CHAPTER I

INTRODUCTION
The interaction of two separate branches of science viz., Fluid dynamics and Electromagnetism gave birth to a new discipline called Magnetohydrodynamics or MHD in short. This subject evoked lot of interest in the recent past owing to its variety of applications arising from the interaction of flow of electrically conducting fluids permeated by a magnetic field. The equations governing MHD are the well known fluid dynamic equations combined with Maxwell's equations. Since Maxwell's equations are linear, there is hope of solving the coupled equations of MHD even when the equations of fluid dynamics are non-linear.

This new branch of science not only provides interesting results but also throws light into many aspects of Geophysical and Astrophysical phenomena. It has more interesting applications in Astrophysics because much of the universe is filled with widely spaced charged particles and permeated by magnetic fields. Because of large scale motions, continuum hypothesis is applicable. Some of the applications of cosmical nature of this subject pertaining to sunspots which are seats of strong magnetic field.
Hale, in 1908 established this fact by Zeeman's effect. Curiously, the magnetic field of the earth did not excite similar enquiry for, the temperature of earth's interior, is well above the critical temperature and that a large fraction of the earth's interior is found to be in a liquid state. Not only the Earth and the Sun, it is probably safe to state that a magnetic field is a normal subsidiary part of any cosmic body that is both fluid and rotating. In 1919, Larmor put forward the tentative suggestion which was fundamental to hydromagnetic dynamo theory that motion of the electrically conducting fluid within the rotating body might by inductive action acted as armature and stator of a self existing dynamo. The general solar field, stellar magnetic field, the existence of the general galactic magnetic field and supernova remnants are some more illustrations of the applications of the subject.

Also geophysicists encounter MHD phenomena in the interactions between conducting fluids and magnetic fields that are present in and around heavenly bodies. The presence of the dense fluid core, which is electrically conducting in the earth and the origin of the main magnetic field of the earth can be very well understood with the knowledge of magnetohydrodynamics.
The subject of MHD has got its unique importance in engineering applications, although it had been studied for many years by physicists. The MHD generator, electromagnetic pump, flow meters are some of the practical applications of recent origin in MHD technology. Another application is the attenuation and reflection of microwaves by rocket exhausts, where the electron density becomes large enough for the exhaust flame to become electrically conducting. Yet another area where MHD find its use is electrical propulsion, which is one of the several proposed methods for achieving interplanetary travel. The dramatic achievements of launched satellite programmes over the last few years, now make it possible to see the earth and its magnetic field in the proper context of planetary magnetic field in general.

During the period, 1930 to 1940 many simple experiments were performed by astrophysicists particularly Cowling (1934) and Ferraro (1937) and scientists notably Williams (1930) and Hartmann (1937) on the flow of conducting fluids. In the year 1942, Alfven published the results obtained by him in the study of the motion of a charged particle in a magnetic field, including the celebrated Alfven's theorem. This is closely analogous to Kelvin's circulation theorem in inviscid fluid dynamics and he was
the originator to the nomenclature Magnetohydrodynamics to the subject. His findings were concerned with a gas which is highly conducting in the presence of magnetic field and found that propagation of a new wave is possible known as Alfven wave. The Alfven wave velocity is equal to the ratio of the geometric mean ion-and-gyro frequencies to the electron plasma frequency, times the speed of light. The Magnetohydrodynamic shock in an incompressible fluid is also named after Alfven as 'Alfven shock', which in general is a three dimensional one. His work was later investigated by Walen (1944) and others.

Since then there has been a flood of papers developing different aspects of these theories and their applications to the Earth, Sun and other systems. Some of the important ones are by Elsasser (1946), Bullard and Gellman (1954), Backus (1958), Chandrasekhar (1956) and Cowling (1962).

Benard Convection

The phenomenon of Rayleigh-Benard convection in a horizontal layer of fluid heated uniformly from below has been studied extensively over the last seventy five years. Agreement between theory and experiment is often very good, and the interaction between them has produced many fruitful
results which have contributed to our understanding of basic fluid dynamical processes but also to our understanding of physical processes in astrophysical and geophysical contexts.

