Chapter III

Database, Concepts and Methodology

Investments in Infrastructure including transport and its related facilities (roads, railroads, ports and airports), water and waste water treatment, telecommunications, energy generation, transmission and distribution, is often mentioned as a prerequisite for the success of development policies and has therefore important issue of discussion and need to be addressed in present times. With ever increasing demand for basic infrastructure facilities owing to increasing population and rural-urban migration it becomes imperative to undertake a study which encompasses the issues: i) to what an extent infrastructure have spillovers on industrial performance in India; ii) is the infrastructure contributing towards productivity growth; iii) whether the infrastructure and foreign direct investment have cause and effect relationship; and iv) what kind of nexus exists between infrastructure, economic growth and pollution. Therefore, the measurement of productivity of Indian manufacturing sector and its relationship with infrastructure along with the presence of externalities assumes great importance in the liberalised regime.

In the present chapter, an attempt has been made to outline the database and construction of variables used in the study. The chapter also deals with the theoretical underpinnings of the concept and methodology to work out technical efficiency and total factor productivity growth in Indian manufacturing sector. Moreover literature and methodological framework regarding empirical estimation of relationship between infrastructure and other economic variables has also been discussed. For this purpose, the chapter has been divided into four distinct sections. Section-I provides the sources of the data and explains the procedure of constructing infrastructure index and the classification of other economic variables Section-II provides a brief overview of the existing parametric techniques along with a detailed discussion on non-parametric Data Envelopment Analysis (DEA). Further, Section –III discusses about methodology pertaining to the long run relationship between the infrastructure development and foreign direct investment and finally Section-IV explains the application of Environment Kuznets Curve on major infrastructure and pollution variables.
Section I

Data-base and Construction of Variables

It is well acknowledged that a well defined set of the economic variables is required to evaluate the empirical relationship between variables based on the underlined objectives of the study. The required data for present study have been culled out from the various reports of Center for Monitoring Indian Economy (CMIE), International Financial Statistics (IFS) of International Monetary Fund (IMF), Balance of Payments Statistics (BOPS) by IMF, Economic Surveys, Handbook of Statistics on Indian Economy and World Development Indicators.

In detail, the present study is confined to the period from 1970-71 to 2009-10. The choice of terminal year is governed by the availability of latest data from the different data sources. However, the period pertaining to the study of industrial performance of major manufacturing industries is confined to 1990-91 to 2008-09 because of data availability from CMIE industry reports for the same period. Further, in order to investigate the presence of infrastructure spillovers on industrial performance the data related to control variable such as expenditure on R&D in given industry group has been squeezed out from “Prowess” data source of CMIE. However, the data on FDI inflows have been collected from “International Financial Statistics (IFS)” published by International Monetary Fund (IMF). In order to convert the figures which are provided in US $, the exchange rate of Indian rupee per US $ has been utilized and for the same “Balance of Payments Statistics (BOPS)” has been utilised. Moreover, the data pertaining to pollution variables has been made available from various issues of World Development Indicators Report, published by World Bank.

| Table 3.1: Description of Variables for Calculating Efficiency and TFP Growth |
|------------------|------------------|------------------|
| **Variable**     | **Description**  | **Nature**       |
| 1) **Output:**   |                  |                  |
| Gross Value Added| Net Output Added + Depreciation | ... |
| 2) **Inputs:**   |                  |                  |
| Labour           | Wage Expenditure | Variable         |
| Intermediate Inputs | Expenditure on Raw Material | Variable |
| Gross Fixed Capital | Net Fixed Capital + Depreciation | Fixed |

**Source:** Authors’ Elaboration

The Table 3.1 provides detailed information regarding the variables used for estimating the efficiency and productivity of Indian manufacturing industries and
further, the definitions of inputs and outputs and other control variables have been given as follows:

**Infrastructure index**: Since it is difficult to incorporate entire infrastructure variables in study and to establish their relation with different policy variables. Therefore, a collective index of all variables is required to represent the entire spectrum of infrastructure variables. The infrastructure index of different infrastructure variables i.e., railways passengers per km, Roads paved, ports traffic handling capacity (commodity wise), telephone subscribers, air freight handling capacity, energy use per capita, electricity installed capacity in MW has been calculated by using the method of Principal Components Analysis. Principal Component Analysis (PCA) is a standard tool for dimensionality reduction, applied in regression, classification and many other data analysis tasks in a variety of fields. PCA finds orthonormal directions with maximal variance of the data and allows its low-dimensional representation by linear projections onto these directions. This dimensionality reduction is a typical preprocessing step in many problems. In order to work out the principal component consider the following data matrix:

\[
X = \begin{bmatrix} X_{ij} \end{bmatrix}
\]

in which the columns represent the \( p \) variables and rows represent measurements of \( n \) objects or individuals on those variables. The data can be represented by a cloud of \( n \) points in a \( p \)-dimensional space, each axis corresponding to a measured variable. There exists a line (say) \( OY_1 \) in this space such that the dispersion of \( n \) points when projected onto this line is a maximum. This operation defines a derived variable of the form

\[
Y_1 = a_1x_1 + a_2x_2 + a_3x_3 + \ldots \ldots a_px_p
\]

with coefficients \( a_i \) satisfying the following condition

\[
\sum_{i=1}^{p} a_i^2 = 1
\]

After obtaining \( OY_1 \), consider the \((p-1)\) - dimensional subspace orthogonal to \( OY_1 \) and look for the line \( OY_2 \) in this subspace such that the dispersion of points when projected onto this line is a maximum. This is equivalent to seeking a line \( OY_2 \) perpendicular to \( OY_1 \) such that the dispersion of points when they are projected on to this line is the maximum. Having obtained \( OY_2 \), consider a line in the \((p-2)\) - dimensional subspace, which is orthogonal to both \( OY_1 \) and \( OY_2 \), such that the
dispersion of points when projected on to this line is as large as possible. The process
can be continued, until \( p \) mutually orthogonal lines are determined. Each of these
lines defines a derived variable:

\[
Y_i = a_{i1}X_1 + a_{i2}X_2 + a_{i3}X_3 + \ldots + a_{ip}X_p
\]

(3.3)

where the constants \( a_{ip} \) are determined by the requirement that the variance of \( Y_i \) is a
maximum, subject to the constraint of orthogonality for each \( i \).

\[
\sum_{k=1}^{p} a_{ik}^2 = 1
\]

(3.4)

The \( Y_i \) obtained are called *Principal Components* of the system and the process of
obtaining them is called *Principal Components Analysis*.

Moreover it is essential to constrain the size of \( a \), otherwise the variance of the
linear composite can become arbitrarily large by selecting large weights. It is
important to note that principal components decomposition is not scale invariant. We
would get different decompositions, depending upon whether the principal
components are calculated from the un-scaled cross-products matrix (SSCP) or
covariance matrix. The magnitudes of the diagonal elements of a cross-products
matrix or a covariance matrix influence the nature of the principal components and
the problem can be avoided by using the standardized variables.

Since first component explains the maximum variance of all the variables, thus
to derive the composite index of all above mentioned variables, the square of these
coefficients (i.e. \( a_{ij}^2 \)) can be used as weights to the respective variables. By using the
square of the first principal component the following weights are assigned to different
infrastructure variables and a series of infrastructure index has been calculated from
1970-2010.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Variables</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Electricity (Installed capacity)</td>
<td>0.147</td>
</tr>
<tr>
<td>2</td>
<td>Telephone (only Landline) '000 lines</td>
<td>0.132</td>
</tr>
<tr>
<td>3</td>
<td>Railways (Passengers carried)</td>
<td>0.148</td>
</tr>
<tr>
<td>4</td>
<td>Energy</td>
<td>0.147</td>
</tr>
<tr>
<td>5</td>
<td>Roads (Surfaced Roads)</td>
<td>0.138</td>
</tr>
<tr>
<td>6</td>
<td>Freight (Air Transport) Lakh Per Tonne Km</td>
<td>0.137</td>
</tr>
<tr>
<td>7</td>
<td>Ports (Commodity wise) '000 tonnes</td>
<td>0.148</td>
</tr>
</tbody>
</table>

*Source: Author’s Calculations*
Labour Input

The figures of the first input labour have been obtained from CMIE industry reports. As per the definition the data is given in form of total wages and salaries expenses in monetary terms, therefore to find the value of wages and salaries at the constant prices the Consumer Price Index for industrial workers has been used. Further, in order to examine the impact of infrastructure on total factor productivity, the use of labour in man days is used. However, the CMIE data base does not provide the exact information regarding labour in terms of man days. Thus, in order to calculate man days per industry (two digit classification) the salaries and wages paid by industry are divided by the average wage rate of an industry. Thus, the man days are as given below:

\[ \text{Number of man days} = \frac{\text{salaries and wages expenses}}{\text{average wage rate}} \]

To get the average wage rate, we used the information from Annual Survey of Industries (ASI) data. ASI contains information on total emoluments and total man days for the relevant industry groups. The average wage rate can be obtained by dividing the total emoluments to the total man-days for relevant industry groups.

