample size than the matched RGS attributes plan.

Tables for designing \(\text{RGSVP}(n; k_1, k_2)\) indexed by AQL and AOQL, when rejected lots are 100% inspected for replacement of defective units, are constructed and a procedure for selection of the plan parameters is indicated.
Repetitive group sampling variables inspection plan indexed by indifference quality level is presented. A procedure for designing a RGSVP, which makes use of the indifference quality level as quality index and the relative slope at this point as the measure of sharpness of inspection is given. A table is also constructed using which one can select the sampling plan for given values of $p_0$ and $h_0$ under both $\sigma$ and $S$-methods.

Repetitive group sampling variables inspection plan indexed by MAPD is presented. The expressions for finding inflection point for the repetitive group sampling variables inspection plan for known and unknown $\sigma$ cases have been derived. A table is also constructed using which one can select the sampling plan for given values of $p$, and $h$, under both $\sigma$ and $S$-methods.

A method of finding $\text{RGSVP}(n_0; k_1, k_2)$ that is matched with a given RGS attributes plan, $\text{RGS}(n; c_1, c_2)$, is provided. RGS variables plan is also compared with single sampling attributes and variables plan and it is shown that the RGS variables plan provides a smaller sample size than the matched RGS attributes plan.
Section 4

In this section, a brief review of chain sampling plans of Dodge (1955), and Dodge and Stephens (1966) is given. At the end, the tables and procedures relating to Tightened-Normal-Tightened Chain Sampling Scheme (TNTChSS) developed by the author and reported in chapter V are mentioned.

When a manufacturing concern produces materials which involve destructive or costly tests for attributes, it is the usual practice to use a small sample plan so that the cost of inspection is minimum. Often a single sampling plan with zero acceptance number is taken for economic consideration but this has the following disadvantages:

1. A single occasional nonconforming unit may call for rejection of the lot.
2. The power of discrimination of the plan between good and bad lots, as revealed by the OC curve, is uniquely poor. That is, the probability of acceptance drops rapidly even for small values of percent nonconforming, $p$.

In contrast, single sampling plans having $c = 1$ or more, as well as double and multiple sampling plans lack this undesirable property, but require a large sample size. For such a situation, Dodge (1955) developed a chain sampling plan of type ChSP-1 which is an answer to the question whether
anything can be done to improve the discriminating power of the \( c = 0 \) single sampling plan without appreciably increasing the sample size. The operating procedure for ChSP-1 plan is as follows.

1. Select a random sample of \( n \) units from each lot and test each unit for conformance to the specified requirements.

2. If the observed number of nonconformities, \( d \), is zero, accept the lot. Reject the lot, if \( d > 1 \).

3. If \( d = 1 \) and if no nonconformities are found in the immediately preceding \( i \) samples of size \( n \), accept the lot.

Thus, the ChSP-1 plan has two parameters viz., \( n \), the sample size, and \( i \), the number of preceding samples of size \( n \) required to be free from nonconformities.

The OC function of ChSP-1 plan derived by Dodge (1955) is

\[ P_s(p) = P(0; n) + P(1; n) P(0; n)^i \]

where \( P(d;n) \) = probability of getting exactly \( d \) nonconforming units in a sample of size \( n \) for given product quality \( p \) (\( d = 0,1 \)).

When \( i = \infty \), the OC function of a ChSP-1 plan reduces to the OC function of the single sampling plan with \( c = 0 \); similarly when \( i = 0 \), the OC function of a ChSP-1 plan reduces to the OC function of the single sampling plan with
c = 1. A more complete discussion of ChSP-1 plan can be found in Schilling (1982) and Stephens (1982).

In chapter V, the author has given procedures and tables for the construction and selection of a new scheme of sampling inspection plans known as tightened-normal-tightened chain sampling scheme designated as

i) TNTChSS(n;i_N,i_T) and

ii) TNTChSS(n_T,n_N;i)

for specified parameters.