In geophysical and astrophysical applications convection theory, the variety of different boundary conditions occurring in nature is a strong motivation to study their influence on properties of convection. Although the earth's core represents only about one sixth of the earth's mass, it is the origin of a variety of geophysical phenomena. By far the most important is the phenomenon of geomagnetism. Because of the obvious mathematical difficulties, the hydro-magnetic dynamo problem has been attacked only in the past few years. Difficulties start with the solution of purely hydrodynamic problem. For this reason Busse (1973) and Soward (1974) have solved the hydrodynamic dynamo problem in the case of convection in a layer heated from below which has the additional advantage that it can be related to geophysical processes.

The equations of hydrodynamics, inspite of their complexity allow some simple patterns of flow as stationary solutions. These flow patterns can be realised only for certain range of parameters, characterizing the flow. These parameters may include geometrical parameters such as the dimensions of the system; parameters characterizing the
velocity field prevailing in the system; the magnitudes of the forces acting on the system, such as pressure gradients, temperature gradients, magnetic fields and rotation etc. When we speak of stability of such a system we wish to determine the reaction of the system to small disturbances. That is if the system is disturbed, will the disturbance gradually die down or will the disturbance grow in amplitude such that the system progressively departs from the initial state. In the former case, the system is said to be stable with respect to the particular disturbance and in the latter case it is unstable. A system is considered stable if there exists no mode of disturbance for which it is unstable. The locus which separates the stable and unstable states defines the state of marginal stability of the system. To determine the stability of a hydrodynamic system all parameters except one are kept as constant while the chosen one is varied continuously. When we pass from stable to unstable state this particular parameter attains a critical value and we say that instability sets in at this value of the chosen parameter while all other parameters have preassigned values.

States of marginal stability are of two kinds. One corresponding to amplitude of a small disturbance growing (or damping) aperiodically and the other by oscillations of increasing (or decreasing) amplitude. In the first case
transition takes place via a marginal state exhibiting a stationary pattern of motions and in the second case transition takes place via a marginal state exhibiting oscillatory motions with a certain definite characteristic frequency. If at the onset of instability a stationary pattern of motions prevails, then we say that the principle of exchange of stabilities is valid and instability sets in as cellular convection. While if at the onset of stability oscillatory motions prevail we have overstability. The instability that occurs in a static layer of fluid heated from below and cooled above has long been a source of fascination (Benard 1900; Rayleigh 1916; Pellow and Southwell 1940; Chandrasekhar 1961).

The basic theoretical foundations for a correct interpretation of the convection phenomenon was given by Lord Rayleigh in a fundamental paper. Rayleigh showed that the stability of a layer of fluid heated from below is decided by the numerical value of the Rayleigh number $R$ (a non-dimensional number characterising the ratio of buoyancy force to viscous force). Rayleigh further showed that instability must set in when $R$ exceeds a critical value $R_c$ and that when $R$ just exceeds $R_c$, a stationary pattern of motions must occur. The determination of $R_c$ and the corresponding critical wave number will therefore be the
chief interest in any hydrodynamic or hydromagnetic convective flows. Such problems of hydrodynamic stability has been enlarged by the interest in hydrodynamic flows of electrically conducting fluids in the presence of magnetic fields.

**Rotating Fluids**

The fact that most of the heavenly bodies including the planets are in a state of rotation, necessitates the study of rotating fluids as one of the important topics. Whenever rotation is included as an absolutely essential factor, there must occur large-scale circulations in the atmosphere and oceans. In many small scale motions rotation plays an important role such as those occurring in centrifugal pumps and hydraulic turbines. Variability in the earth's atmosphere through the occurrence of weather systems, is an almost universal characteristic of natural flows. The oceans also contain weather system, and indeed sea-surface temperature fluctuations may have effects on atmospheric weather comparable with the atmosphere's intrinsic variability and changes in the earth's magnetic field reflect the changes in the earth's fluid core. The recent dramatic growth in our information about the atmospheres of other planets tells the same story. The appearance of Jupiter has changed far more than anticipated.
between the visits of the Pioneer probes in 1973-74 and the Voyager probes in 1979. Mars has intermittent dust storms, fluctuating winds, occasional clouds indicative of large cyclones. Ultraviolet images of Venus give every indication of a complex meteorology. The major reason why these systems, extremely varied in their dynamical processes, all show this characteristic is the pervasiveness of instability of fluid motion. Variation in density of atmosphere and ocean in the vertical direction occurring due to temperature changes or some other cause, also play a significant role through the action of buoyancy force. Fluids in which such density variation occur are classified as stratified fluids.