Capital Input

In the present study, we use the gross fixed capital stock as a measure of capital input. As the data available is at current prices therefore an asset price deflator has been used to find the gross fixed capital at constant prices. The average price of the term lending institutions has been used as the price of capital.

Gross Value Added

The gross value added figures at constant price have been utilized as an index of output. As per the definition of GVA given by CMIE, the figures of Gross Value Added are derived by deducting total input from total output. The figures of ‘total output’ comprises total ex-factory value of products and by-products manufactured as well as other receipts from non industrial services rendered to others, work done for others on material supplied by them, value of electricity produced and sold and value of own construction. However, ‘total inputs’ comprises total value of fuels, materials consumed as well as expenditures such as cost of contract, cost of materials consumed for repair, insurance charges, banking charges, cost of printing and stationery, etc. Since the data is available in monetary terms, thus GDP deflator has been used to find out the gross value added at constant prices.
**Capital Intensity** \((K/L)\): Capital intensity can be obtained by dividing the gross capital to the labour. In order to get capital intensity as an industry-specific effect, we simply divide the summation over all firms’ capital stock to the summation over all firms’ labour of an industry (two digit classification)

**R&D Intensity** \((R)\): The R&D intensity is measured by the share of R&D expenditure to total sales in an industry.

### Section II

Productivity and efficiency are the two most important concepts in measuring performance and are frequently used. The concept of efficiency in the literature refers to an industry's ability to maximise outputs (such as gross value added) for a given set of inputs (such as labour, raw material and gross fixed capital, etc.), or to minimise the use of inputs given a set of outputs. An industry is said to be technically efficient if it cannot increase its outputs without some corresponding augmentation of inputs given the current state of production technology. In other words, a technically efficient industry would be one that produces the maximum possible output(s) from a given set of inputs or one that produces a certain level of output(s) with the minimum amount of inputs. Koopmans (1951) defines the efficiency as “A possible point in the commodity space is called efficient whenever an increase in one of its coordinates (the net output of one good) can be achieved only at the cost of a decrease in some other coordinate (the net output of another good).” The term commodity space is more commonly referred to as the production possibility set, meaning a set of all points, representing input (vector) and output (vector) pairs such that the input can be used to produce the output. Thus a technically inefficient producer could, by improving its performance, produce its output with less of at least one input, or could use its inputs to produce more of at least one output.

Farell (1957) defined efficiency as the ratio of observed output to the maximum potential output that can be attained from given inputs. If an industry’s actual output is below the maximum potential output, the shortage is regarded as an indicator of inefficiency. This interpretation may be ruled out as there is no inefficiency in a competitive market and inefficiency may exists in developing countries where market failure is prevalent and government deeply intervenes in the market.
Technical efficiency is defined as relative productivity over time or space, or both. For instance, it can be divided into intra-and inter-industry measures of efficiency. The former involves measuring the industry’s own production potential relative to a highest level of productivity and in contrast, the latter measures the performance of a particular industry relative to its best practice industry or industries available in the sample (Lansink et al., 2001). The concept of technical efficiency is closely associated with the ‘production frontier’ and the ‘cost frontier’ in economics. The former refers to the set of maximum outputs given the different levels of input, while the latter indicates the set of minimum inputs given the different levels of outputs. The production frontier reflects the current state of technology in the industry. Corresponding to the production or cost frontier, technical efficiency can be differentiated respectively as output- and input-oriented technical efficiencies; i.e., the producer can either improve output(s) given the same level of input(s) or reduce the input(s) given the same level of output(s).

Theoretically, the technical efficiency can be decomposed into scale and pure technical efficiency. Scale efficiency refers to the ratio of productivity measured when the industry produces at the actual, relative to ideal, production size, while the pure efficiency or managerial efficiency is a measure of managerial performance and is a devoid of scale effect and can be measured subject to the assumption of varying returns to scale.

The difference between technical efficiency, scale efficiency and productivity is illustrated, in Figure 3.1.

**Figure 3.1: Illustration of Efficiency and Productivity**

Source: Arora (2010)
The production frontier (OF) represents that all the points are technically efficient, while the points lying below or to the right of the efficient frontier are technically inefficient. At point A, the productivity can be measured by the ratio of DA/OD which can be further improved by moving from point A to B and the new level of productivity is then given by DB/OD. Therefore, productivity can be represented, by the slope of the ray through the origin, which joins the specific point. The output-oriented technical efficiency of industry A can be measured by the ratio of the productivity of point A to that of point B, i.e., (DA/OD)/(DB/OD)=DA/DB, which indicates the ratio of the actual output to potential output, given the input level. Similarly, the input-oriented technical efficiency of industry A can be measured as OH/OD, which indicates the ratio of inputs needed to produce output y, relative to the input, actually used to produce output y. The ray through the origin at a tangent to point C, in the above figure, defines the point of maximum possible productivity. The scale efficiency of industry B is thus defined as (DB/OD)/(GC/OG).

Technical inefficiency can be viewed from two different perspectives (Viton 1997). Input-oriented technical inefficiency focuses on the possibility of reducing inputs to produce a given output level and output oriented technical efficiency refers the possibility of increasing output given a certain level of inputs. This concept is illustrated in following figure.

**Figure 3.2: Input and Output Oriented Efficiency**

In figure 3.2, panel-A represents the input oriented approach using two inputs (X₁ and X₂) and one output (Y). The area above curve Y represents the set of infeasible inputs (X₁,X₂) that produce a given output Y. All the points on the curve itself represent the set of efficient inputs (the efficiency frontier). Any deviation from
the curve leads to inefficiency. A simple method to measure the degree of technical efficiency is to calculate the scalar \( \frac{OA'}{OA} \). The results of course lie in a ratio between 0 and 1. Output oriented technical inefficiency implies the possibility of increasing the output given the fixed inputs. Panel-B of above figure, represents two outputs and one input case and the area below curve DE represents the set of feasible output \((Y_1, Y_2)\). The curve itself represents the set of efficient output (the efficiency frontier) and the observation points on this frontier are efficient.

Further, the concept of technical efficiency can also be explained with the help of production function. The production function of any industry producing a single output with multiple inputs following the best practice techniques can be defined as:

\[
Y^*_i = f(x_{i1}, x_{i2}, \ldots, \ldots, x_{im}) / A
\]

where \(x_{ik}'s\) and \(Y^*_i\) are the \(k^{th}\) input and frontier output of the \(i^{th}\) industry respectively, and \(A\) is the given technology that is common to all industries in the sample.

Consider the situation where the industry is not producing its maximum possible output owing to some slackness in production induced by various non-price and socio-economic organizational factors. The production function of the industry can be written as:

\[
Y^*_i = f(x_{i1}, x_{i2}, \ldots, \ldots, x_{im}) \exp(u_i)
\]

Here \(u_i\) represents the combined efforts of various non-price and socio-economic organizational factors which constrain the industry from obtaining its maximum possible output. In other words, \(u_i\) which is industry-specific, reflects the industry’s ability to produce at its present level, which is otherwise called the industry’s technical efficiency. When the industry is fully technically efficient, then \(u\) takes the value of 0 and when the industry faces constraints, \(u\) takes a value less than 0. The value of \(u\) reflects the extent to which the industry is affected by the constraints. A measure of technical efficiency of the industry can be defined as:

\[
\exp(u_i) = Y_i / Y^*_i = \text{Actual output/ Maximum possible output} \quad (3.7)
\]

Where, the actual or realized output is observed output for a given set of inputs and the potential output is the technologically feasible maximum output for the same set of inputs under the production environment faced by industries.
Diagrammatically, $Y_{i}^{*}$ is the frontier output and $Y_{i}$ is the observed output if industry uses $X_{i}$ input combination. Thus, technical inefficiency is represented by the gap between $Y_{i}^{*}$ and $Y_{i}$.

**Approaches to Measure Technical Efficiency**

The literature on measurement of technical efficiency provides the two competing approach:

1. Parametric Frontier Approach
2. Non-Parametric Frontier Approach

The parametric frontier approach involves the estimation of certain parameters based on econometric models. The parametric approach can be further classified into one error structure approach which includes deterministic approach (Aigner and Chu (1968)) and statistical approach (Richmond (1974), Olson et.al. (1980)).

The one-error structure assumes that there are no random errors and that all of the noise in the error structure is technical inefficiency arising from industry-specific factors. The two-error structure on the other hand allows both technical inefficiency as well as random errors due to uncontrolled factors and measurement errors. There are various models available for estimation of technical efficiency under these assumptions but the discussion is on selective and only on core models used for measuring technical efficiency and changes and improvements in the measurement of technical efficiency over time.
The One-Error Structure

(a) **Deterministic Approach**

This was conceived by Aigner and Chu (1968) who specified the following Cobb-Douglas production function:

\[
\ln Y = \ln f(X) - u
\]

\[
= \alpha_0 + \sum_{j=1}^{m} \alpha_i \ln X_{ij} - u, \quad u \geq 0
\]

Where, \(i = 1, 2, \ldots, n\) and \(j = 1, 2, \ldots, m\)

Where the error term forces \(f'(x) \geq y\) as the frontier output is defined as the maximal output attainable by industry given its inputs and technology. The elements of the parameter vector \(\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_N)\) can be estimated either by linear programming (minimizing the sum of the absolute values of the residuals subject to the constraint that each residual be non-positive), or by quadratic programming (minimizing the sum of squared residuals subject to the same constraint).

(b) **Statistical Approach**

Consider the following production function:

\[
\ln Y_i = \alpha_0 + \sum_{j=1}^{m} \alpha \ln X_{ij} - u
\]

Where, \(\alpha_0\) is the intercept term;

\(\alpha\) is the vector of parameters; and

\(u\) is the residual.

The estimation of equation (3.9) by ordinary least squares (OLS) provides a best linear unbiased estimate of \(\alpha_{ij}\). In this model \(\alpha_0\) is part of technical efficiency as the potential output is given by \((\hat{\alpha}_0 + \hat{\alpha}' \ln X_i)\). Thus, for the measure of technical efficiency, it is important that the above model is corrected by Richmond’s (1974) method of corrected ordinary least squares (COLS). Thus, we can write equation (3.9) as:

\[
\ln Y_i = (\alpha_0 - \mu) + \sum_{j=1}^{m} \alpha_i \ln X_{ij} - (u_i - \mu)
\]

Where, \(\mu\) is the mean of \(u\) and the new error term \((u - \mu)\) satisfies all the standard OLS conditions of zero mean and constant variance but not the normality condition.
Now, $u$ can assume any distributional specification and we can estimate the parameters from the higher-order (second, third and so on) moments of the distribution of the OLS residuals. Since $\mu$ is a function of these parameters, $\mu$ can then be estimated consistently and used to ‘correct’ the OLS constant term.

(c) The Two-Error Structure

The two-error structure was developed to take explicit account of statistical errors as efficiency scores may be affected by noise or measurement error. This model eases off the strong assumption of the single error structure that all observations in the data set are accurate. The stochastic frontier with the two-error structure is given by:

$$\ln Y_i = \beta_0 + \sum_{j=1}^{m} \alpha_i \ln X_{ij} + u_i + \nu_i$$

(3.11)

Where, $u$ is the difference between the individual industry’s practice and the best practice technique; and $\nu$ represents statistical errors and other random factors. The two-error structure allows us to find out whether the deviation of industry’s actual output from its potential output (that is, technical inefficiency) is mainly because the industry did not use the best practice technique or because of external random factors.

Non-Parametric Approach i.e. Data Envelopment Analysis

The measurement of technical efficiency with DEA is based upon deviations of observed output or input vectors from the best production or efficient production frontier. If a production units’ actual production point lies on the frontier it is perfectly efficient. If it deviates from the frontier then it is technically inefficient, with the ratio of the actual to potential production defining the level of efficiency of the individual industry. This measure of technical efficiency provides an indication of how the use of all inputs can be minimized in the production process of a given industry, while continuing to produce the same level of output. Additionally, we consider the possible reduction of a subset of inputs while keeping other inputs and the output constant.

The technique of DEA was proposed by the Charnes, Cooper and Rhodes (1978) as a mathematical programming technique to evaluate the relative efficiency of various kinds of homogeneous organizational units termed as decision making units (in present study decision making units are major infrastructure industries). DEA is useful to assess the relative effectiveness among decision making units having
multiple inputs and multiple outputs. DEA is useful in cases where industries’ input-output transformation relationships are not well established.

Using DEA approach, each industry is optimised to its fullest possibility by calculating an optimal performance for each industry relative to all other industries in the sample. DEA seeks to describe which of the ‘n’ industries determine an envelopment surface or efficient frontier. Any industry that lies on the envelopment surface is believed to be efficient. Industries that lie below the efficient frontier are deemed inefficient. The relative efficiency of each industry is calculated using all of the industries input and output variables. For each inefficient industry, DEA identifies the sources and level of inefficiency for each of the inputs and outputs, and suggests how to reach the efficient frontier with the given level of inputs and outputs. Further, the estimated production frontier can reveal some characteristics of the production technology, such as returns to scale and rates of factor substitution.

**CCR and BCC Models**

As Farrell (1957) introduces the concept of relative efficiency, according to which, the efficiency of an industry can be evaluated by comparing it to the other industries in a given sample. This concept was extended by Charnes et al. (1978) who developed the first DEA model, called CCR (Charnes, Cooper and Rhodes), to incorporate many inputs and outputs simultaneously. In this way, DEA provides a straightforward approach for calculating the efficiency gap between the actions of each producer and best practices, inferred from observations of the inputs used and the outputs generated by efficient industries.

In DEA models, we evaluate ‘n’ industries, where each industry takes ‘m’ different inputs to produce ‘s’ different outputs. The essence of DEA models in measuring the efficiency of industry lies in maximizing its efficiency rate; subject to condition that the efficiency rate of any other industry in the sample must not be greater than one i.e. the weights of inputs and outputs must be greater than zero. Such a model is defined as:

$$\text{Maximize} \quad \frac{\sum u_i y_{ij}}{\sum j v_j x_{ij}} \quad (3.12)$$
Subject to
\[
\frac{\sum_i u_i y_{ik}}{\sum_j v_j x_{jk}} \leq 1 \quad k=1,2,\ldots,n
\]
\[
u_j \leq \epsilon \quad j=1,2,\ldots,m
\]
\[u_i \geq \epsilon \quad i=1,2,\ldots,s\]

This model can be converted into a linear programming model and transformed into a matrix:

\[
\text{Maximize } TE_{CRS}^k = \theta_k
\]
subject to:
\[
\sum_{j=1}^{n} \lambda j x_j \leq x_{ik} \quad i = 1,2,\ldots,m;
\]
\[
\sum_{j=1}^{n} \lambda j y_j \geq \theta_k y_{rk} \quad r = 1,2,\ldots,s;
\]
\[
\lambda j \geq 0, \quad j = 1,2,\ldots,n.
\]
(3.13)

Where, \(\theta\) represents the technical efficiency and hence percentage of radial increase to which output is subjected; \(\lambda_k\) represents the influence of \(k^{th}\) industry in determining technical efficiency; \(X_{i,k}\) and \(Y_{i,k}\) are \(i^{th}\) and \(r^{th}\) input and output variables of the \(k^{th}\) industry respectively.

The first DEA CCR model assumed constant returns to scale (CRS) which means an industry producing an output \(Y\), using an input \(X\), it is feasible for an industry to produce \(a*Y\) using \(a*X\) amount of input (\(a\) is a scalar). However, in practice this may not always be observed, as increasing the input does not usually result in a proportionate increase in output. For instance, in industry, when the amount of labour and capital is increased, there is not always an equiproportional increase in output. For this reason, a variable returns to scale (VRS) option might also be more considered for technical efficiency measures in the industry. The first DEA model used to assess technical efficiency under the VRS assumption was developed by Banker et al. (1984) and was called the BCC (Banker, Charnes and Cooper) model.