The important effect within the geophysical flows which can influence the flow besides stratification is the rotation (Greenspan (1968), Light Hill (1966)). The following three principles which give the theory of homogeneous rotating fluids its distinctive character are found to be essential for an understanding of a wide variety of phenomena. The first principle is the Taylor-Proudman (1917) theorem, which is valid for a homogeneous fluid in which the viscous and inertial forces are small compared to the coriolis force. It not only states that the velocity does not vary in the direction of $\mathbf{n}$, where $\mathbf{n}$ is the
rotation vector, but also implies that very slight extensions of fluid columns parallel to \( \vec{\Omega} \) can induce appreciable vorticity throughout the fluid. The second important principle is that the extension of fluid columns in the fluid's inviscid interior can be produced by the suction of the fluid into their viscous boundary layers (called Ekman layers) existing along surfaces which are not parallel to \( \vec{\Omega} \). Thirdly, the circulations produced by the Ekman layer suction must frequently be closed by boundary layers parallel to \( \vec{\Omega} \) which were first discussed by Stewartson (1957). These boundary layers have a double structure and can either be free or attached to a rigid wall which is parallel to the rotation vector.

A conceptually simple problem, which has provided much valuable insight into the dynamics of homogeneous rotating fluids in recent years, is the so called spin-up problem. In the simplest case it concerns the manner in which a contained fluid adjusts from one state of rigid body rotation to another (with the rotation axis fixed in direction).

In the spin-up problem treated by Greenspan and Howard (1963) the fluid is 'contained' between two infinite flat parallel plates perpendicular to the rotation axis and only an infinitesimal change in rotation speed is allowed so that the mathematical problem becomes linear. Extensions of this work are required before it can be applied directly
to geophysical or astrophysical problems of interest. One such extension of importance is to consider the effects of a stable density stratification as has been done by Pedlosky (1967). Loper and Benton (1969) studied the prototype hydromagnetic spin-up problem for a homogeneous fluid. The main motivation for their work was the slow westward drift of the geomagnetic field and the solar spin-down problem (e.g., Howard, Moore and Spiegel (1967)) which suggest the relevance of spin up problems for electrically conducting fluids.

The influence of magnetic field over rotation has drawn considerable attention for the past so many years because it is fundamental to various geophysical phenomena such as geomagnetic dynamo, solar spindown. There are many important contributions in this field and the papers of Hide (1969), Taylor (1963), Acheson and Hide (1973) give a very good account of the available literature. Similar to Ekman layer in rotating flows, the boundary layer at a rigid boundary surface of an electrically conducting rotating incompressible fluid due to the combined effects of rotation and magnetic field is the Ekman-Hartmann layer. The motivation for the study of these layers is to understand the character of the boundary layer likely to exist at the core-mantle interface of the earth where rotation and magnetic field act simultaneously. Further the knowledge about this
layer stands as a tool to know, how the geophysically important Ekman suction velocity is affected by magnetic fields [Gilman and Benton (1968)] and are called sometimes as Ekman-Hartmann layer or MHD-Ekman layer.

There have been a number of studies of hydromagnetic convection in rapidly rotating Boussinesq fluid layers. A recent study has been made by Eltayeb (1972, 1975), by further restricting to large magnetic field. Two applications of his stability problem leads to the conclusion that magnetic stars should be rapid rotators. In general this appears to be the case, though it seems to be true that many rapid rotators are not magnetic. The Earth provides a second illustration of the stability criterion and this leads to some of the ingredients required for the operation of the geodynamo.

Another important area where rotation plays an important role is the dynamics of oceans and atmosphere where large scale motions occur. The phenomenon of weather in the atmosphere is in fact nothing more the existence of large scale wave like fluctuations in the circulations of the atmosphere whose occurrence cannot be predicted. Observations of oceanic motions have also revealed fluctuations at periods which bear no evident relationship with the astronomical periods which characterize the externally
impressed forces. The existence of fluctuations in the circulations of the atmosphere and oceans can be attributed to the instability of the dynamical steady equilibrium state to very small wave-like disturbances. A truly major contribution of the early pioneering work in the field of atmospheric instability was done by Charney (1947) and Eady (1949). They demonstrated that the mode of instability of conceptually reasonable initial states possessed time and space scales and a physical structure remarkably close to the observed weather waves in the atmosphere. The notion that the observed fluctuations in the atmosphere could be explained in terms of the small-amplitude stability analysis of a highly idealized flow is not an obvious one, and its subsequent verification is a tribute to the profound physical insight of the early investigators. The characteristic disturbances of certain types of initial states are identified by Eady as the ideal forms of the observed cyclone waves and long waves of middle and high altitudes and the implications regarding the ultimate limitation of weather forecasting.

Previous studies of the stability of a zonal current have revealed two different types of instabilities. The first one is an interesting form of stability that can occur in a rotating stratified fluid called baroclinic instability.
The atmosphere which is heated by the sun more strongly at the equator than the poles, is a system that is inherently unstable. The mechanism of baroclinic instability gives a method whereby a small perturbation of the basic steady flow can generate large scale waves known in the atmosphere as cyclones. Three main ingredients are namely, rotation, stratification and a horizontal temperature gradient. The underlying principle of baroclinic stability is, where the initial state is the one in which heavier fluid always lies below lighter fluid. For baroclinic instability, the available potential energy comes from the horizontal temperature gradient. The other is a symmetric instability (no longitudinal variations) which draws its energy from the kinetic energy of the unperturbed zonal flow. Since the symmetric instabilities can only occur if the Richardson number $R_i$, is less than unity, an analysis of the non-geostrophic baroclinic stability problem is required to answer the question.

The published studies of the conventional baroclinic instabilities under non-geostrophic conditions have neglected the latitudinal variations of the perturbations [Philips (1964)] and therefore exclude a priori any symmetric instabilities. Similarly the published studies of symmetric instabilities have neglected the longitudinal variations of the perturbations and therefore also exclude a priori the conventional
baroclinic instabilities. Both latitudinal and longitudinal variations must be taken into account if one is to determine which type of instability has the largest growth rates. Growth rates for various types of perturbations are found as a function of $R_i$. In general the symmetric instabilities play an important role in the atmospheres of the major planets of the solar system.

The large scale atmospheric motions have long been known to be driven by the latitudinal variation of the heating of the earth by the sun. However the predominant feature of the resulting circulation is its longitudinal component. Hadley (1735) first related this phenomenon to the constraint of the earth's rotation in one of his papers. The general circulation of the real atmosphere is of course much more complicated than a simple Hadley cell. Furthermore, theoretical studies are hampered not only because of the complexity of the governing mathematics, but also meaningful quantitative data on real geophysical flows are very difficult to obtain, especially on the large scale. Recently, theoretical geophysical fluid dynamics has been stimulated by the initiation of controlled laboratory experiments in rotating tanks [Long (1953)]. A class of experiments has been performed with a rotating fluid differentially heated in the horizontal. Meteorological
interest in these experiments lies in the apparent relation of the wave and eddy regimes to the general circulation patterns in the atmosphere. However, to gain understanding of these complicated regimes by existing theoretical techniques, it is necessary first to have a complete theory of the symmetric state. Then by the study of the symmetric state, the criterion for the initial onset of the wave regime can be found.