The main difference between CCR and BCC model lies in their assumption of production technique. As shown in Table 3.3, whereas CCR model produces a constant return to scale (CRS) envelopment surface, the BCC model produces a variable return to scale (VRS) envelopment surface. In BCC model, we add one more constraint \(\sum \lambda_j = 1\).
Table 3.3: Representative DEA models and Their Application

<table>
<thead>
<tr>
<th>Model</th>
<th>Envelopment Surface</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCR</td>
<td>CRS</td>
<td>Input Oriented</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Output Oriented</td>
</tr>
<tr>
<td>BCC</td>
<td>VRS</td>
<td>Input Oriented</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Output Oriented</td>
</tr>
</tbody>
</table>

**Source:** Lewin and Seiford (1994)

Each model is further broken down by its approach to achieve efficiency. In input orientation models, proportional decrease in the input variables is used as a means to achieve efficiency. In output orientation models, proportional increase in the output variables is used to achieve efficiency. (Lewin and Seiford (1994))

The input oriented CCR model can be written as:

\[
\text{Minimize } TE_{\text{CCR}}^k = \theta_k \\
\text{Subject to: } \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_k x_{ik} \quad i = 1, 2, \ldots, m; \quad \text{(3.14)}
\]

The VRS DEA is more flexible and envelopes the data in a tighter way than the CRS DEA. Moreover, to identify whether CRS or VRS applies to production, the DEA technique has been applied to calculate and compare the efficiency values under both assumptions. The use of the VRS specification permits the calculation of technical efficiency (TE) without the scale efficiency (SE) effects (Coelli, 1998). As the scale efficiency can be obtained by the ratio \( TE_{\text{CCR}} / TE_{\text{VRS}} \) thus the values of efficiency under CRS and VRS are required to calculate the scale efficiency. The measurement of scale efficiency can be illustrated as:

**Figure: 3.4: Measuring Scale Efficiency**
In this figure, we have two efficiency frontiers. Efficiency frontier $BG$ assumes CRS technology and efficiency frontier $AH$ assumes VRS technology. The $CD$ part of frontier has both CRS efficiency and VRS efficiency because both frontiers merged within each other. Thus, at this level overall technical efficiency (OTE) and pure technical efficiency (PTE) scores are unity and consequently scale efficiency score ($=OTE/PTE$) also equal to unity. Further, at point $F$, decision making unit (industry) ‘$s$’ is efficient assuming VRS technology but inefficient assuming CRS technology and consequently scale efficiency is given by ($=IF/IE$) $=EF$. The segment $EF$ is the measure of scale inefficiency.

It is pertinent to note that even though the both approaches are used to calculate the efficiency but the application on the same data set will report different results. This is due to the underlying assumptions that are assumed by each approach. For this same reason, there has not been a consensus on the best approach to be used when assessing efficiency. The main differences between the efficiency approaches could be presented in the following points:

1. The parametric approaches were initially designed to measure economic efficiency while the nonparametric approach was initially designed to measure technical efficiency. Technical efficiency requires the industry to produce the maximum outputs from a given set of inputs or the minimum inputs for a certain level of outputs. Economic efficiency requires the industry to choose its inputs/outputs combinations that will maximize profit based on the prevailing market prices. The economic theory suggests that economic efficiency implies technical efficiency; however, technical efficiency does not imply economic efficiency.

2. The shapes of the different efficiency frontier imposed by the both approaches are different. The parametric approach imposes more structure to the frontier versus less structure by the non-parametric approach. The structure imposed by the parametric approaches could be thought of as an advantage and a disadvantage over the non-parametric approach. An advantage because they allow for an error term to exist in the cost function and a disadvantage because they force a specific functional form on the efficient frontier.

3. The parametric approaches differ in the way they separate the unobserved random error from the unobserved inefficiency factor. To make this
distinction, they impose different assumptions on the distributions of the error term and the inefficiency factor. Further, the linear programming based DEA approach focuses on best practice frontiers rather than on central tendency properties of empirical data.

Section III
Productivity Measurement: A Theoretical Approach

Productivity refers to the relationship between outputs and inputs in real terms and is often measured as a partial measure, a total factor productivity measure, or a multifactor productivity measure. The partial measure is calculated as labour or capital productivity, i.e., net or gross output per unit of the respective inputs. It is given by:

\[
\text{Labour Productivity} = \frac{Q}{L} \tag{3.15}
\]

\[
\text{Capital Productivity} = \frac{Q}{K} \tag{3.16}
\]

Where Q, K and L are the aggregate level of output, capital and labour respectively.

But this measure only considers the use of a single input and ignores all other inputs, thereby causing misleading analysis. Thus the partial measure does not evaluate overall change in productive capacity since it is affected by the changes in compositions of inputs. For instance, improvements in labour productivity could be due to capital substitution or changes in scale economies, both of which may be unrelated to the more efficient use of labour. However, Kendrick (1961) maintains that these measures are useful in estimating the saving in the input per unit of output. Sargent and Rodriguez (2000) advocated the use of labour productivity to examine trends over a period that is less than a decade given the biases in estimating capital stock to obtain total factor productivity growth.

Unlike the partial measure, the multifactor and Total Factor Productivity (TFP) measure consider the joint use of the inputs in production and mitigate the impact of factor substitution and scale economies. They are given by:

\[
\text{TFP index} = \frac{Q}{aL + bK} \tag{3.17}
\]
\[ MFP_{\text{index}} = \frac{Q_2}{aL + bK + cM} \]  

(3.18)

Where \( Q_1 \) value added output, \( Q_2 \) is gross output, \( M \) is intermediate inputs and \( a, b \) and \( c \) are weights given by input shares.

These measures are the ratio of output to the weighted average of inputs. The distinction between TFP and multifactor productivity is that the latter includes the joint productivity of labour, capital and intermediate inputs and the former considers the joint productivity of labour and capital only. Intermediate inputs comprise of materials, supplies, energy and other purchase services. The multifactor productivity measure may also include other inputs such as land and other natural resources used in the production process.

**Approaches to TFP measurement**

The total factor productivity growth (TFP) defined as the residual growth in outputs not explained by the growth in input use is often measured by two approaches non frontier approaches vis-à-vis frontier approach. The former includes growth accounting formula (Solow, 1957) and index number approach (Laspeyres, Paasche, Fisher and Tornquist) whereas later one is based on ideas of Malmquist (1953) and Farrell (1957). All the index numbers measure the changes in the level of a set of variables between a base period and current period. Alternatively, the productivity change can be measured by using frontier approach i.e. Malmquist Productivity Index (MPI) (Malmquist 1953). The index was introduced into DEA literature by Caves, Christensen and Diewert (1982) and is based on Malmquist proposal to construct indices as a ratio of distance functions. Distance functions are representations of multi-output and multi-input technologies which require data only on input and output quantities (Fare et al. 1994).

The crucial distinction between these approaches lies in the very definition of the word ‘Frontier’. A frontier refers to a set of best obtainable position. Thus a production frontier traces the set of maximum outputs obtainable from a given set of inputs and technology, and cost frontier traces the minimum achievable cost given input prices and output. The production frontier is an unobservable function that is said to represent the ‘best practice’ function as it is a function bounding or enveloping the sample data. This is different from the average function, which is often estimated by the Ordinary Least Square (OLS) regression as a line of best fit through the sample.
data. The frontier and non-frontier categorization is of methodological importance since the frontier approach identifies the role of technical efficiency in overall industry performance, while the non-frontier approach assumes that the industries are technically efficient. Technical efficiency which is presented by a movement towards the frontier refers to the efficient use of inputs and technology due to the accumulation of knowledge in the learning-by-doing process, diffusion of new technology, improved managerial practice and so on. The frontier TFP growth measure consists of outward shift of the production function resulting from technological progress due to the technological improvements incorporated in inputs, as well as technically efficiency related to movement towards the production frontier. The non-frontier approach on the other hand only considers technological progress as a measure of TFP growth. However, another difference between frontier and non-frontier approach is that the former is best suited to describe industry or industry behavior. This is due to the benchmarking characteristic of the frontier approach, whereby an industry’s actual performance is compared with its own maximum potential performance or as defined by best-practice or efficient industries in the sample. Moreover, the parametric technique is an econometric estimation of a specific model and since it is based on the statistical properties of error terms, it allows for statistical testing and hence validation of the chosen model. However, the choice of the functional form is crucial for modeling the data as different model specifications can give rise to very different results. The non-parametric technique on the other hand does not impose any functional form on the model but has the drawback that no direct statistical test can be carried out for validation. Both of these approaches have been discussed as follows

The Non–Frontier Approach

The non-frontier approach uses the standard growth accounting framework which separates the growth of real output into an input component and a productivity component. It is given by:

\[
Output \ growth = Input \ growth + TFP \ growth
\]

\[
=> \quad TFP \ growth = Output \ growth - Input \ growth
\]

Where, input growth consists of the sum of the increase in the use of inputs and/or increase in productivity. This framework is able to provide the contribution of output to growth of each of the inputs used. TFP growth under this framework is
estimated as a residual measuring ‘everything and anything’ of output growth that is not accounted for by input growth and because the determination of TFPG is yet to be proved, this measure is often called a ‘measure of ignorance’ (Abramowitz, 1956).

The growth accounting is a step towards reconciliation of the economic balance sheet, as it provides a filing system that is complete in the sense that all phenomenon that affect economic growth must do so through input factor qualities and relative factor intensiveness (Nadiri, 1970). But, in spite of its limitations, the results from growth accounting have proven to be useful policy parameters, and the residual has provided the theory to guide a considerable body of economic measurement.

Let us discuss non-parametric index number method and then move on to parametric average response function to measure TFPG.

**a) Arithmetic indices of TFP**

In the arithmetic indices, it is taken as a weighted arithmetic mean of factor inputs with weights being the respective income share:

\[
TFP_t = \frac{\left(\frac{y_i}{y_0}\right)}{\sum_{i=1}^{k} S_i \left(\frac{X_{i,t}}{X_{i,0}}\right)}
\]

where, \(S_i\) is income share of the input \(i\). The most important widely used variant of arithmetic indices is Kendrick index. Kendrick index (1961) of TFP is based on a linear production function, which assumes infinite elasticity of substitution between factors of production. The Kendrick index is defined as:

\[
TFP_t = \frac{y_i}{\sum_{i=1}^{k} W_{i,0} X_i}
\]

where, \(W_{i,0}\) refers to the reward of the input in the base year. The arithmetic index of TFP rate from base year 0 to period 1 is expressed as:

\[
\frac{TFP}{TFP} = \frac{\left(\frac{y_{1,1}}{y_0}\right)}{\sum_{i=1}^{k} \frac{W_{i,0} X_{i,1}}{W_{i,0} X_{i,0}}}
\]
where $TFP$ indicates the rate of change of TFP with respect to time (i.e., $dTFP/dt$). Nadiri (1970) points out that such an index of TFPG rate is consistent with a production function of the form:

$$
y = \frac{\beta \prod_{i=1}^{k} X_i}{\left[ \sum_{i=1}^{k} \alpha_i X_i^\rho \right]^{-1/\rho}} \quad (3.24)
$$

\[b) \text{The Translog-Divisia Index:}\]

The most commonly used index for productivity measurement is the Theil-Törnqvist Index or the Translog-Divisia index defined for two time periods $s$ and $t$:

$$
\ln \frac{TFP_t}{TFP_s} = \ln TFP_t - \ln TFP_s
$$

$$= \frac{1}{2} \sum_{i=1}^{N} (W_i + W_i') (\ln y_i' - \ln y_i) - \frac{1}{2} \sum_{j=1}^{k} \left( \gamma_j + \gamma_j' \right) (\ln x_j' - \ln x_j) \quad (3.25)
$$

Where the $y$’s and $x$’s represent the value of output and input, and the $w$’s and $\gamma$’s represent value shares of outputs and inputs respectively. For the labour input, $\gamma$ is the labour income as paid out to workers, and for the capital input, the share is given by income earned by capital. The above index is easy to compute and can be calculated with just two data points, but it is appropriate only under the assumption of CRTS with the imposition of Marginal Productivity conditions where each input is assumed to be paid the value of its marginal product. However, unlike the parametric estimation, the index number does not assume constant weights as the periodic variations in factor shares given by the $\gamma$’s are directly taken into account.

c) \text{Econometric Estimation of Production and Cost Functions:}\]

The advantage of production function approach is that the parameters of production function (like returns-to-scale, elasticity of substitution between inputs, elasticity of inputs etc.) are estimated at the same time as the technical progress term by using suitable econometric techniques. The most important thing in this method is to find some suitable proxy for the disembodied technical progress and often time is used. The proxy time alone cannot alter production relations, and therefore, technical progress is treated as a residual part of output. This approach captures TFP growth as
the shift in production function and assumes that there is no inefficiency in production, and hence, therefore, TFP growth is equal to technical change.

In econometric analysis of production function, TFP can be measured by incorporating technical progress by introducing a time variable. In case of two input framework, the production function with technical progress may be written as:

\[
Y = f(K, L, t)
\]

where \( Y, K \) and \( L \) denote output, capital and labour inputs, respectively, \( t \) is the time variable which is explicitly incorporated in the production function as proxy for technical progress.

An alternative of technical progress is efficiency and if we regard efficiency parameter as time dependent variable then production function (3.26) is as follows:

\[
Y = A(t) f(K,L)
\]

Where \( A(t) \) represents effect of technical change and is an increasing function of time. Hence, \( A(t) \) denotes change in output due to factors other than changes in the quantities of inputs. Some of important specifications of production function to measure TFP growth are discuss below:

**c1) Cobb-Douglas Production Function**

The Cobb-Douglas production function is simplest and commonly used specification of production function for TFP measurement. Assuming Hicks-neutral technical progress grows at a constant exponential rate, CD Production function can be stated as follows:

\[
Y = A_0 e^{\alpha t} L^\alpha K^\beta
\]

Where \( Y, L \) and \( K \) are output, labour and capital respectively, \( t \) is time, \( \alpha \) and \( \beta \) are output elasticities of labour and capital respectively and \( \lambda \) is the rate of Hicks-neutral disembodied technical progress. This specification of CD production function can be estimated by ‘Maximum Likelihood Estimation (MLE) technique’. The log form of CD production function is:

\[
\log Y = \log A_0 + \alpha \log L + \beta \log K + \lambda t
\]

In the form ‘\( \alpha + \beta \)’ represents returns to scale as: i) \( \alpha + \beta = 1 \), implies Constant Returns to Scale; ii) \( \alpha + \beta > 1 \), implies Increasing Returns to Scale; and iii) \( \alpha + \beta < 1 \), implies Decreasing Returns to Scale. An alternative version of the equation (3.29) can be derived by subtracting \( \log L \) from both sides
\[
\log \left( \frac{Y}{L} \right) = \log A_0 + \beta \log \left( \frac{K}{L} \right) + (\alpha + \beta - 1) \log L + \lambda t \quad (3.30)
\]

According to equation (3.30), the coefficient of \( \log L \) equals the sum of output elasticities minus one, and its sign therefore indicates returns to scale. The advantage of this equation is that the statistical significance of returns to scale can be tested at a given level of confidence i.e., whether the coefficient of \( \log L \) significantly differs from zero. The null hypothesis that \( \alpha + \beta - 1 \neq 0 \) is established against the alternative of \( \alpha + \beta - 1 = 0 \). If returns-to-scale is constant, i.e., \( \alpha + \beta = 1 \) then coefficient of \( \log L \) should be insignificant.

A restrictive form of Cobb-Douglas production function explicitly assumes constant returns to scale (i.e. \( \alpha + \beta = 1 \)) and is written as:

\[
\log \left( \frac{Y}{L} \right) = \log A_0 + \beta \log \left( \frac{K}{L} \right) + \lambda t \quad (3.31)
\]

Above equation is known as constrained CD production function and parameters are generally estimated by OLS procedure.

c2) Constant Elasticity of Substitution (CES) Production Function

The Sollow, Minhas, Arrow and Chenary in (1961) developed CES production function as

\[
Y = A_0 \left[ \alpha L^{-\rho} + (1-\alpha)K^{-\rho} \right]^{\frac{1}{\rho}} \quad (3.32)
\]

where \( Y, L, \) and \( K \) are output, labour and capital respectively, \( \alpha \) is distribution parameter, \( \rho \) is substitution parameter related to elasticity of substitution assumed to be constant. After modifying for Hicks-neutral disembodied technological progress and non-constant returns to scale can be written as:

\[
Y = A_0 e^{\lambda t} \left[ \alpha L^{-\rho} + (1-\alpha)K^{-\rho} \right]^{\frac{1}{\rho}} \quad (3.33)
\]

where \( t \) is time parameter \( \nu \) is a scale parameter which was equal to unity in (3.33) because of assumption of CRTS and \( \lambda \) is rate of Hicks-neutral disembodied technical change. Taking log of both sides

\[
\log Y = \log A_0 + \lambda t - \frac{\nu}{\rho} \log \left[ \alpha L^{-\rho} + (1-\alpha)K^{-\rho} \right] \quad (3.34)
\]
Equation (3.34) cannot be estimated by OLS method, hence different methods has
been suggested by different persons. Some well known methods are SMAC (1961)
and Kmenta’s Approximation.

The SMAC form has the advantage that capital data are not required in its
estimation. This proves to be extremely helpful when capital data are unreliable or not
available whereas Kmenta’s single equation method based upon linear estimation of
CES by converting non-linear CES into linear CES using Taylor series.