The mathematical description of the axisymmetric flow, which resembles the trade wind circulation occurring in the atmosphere in the tropics, is a necessary preliminary to baroclinic instability analyses, in addition, to being an important problem in its own right. In most of the cases the Peclet number $P$ (a general measure of the ratio of convective effects to conductive effects) is so high that the effect of the hydrodynamic motion on the impressed temperature field is considerable and the governing mathematical equations are essentially non-linear. When the Peclet number is low, the effect of hydrodynamic motion on the temperature field is only small, and the governing equations can be linearised and solved analytically [Robinson (1959), Hunter (1967)].

Experiments on thermal convection in a rotating fluid subject to a horizontal temperature gradient (reviewed by
Hide and Mason (1975) have shown that simulating aspects of the oceanic or atmospheric circulation can be reproduced in a simple laboratory model consisting of a rotating fluid annulus bounded above and below by horizontal planes. In the experiments the two vertical surfaces are maintained at different temperatures while the horizontal surfaces are insulated. A theoretical study of the resulting steady axisymmetric fluid motion in the limit of small Ekman number (which measures the dissipative effects of viscosity) by Daniels (1976) was to understand further the properties of differentially heated rotating system and its relevance to problems of oceanographic and meteorological significance.

**Basic Equations**

The basic equations governing the flow of an electrically conducting fluid in the presence of a magnetic field, when referred to a frame which rotates with angular velocity \( \Omega \) relative to an inertial frame are as follows.

The equation of momentum

\[
\rho \left[ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} + 2 \vec{\Omega} \times \vec{u} \right] = -\nabla p + \rho \vec{F} + \gamma \rho \nabla^2 \vec{u} + \vec{j} \times \vec{B}
\]

... (1.1)

Here \( \vec{u} \) denotes the Eulerian flow velocity, \( \rho \) density, \( p \) pressure, \( \vec{F} \) the body force per unit mass, \( \vec{j} \) current density,
\( \vec{B} \) the magnetic field, \( \nu \) the coefficient of kinematic viscosity and \( t \) time.

The equation of continuity

\[
\frac{DP}{Dt} + \rho \nabla \cdot \vec{u} = 0 \tag{1.2}
\]

Equation of heat flow

\[
\frac{\partial T_e}{\partial t} + (\vec{u} \cdot \nabla) T_e = \kappa \nabla^2 T_e + Q \tag{1.3}
\]

where \( \kappa \) is thermometric diffusivity, \( T_e \) temperature and \( Q \) represents effects due to adiabatic compression and to any heat sources, associated with radiative or chemical processes, mechanical friction, phase changes, dissipation of electromagnetic energy, etc., that might be present in the fluid.

The electrodynamic equations are as follows:

The magnetic induction

\[
\vec{B} = \mu \vec{H} \tag{1.4}
\]

The electrical displacement

\[
\vec{D} = \varepsilon \vec{E} \tag{1.5}
\]

Ampere's law

\[
\nabla \times \vec{B} = \mu \vec{j} \tag{1.6}
\]
Faraday's law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1.7)$$

Ohm's law

$$\vec{j} = \sigma_c (\vec{E} + \vec{u} \times \vec{B}) \quad (1.8)$$

Gauss's law

$$\nabla \cdot \vec{B} = 0 \quad (1.9)$$

where $\mu$ denotes the magnetic permeability, $\varepsilon$ dielectric constant and $\sigma_c$ the electric conductivity.

The generalized Ohm's law [Cowling (1957)] with ion slip neglected is given by

$$\frac{\vec{j}}{\sigma_c} = (\vec{E} + \vec{u} \times \vec{B}) - \frac{1}{en_e} \vec{j} \times \vec{B} \quad (1.10)$$

where $e$ is the charge of an electron and $n_e$ is the electron charge density.

**Hall Effect**

The electrical current density $\vec{j}$ represents the relative motion of charged particles in a fluid. The equation of electric current density can be derived from the diffusion velocities of charged particles. The major forces on charged
particles are electromagnetic forces which give rise to the generalized Ohm's law. However the deduction from the diffusion of velocities of charged particles is more complicated than the generalized Ohm's law because, when we apply an electric field \( \vec{E} \), there will be an electric current in the direction of \( \vec{E} \). If the magnetic field \( \vec{H} \) is perpendicular to \( \vec{E} \), there will be an electromagnetic force \( j \times \vec{B} \) which is perpendicular to both \( \vec{E} \) and \( \vec{H} \). Such a force will cause the charged particles to move in the direction perpendicular to both \( \vec{E} \) and \( \vec{H} \). Then we will have a new component of electric current density in the direction perpendicular to both \( \vec{E} \) and \( \vec{H} \) which is known as Hall current. This phenomenon is generally called the Hall effect and occurs in solids and liquid conductors as well as in ionized gases.