3) Transcendental Logarithmic (Translog) Production Function

The Translog production function has been developed by Christensen,
Jorgenson and Lau (1973). The Translog production function specification is a
flexible functional form imposing relatively few a-priori restrictions on the properties
of the underlying technology. It allows for variable elasticity of substitution (VES),
variable scale elasticity and non-neutral technological progress. Homotheticity,
separability and CRTS can be imposed by testable restrictions on the parameters, and
the form reduces to the multiple input Cobb-Douglas specification as a special case.
The Translog production function with \( n \) inputs and general factor-augmenting
technical progress takes the form.

\[
Y = \exp \left[ \alpha_0 + \alpha_t t + \beta_n t^2 + \sum_{i=1}^{n} \alpha_i \log X_i + 0.5 \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \log X_i \log X_j + \sum_{i=1}^{n} \beta_n t \log X_i \right]
\]  

(3.35)

Where, \( Y \) is index of output, \( \text{‘} X_i \text{’} \) represents the ‘\( i^{th} \)’ input, \( t \) is time variable allowing
non-neutral technological change and \( \alpha_s \) and \( \beta_s \) are the parameters of the production
function. In Translog production function specification, the elasticity of output with
respect to inputs and capital are not constant and changes with input levels and time.
The elasticity of output of each variable input is:

\[
\frac{\delta \log Y}{\delta \log X_i} = \alpha_i + \sum_{j=1}^{n} \beta_{ij} \log X_j + \beta_n t
\]  

(3.36)

The expression for the rate of technical progress on TFPG in a Translog production
function is given by:

\[
\frac{\delta \log Y}{\delta t} = \alpha_i + \beta_n t + \sum_{i=1}^{n} \beta_n X_i
\]  

(3.37)
where $\alpha_t$ is rate of autonomous TFPG, $\beta_{lt}$ is the rate of change in TFPG and $\beta_{it}$ defines the bias in TFPG. If $\beta_{lt} = 0$ then TFPG is the Hicks-neutral type. If $\beta_{lt} > 0$, then the share of $i^{th}$ input rises with time and there is $i^{th}$ input using bias.

c4) Cost Function Estimation

A fundamental result of the duality theory is that under certain weak regularity conditions, a unique correspondence exists between production and cost functions. The specification of production function implies a particular cost function, and vice-versa. Let the production function be

$$Y = f(X, t)$$

(3.38)

Where, $Y$ denotes output and $X$ is the vector of inputs then under the assumption of cost minimization on the part of the industry, cost function can be derived as:

$$C = g(q, p, t)$$

(3.39)

Where ‘$C$’ denotes total cost, ‘$p$’ is the vector of input prices and ‘$t$’ is the time. A set of cost share equations can also be derived as (by using Shephard’s Lemma):

$$S_i = \frac{P_i X_i}{C} = \frac{\delta \log C}{\delta \log P_i} ; i = 1, 2, 3$$

(3.40)

The rate of technological change ($V_t$) may be obtained as:

$$V_t = \frac{\delta \log C}{\delta t}$$

(3.41)

And the bias in technological change for the use of the $i^{th}$ input ($V_{pit}$) may be obtained as:

$$V_{pit} = \frac{\delta \log S_i}{\delta t} = \frac{\delta^2 \log C}{\delta \log P_i \delta t}$$

(3.42)

Technical change is Hicks-neutral if $V_{pit} = 0$.

**Frontier Approach of TFP measurement:**

In the frontier approach TFPG can be decomposed into: i) Technological Progress; and ii) efficiency change. The Data Envelopment Analysis (DEA) based Malmquist Productivity Index (MPI) is a non-parametric technique that converts multiple input and output measures into a single comprehensive measure of productivity. This is done by linear programming which constructs the frontier technology from data and calculates the distance to that frontier for individual observations (industries). The frontier technology is formed as linear combinations of observed extreme activities, yielding a frontier consisting of facets, and the
performance of each industry is evaluated by comparing against a composite industry that is constructed by floating a piece wise flexible linear surface on the observations. Thus only part of the entire frontier is relevant when evaluating an industry's performance and this relevant portion is called a facet.

It should be noted that industries identified as efficient are only efficient in relation to other industries in the same sample. It is conceivable for an industry outside the sample to obtain higher efficiency score than the best practice industry in the sample. Thus, DEA labels an industry as efficient or inefficient by comparing to its reference set, which consists of efficient industries most similar to that industry in their levels of inputs and outputs.

Following Fare et al. (1998) for the output-based Malmquist index we assume the production technology describes the possibilities for the transformation of inputs $X^t$ into outputs $Y^t$ for each time period $t=1, \ldots, T$. This is the set of output vectors that can be produced with input vector $X$ for the technology in period $t$ and with $y^t \in R^n$ outputs and $x^t \in R^m$ inputs:

$$P^t(x) = \{y^t: \text{Such that } x^t \text{ can produce } y^t\} \quad (3.43)$$

The output distance function is defined at $t$ as the reciprocal of the maximum proportional expansion of output vector $y^t$ given input $x^t$:

$$D^t_0(x^t, y^t) = \inf \left\{ \phi: \left( x^t, \frac{y^t}{\phi} \right) \in P(x^t) \right\} \quad (3.44)$$

where $\phi$ is the coefficient dividing $y^t$ to get a frontier production vector at period $t$ given $x^t$.

The Malmquist index is defined using distance functions. Depending on the technology used as the reference, we can define a period $t$-based or a period $(t + 1)$-based Malmquist index. The period $t$-based Malmquist index is defined as

$$M^t_0 = \frac{D^t_0(x^t, y^{t+1})}{D^t_0(x^t, y^t)} \quad (3.45)$$

Using the technology at $t + 1$ as the reference, the period $(t + 1)$-based Malmquist index is defined as:

$$M^{t+1}_0 = \frac{D^{t+1}_0(x^{t+1}, y^{t+1})}{D^{t+1}_0(x^t, y^t)} \quad (3.46)$$
In order to avoid choosing the MPI of an arbitrary period Färe et al. (1994) specified the Malmquist productivity change index as the geometric mean of equations (1) and (2) referred as the “Fare index”

\[
M_0 = \sqrt{M_0^* M_0^*} = \sqrt{\frac{D_0(x_t^{*t}, y_t^{*t})}{D_0'(x', y')}} \frac{D_0(x_t^{*t+1}, y_t^{*t+1})}{D_0'(x', y')} = (3.47)
\]

From above discussion, it is evident that the DEA based techniques of efficiency and productivity measurement are best suited for the small sample datasets used by the present study. The technical efficiency and productivity of major Indian manufacturing industries at 2-digit level of aggregation, has been estimated using the CMIE dataset over the period 1990-91 to 2008-09. The use of the DEA technique over the econometric based stochastic frontier models has been preferred to avoid the risk of imposing a rigid parametric functional form. The rigidity of the functional form also involves the selection of a functional form of cost and production frontier. The selection of appropriate functional form is also a complicated task and need the thorough scrutiny of the researchers. Further, the DEA based MPI has advantage to decompose the TFP growth into: i) efficiency change (a measure of frontier catching-up); and ii) technical progress (a measure of shift in production frontier).

Section IV

Co-integration and the Error Correction Mechanism (ECM)

The non-stationary of variables in a regression model results in spurious estimators. Therefore, one way of resolving the problem of non-stationarity in time series data is to difference the series successively until stationarity is achieved and then use stationarity series for regression analysis. But there are two main problems using first differences.

1) If the model is correctly specified as a relationship between variables (say Y and X) and we difference both variables then implicitly we are also differencing the error process in the regression. This would then produce a non-invertible moving average error process and would present serious estimation problems.

2) The second problem is that if we difference the variables the model can no longer give a unique long run solution.

Thus, if we have \( Y_t \) and \( X_t \) that are both \( I(1) \), then if we regress:
We will not generally get satisfactory estimates of \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \). One way of resolving this is to difference the data in order to ensure stationary of our variables. Therefore, after that we will have that \( \Delta Y_t \in I(0) \) and \( \Delta X_t \in I(0) \), and the regression model will be:

\[
\Delta Y_t = a_1 + a_2 \Delta X_t + \Delta u_t
\]  

In this case the regression model may give us correct estimates of \( \hat{a}_1 \) and \( \hat{a}_2 \) parameters and the spurious equation problem has been resolved. However, what we have from equation (3.49) is only the short-run relationship between the two variables and the long run equation is defined as:

\[
Y_t^* = \beta_1 + \beta_2 X_t
\]

So \( \Delta Y_t \) gives us no information about the long run behavior of our model. As in the present study we are mainly interested in long run relationship, therefore concept of co-integration and ECM are very useful. As \( Y_t \) and \( X_t \) are both are now integrated of order one i.e. \( Y_t \in I(1) \) and \( X_t \in I(1) \). In the special case that there is a linear combination of \( Y_t \) and \( X_t \), that is I(0), then \( Y_t \) and \( X_t \) are co-integrated. Thus, if this is the case the regression of equation (3.55) is no longer spurious, and it also provides us with the linear combination:

\[
\hat{u}_t = Y_t - \hat{\beta}_1 - \hat{\beta}_2 X_t
\]

That connects \( Y_t \) and \( X_t \) in the long run.

**The Error-Correction Mechanism (ECM)**

If, then \( Y_t \) and \( X_t \) are co-integrated, by definition \( \hat{u}_t \in I(0) \). Thus, we can express the relationship between \( Y_t \) and \( X_t \) with an ECM specification as:

\[
\Delta Y_t = a_0 + b_1 \Delta X_t - \Pi \hat{u}_{t-1} + Y_t
\]

This will now have the advantage of including both long run and short run information. In this method, \( b_1 \) is the impact multiplier (the short run effect) that measures the immediate impact that a change in \( X_t \) will have on a change in \( Y_t \). On the other hand \( \Pi \) is the feedback effect, or the adjustment effect, and shows how
much of the disequilibrium is being corrected, i.e. the extent to which any
disequilibrium in the previous period effects any adjustment in $Y_t$.
Thus,

$$
\hat{u}_{t-1} = Y_{t-1} - \hat{\beta}_1 - \hat{\beta}_2 X_{t-1},
$$

(3.53)

Where $\beta_2$ expresses the long run response

Equation (3.53) now emphasizes the basic approach of co-integration and error
correction models.

**Testing for co-integration**

There are two approaches to test the co-integration in a given equation:

1) **Co-integration in single equations: the Engle-Granger approach**

Engle and Granger (1987) further formalized this concept
by introducing a very simple test for the existence of co-integrating (i.e. long- run
equilibrium) relationships.

In order to understand this approach (which is often called EG approach)
consider the following two series $X_t$ and $Y_t$, and the following cases:

a) If $Y_t \square I(0)$ and $X_t \square I(1)$, then every linear combination $(\theta_1 Y_t + \theta_2 X_t)$ of

those two series will result in a series that will always be I(1) or non-stationary.

This will happen because the behavior of the non-stationary series will dominate
the behavior of the I(0) one.

b) If we have both series are I(1) or non-stationary, then in general any linear

combination the two series, say $(\theta_1 Y_t + \theta_2 X_t)$ will also be I(1).

Now, the problem is how can we estimate the parameters of the long run
equilibrium relationship and make sure whether or not we have co-integration. Engle
and Granger proposed a straightforward method which involves four steps:

**Step 1:** Test the variables for their order of integration

It is by definition, co-integration necessitates that the variables be integrated of the
same order. The Dickey-Fuller and the Augmented Dickey-Fuller (ADF) tests can be
applied in order to infer the number of unit roots (if any) in each of the variables. We
can differentiate three cases which will either lead us to the next step or will suggest stopping:

a) If both variables are stationary (i.e. \( I(0) \)), it is not necessary to proceed since standard time series methods apply to stationary variables.

b) If the variables are integrated of different order, it is possible to conclude that they are not co-integrated.

c) If both variables are integrated of the same order then we proceed with step two.

**Step 2: Estimate the long run relationship**

If the results of step 1 indicate that both variables are integrated of the same order, the next step is to estimate the long run equilibrium relationship of the form:

\[
Y_t = \beta_1 + \beta_2 X_t + e_t
\]

and obtain the residuals of this equation.

If there is no co-integration, the results obtained will be spurious. However, if the variables are co-integrated, then OLS regression yields ‘super consistent’ estimators for the co-integrating parameters \( \hat{\beta}_2 \).

**Step 3: Check for (co-integration) the order of integration of the residuals**

In order to determine if the variables are actually co-integrated, denote the estimated residual sequence from this equation by \( \hat{e}_t \). Thus, \( \hat{e}_t \) is the series of the estimated residuals of the long run relationship and if these deviations from long run equilibrium are found to be stationary, then \( X_t \) and \( Y_t \) are co-integrated.

In fact we perform a DF test on the residual series to determine their order of integration. The form of the DF test is the following:

\[
\Delta e_t = a_1 \hat{e}_{t-1} + \sum_{i=1}^{n} \delta_i \Delta \hat{e}_{t-i} + v_t
\]

It is pertinent to note that as \( \hat{e}_t \) is a residual therefore, we do not include a constant or a time trend. Further, if we find that \( \hat{e}_t \not\in I(0) \) then we can reject the null that the variables \( X_t \) and \( Y_t \) are not co-integrated. We repeat the same process if we have a single equation with more than just one explanatory variable.

**Step 4: Estimate the error correction model**
If the variables are co-integrated, the residuals from the equilibrium regression can be used to estimate the error-correction model and to analyze the long run effects of the variables as well as to see the adjustment coefficient, which is the coefficient of the lagged residual terms of the long run relationship identified in step 2.

2) Co-integration in multiple equations and the Johansen approach

If we have more than two variables in the model, then there is the possibility of having more than one co-integrating vector. By this we mean that the variables in the model might form several equilibrium relationships governing the joint evolution of all the variables. In general for n number of variables we can have only up to n-1 co-integrating vectors. Therefore, when n=2 which is the simplest case, we can understand that if co-integration exists then the co-integrating vector is unique.

Having n=2 and assuming that only one co-integrating relationship exists, where there are actually more than one, is a very serious problem that cannot be resolved by the EG single equation approach. Therefore, an alternative to the EG approach is needed and this is the Johansen approach for multiple equations.

In order to present this approach, it is useful to extend the single equation error correction model to a multivariate one, let’s assume that we have three variables, $Y_t$, $X_t$ and $W_t$ which can all be endogenous.

$$Z_t = A_1 Z_{t-1} + A_2 Z_{t-2} + \cdots + A_k Z_{t-k} + u_t$$  \hspace{1cm} (3.56)

It can be reformulated in a vector error correction model (VECM) as follows:

$$\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + \Gamma_2 \Delta Z_{t-2} + \cdots + \Gamma_{k-1} \Delta Z_{t-k-1} + \Pi Z_{t-1} + u_t$$  \hspace{1cm} (3.57)

Where \( \Gamma_i = (I - A_1 - A_2 - \cdots - A_k) \) and \( \Pi = -(I - A_1 - A_2 - \cdots - A_k) \).

\( (i=1, 2, \ldots, k-1) \)

Here we need to carefully examine the \( \Pi \) matrix. The \( \Pi \) matrix contains information regarding the long run relationships. We can decompose \( \Pi = \alpha \beta' \) where \( \alpha \) will include the speed of adjustment to equilibrium coefficients while \( \beta' \) will be the long run matrix of coefficients. Therefore the \( \beta' Z_{t-1} \) term is equivalent to the error correction term \((Y_{t-1} - \beta_0 - \beta_1 X_{t-1})\) in the single equation case, except that now \( \beta' Z_{t-1} \) contains up to \((n-1)\) vectors in a multivariate framework.

The steps of the Johansen approach in practice:
Step 1: Testing the order of integration of the variables
As with EG approach, the first step in the Johansen approach is to test for the order of integration of the variables under examination.

Step 2: Setting the appropriate lag length of the model
The most common procedure in choosing the optimal lag length is to estimate a VAR model including all our variables in levels. This VAR model should be estimated for a large number of lags, then reducing down by re-estimating the model for one lag less until we reach zero lags. In each of these models we inspect the values of the AIC and the SBC criteria, as well as the diagnostic concerning autocorrelation, heteroskedasticity. In model that minimizes AIC and SBC is selected as the one with the optimal lag length. If there any conflict arises between the minimum values of AIC and SBC then one should prefer the minimum of SBC to select optimal lag length. This model should also pass all the diagnostic checks.

Step 3: Choosing the appropriate model regarding the deterministic components in the multivariate system
Another important aspect in the formulation of the dynamic model is whether an intercept and/or a trend should enter in either the short run or the long run model, or both models. The general case of the VECM including all the various options that can possibly happen is given by the following equation:

$$\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + \ldots + \Gamma_k \Delta Z_{t-k-1} + \alpha \left( \begin{array}{c} \beta \\ \mu_1 \\ \mu_2 \\ \delta_1 \\ \delta_2 \\ \end{array} \right) \left( \begin{array}{c} Z_{t-1} \\ 1 \\ t \end{array} \right) + \mu_2 + \delta_2 t + u_t$$

(3.58)

The possible cases are; have a constant (with coefficient $\mu_1$) and/or a trend (with coefficient $\delta_1$) in the long run model (the co-integrating equation (CE)), and a constant (with coefficient $\mu_2$) and/or a trend (with coefficient $\delta_2$) in the short run model (the VAR model). In general five distinct models can be considered. Although the first and the fifth are not that realistic, we present all of them for reasons of complementarity.

Model 1: No intercept or trend in CE or VAR ($\delta_1 = \delta_2 = \mu_1 = \mu_2 = 0$). In this case there are no deterministic components in the data or in the co-integrating relations.
However, this is quite unlikely to occur in practice, especially as the intercept is generally needed in order to account for adjustments in the units of measurements of the variables in \( Z_{t-1} \ 1 \ t \).

**Model 2:** Intercept (no trend) in CE, no intercept or trend in VAR \((k \times k)\). This is the case where there are no linear trends in the data, and therefore the first differenced series have a zero mean. In this case the intercept is restricted to the long run model (i.e. the co-integrating equation) to account for the unit of measurement of the variables in \( Z_{t-1} \ 1 \ t \).