**Boundary Conditions**

The boundary conditions are recorded as follows:

Let \( \hat{n} \) denote the unit drawn normal to the interface separating two electrically conducting media with different physical properties and \( \langle \alpha \rangle \) denotes the 'jump' in \( \alpha \) across the boundary. The boundary conditions are

\[
\hat{n} \cdot \langle \vec{u} \rangle = 0, \quad (1.11)
\]

\[
\hat{n} \times \langle \vec{u} \rangle = 0, \text{ if the fluid is viscous.} \quad (1.12)
\]
The magnetic boundary conditions are

\[
\hat{n} \cdot \langle B \rangle = 0 \tag{1.13}
\]

\[
\hat{n} \times \langle H \rangle = j \tag{1.14}
\]

\[
\hat{n} \cdot \langle D \rangle = \phi^* \tag{1.15}
\]

\[
\hat{n} \times \langle E \rangle = 0 \tag{1.16}
\]

where \( j^* \) denotes the surface current and \( \phi^* \) the surface charge density.

**Non-dimensional Numbers**

In general while expressing the results of the theory, it will be often convenient to combine various parameters into certain non-dimensional combinations or numbers.

If the equations of the thermal stability of a horizontal layer of fluid heated from below when cast in dimensionless form, the combinations which occur are

\[
p = \frac{\gamma}{\kappa}, \quad R = \frac{g \alpha \beta d^4}{\kappa \gamma}.
\]

Here \( \gamma \) is the kinematic viscosity, \( \kappa \) the thermal diffusivity, \( \alpha \) the co-efficient of volume expansion, \( g \) the acceleration due to gravity, \( \beta \) the adverse temperature gradient and \( d \) the depth of the layer. \( R \) is called as
the Rayleigh number and $p$ is the Prandtl number.

If the layer is rotated with uniform angular velocity $\vec{\omega}$, a new dimensionless parameter, $T$, known as the Taylor number arises. It is defined by,

$$T = \frac{4|\vec{\omega}|^2 d^4}{\nu^2}.$$

Alternatively, if the fluid is electrically conducting in which a uniform magnetic field $\vec{B}_0$ prevails, two more dimensionless parameters $p_m$ and $M$ appear, where

$$p_m = \frac{\gamma}{\eta}, \quad M^2 = \frac{\frac{\vec{B}_0^2 d^2}{\mu \rho_c \gamma \eta}}{\frac{1}{\mu \sigma_c}}$$

$p_m$ is called the magnetic Prandtl number, $M$ is the Hartmann number, $\rho_c$ is the mean density, $\sigma_c$ is the electric conductivity, $\mu$ is the magnetic permeability and $\eta(=\frac{1}{\mu \sigma_c})$ is the magnetic diffusivity (m.k.s units).

The ratio of the frictional force per unit mass to the Coriolis acceleration is a non-dimensional parameter, $E$, called the Ekman number and given by

$$E = \frac{\gamma}{\Omega L^2}.$$

Though the value of $E$ is quite small it is of great importance in rotating fluids of boundary layers, especially in
finding the thickness of the boundary layer.

Richardson number $R_i$ defined by

$$R_i = \frac{\alpha g L^2 \beta}{U^2}$$

which represents a measure of the strength of stratification, is also the ratio of the buoyancy force to the inertial force.

In problems where the effect of magnetic field is taken into account, the parameter called magnetic interaction parameter ($S^2$) occurs, which is defined depending upon the nature of the situation there. If the effect of Hall current is also considered, an additional parameter known as Hall parameter denoted by $m$ appears in the governing equations.
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