**Model 3:** Intercept in CE and VAR, no trends in CE and VAR \((\delta_1 = \delta_2 = 0)\). In this case there are no linear trends in the levels of the data, but we allow both specifications to drift around an intercept. In this case it is assumed that the intercept in the CE is cancelled out by the intercept in the VAR, leaving just one intercept in the short run model.

**Model 4:** Intercept in CE and VAR, linear trend in CE, no trend in VAR \((\delta_2 = 0)\). In this model we include a trend in the CE as a trend stationary variable in order to take in to account exogenous growth (i.e. technical progress). We also allow for intercepts in both specifications while there is no trend in the short run relationship.

**Model 5:** Intercept and quadratic trend in the CE intercept and linear trend in VAR. this model allows for linear trends in the short run model and thus quadratic trends in the CE. Thus, in this final model everything is unrestricted. However, this model is very difficult to interpret from an economics point of view, especially since the variables are entered as logs, because a model like this would imply an implausible ever-increasing or ever-decreasing rate of change.

So the problem is which of the five different models is appropriate in testing for co-integration. As we know that the first and the fifth are not that realistic therefore the problem reduces to a choice of one of the three remaining models. Johansen (1992) suggests that we need to test the joint hypothesis of both the rank order and the deterministic components, applying the so called *Pantula principle*. The *Pantula principle* involves the estimation of all three models and the presentation of the results from the most restrictive hypothesis (i.e. \( r=\)number of co-integrating relations=0 and model 1) through the least restrictive hypothesis (i.e., \( r=\)number of variables entering the VAR-1=\( n-1 \) and model 4). The model selection procedure then
comprise moving from the most restrictive model, at each stage comparing the trace test statistic to its critical value, stopping only when we conclude for the first time that the null hypothesis of no co-integration is rejected.

**Step 4:** Determining the rank of $\Pi$ or the number of co-integrating vectors

According to Johansen and Juselius (1990), there are two methods for determining the number of co-integrating relations, and both involve estimation of the matrix $\Pi$. This is a $k \times k$ matrix with rank $r$. The procedures are based on propositions about Eigen values.

a) One method tests the null hypothesis, that rank ($\Pi$) = $r$ against the hypothesis that the rank is $r+1$. So, the null in this case is that there is co-integrating vectors and that we have up to $r$ co-integrating relationships, with the alternative suggesting that there is $(r+1)$ vectors. The test characteristics are based on the characteristics roots (also called Eigen values) obtained from the estimation procedure. The test consists of ordering the largest Eigen values in descending order and considering whether they are significantly different from zero. Suppose we obtained $n$ characteristic roots denoted by $\hat{\lambda}_1 > \hat{\lambda}_2 > \hat{\lambda}_3 > ... > \hat{\lambda}_n$. If the variables under examination are no co-integrated, the rank of $\Pi$ is zero and all the characteristic roots will equal zero. Therefore $(1 - \hat{\lambda}_1)$ will be equal to 1 and since $\ln(1) = 0$, each one of the expressions will be equal to zero for no co-integration. On the other hand, if the rank of $\Pi$ is equal to 1, then $0 < \hat{\lambda}_1 < 1$ so that the first expression $(1 - \hat{\lambda}_1) < 0$, while all the rest will be equal to zero. To test how many of the numbers of the characteristic roots are significantly different from zero this test uses the following statistic:

$$
\lambda_{\text{max}}(r,r+1) = -T \ln(1 - \hat{\lambda}_{r+1})
$$

As we said before, the test statistic is based on the maximum eigenvalue and because of that is called the maximum Eigen value statistic (denoted by $\lambda_{\text{max}}$).

b) The second method is based on a likelihood ratio test about the trace of the matrix (and because of that it is called the trace statistic). The trace statistic considers whether the trace is increased by adding more Eigen values beyond the $r^{th}$ Eigen value. The null hypothesis in this case is that the number of co-integrating vectors is less than or equal to $r$. From the previous analysis it should be clear that when
all $\hat{\lambda}_i = 0$, then the trace statistic is equal to zero as well. On the other hand, the closer the characteristic roots are to unity the more negative is the $\ln(1 - \hat{\lambda}_i)$ term and, therefore, the larger the trace statistic. This statistic is calculated by:

$$
\hat{\lambda}_{trace}(r) = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i) \quad (3.60)
$$

The usual procedure is to work downwards and stop at the value of $r$ which is associated with a test statistic that exceeds the displayed critical value. Critical values for both statistics are provided by Johansen and Juselius (1990).

Further, it is of worth mentioning here that the trace statistic and the maximum Eigen value statistic may yield conflicting results. For such cases, it is recommended that one should examine the co-integrating vector and base their choice on the interpretability of the co-integrating relations (Johansen and Juselius, 1990).

**Step 5:** Testing for weak exogeneity

After determining the number of co-integrating vectors we need to proceed with tests of weak exogeneity. Remember that the $\Pi$ matrix contains information about the long run relationships, and that $\Pi = \alpha \beta'$, where $\alpha$ represents the speed of adjustment coefficients and $\beta$ is the matrix of long run coefficients. From this it should be clear that when there are $r \leq n - 1$ co-integrating vectors in $\beta$, then this automatically means that at least $(n-r)$ columns of $\alpha$ are equal to zero. Thus, the typical problem faced, of determining how many $r \leq n - 1$ co-integration vectors exist in $\beta$, amounts to equivalently testing which columns of $\alpha$ are zero.

**Step 6:** Testing for linear restrictions in the co-integrating vectors

An important feature of the Johansen approach is that it allows us to obtain estimates of the coefficients of the matrices $\alpha$ and $\beta$, and then test for possible linear restrictions regarding those matrices. Especially for matrix $\beta$, the matrix that contains the long run parameters, this is very important because it allows us to test specific hypothesis regarding various theoretical predictions from an economic theory point of view.
Section V

In the present study tabular analysis and growth rates have been used to explain the various characteristics of infrastructure development in India. The linear programming based technique of data envelopment analysis has been used to estimate the technical efficiency of the Indian manufacturing industry and to segregate into two mutually exclusive and non-additive components namely, pure efficiency and scale efficiency for the period 1990-91 to 2008-09. The total factor productivity growth in Indian manufacturing sector (major infrastructure industries at 2-digit classification) has been worked out with the help of Malmquist Productivity Index (MPI) which decomposed the productivity growth into two mutually exclusive and non-additive components, namely technical change (TCH) and efficiency change (ECH). The technical change reflects improvement in technological transfer that yields innovations and better technological adoptions among the industries whereas, technical efficiency change is a proxy of catching up which indicates the movement of a decision making unit towards best-practice production technology.

Further, in order to assess the impact of spillovers of infrastructure on economic policy variables, panel data co-integration analysis has been applied. The co-integration technique along with vector error correction mechanism has been applied for analyzing the cause and effect relationship between infrastructure development and foreign direct investment inflows in India. In addition, the long run relationship has been estimated between economic growth and infrastructure development index.

The nature of relationship between the quality of environment and economic growth is tested through Environmental Kuznets Curve (EKC) Hypothesis. The Environmental Kuznets Curve (EKC) states that during the initial phase of economic development, degradation of environment in quality and quantity takes place. Expressed simply, EKC states an inverted- U shape relationship (Curve) between per capita income (GDP) and environmental degradation (or a U shaped relation with level of environmental quality). A typical Environmental Kuznets Curve is depicted as follows:
The earliest EKCs were simple quadratic functions of the levels of income. However, economic activity inevitably implies the use of resources and their deterioration subject to economic laws. The use of resources inevitably implies the production of waste and thereby deterioration of environment. The standard EKC regression model is

\[
\ln(E/P)_{it} = \alpha_i + \beta_1 \ln(GDP/P)_{it} + \beta_2 (\ln(GDP/P))_{it}^2 + \varepsilon_{it} \tag{3.61}
\]

where E is emissions, P is population, and ln indicates natural logarithms. Statistically, the presence of EKC can be checked in equation by just denoting the change of sign in equation. The change of sign from linear term to quadratic form (i.e. from + to −) gives the presence of EKC.

Moreover, in the present study, on Y-axis of EKC, indicator of environmental degradation (i.e. CO$_2$ emissions, area under forest cover and Suspended Particulate Matter (SPM) are measured and X-axis measures the indicator of development (i.e. GDP, infrastructure index, Energy use, motor vehicle). The upswing of the inverted – U illustrates that cetris peribus, greater output per head generates more pollution.

However, to analyze the data empirically, the detailed methodology has been given at appropriate places in the respective chapters and the data has been analyzed, and results are interpreted accordingly to draw relevant policy implications.

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