PART I

17.2 GeV/c $\pi^-$-N and $\pi^-$-Em interactions
On the Interactions of 17.2 Gev $\pi^{-}$ with Light and Heavy Emulsion Nuclei

J. M. KOHLI, I. S. MITTRA and M. B. SINGH

Physics Department, Panjab University, Chandigarh

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257.7 meters of track length was followed and 631 interactions were picked up. These events have been separated into $\pi^{-}$-AgBr and $\pi^{-}$-CNO groups of nuclei on the basis of Lohrmann et al. selection criterion and also on the assumption that the overall numbers of events belonging to AgBr and CNO nuclei are governed by the geometrical cross-sections. The results regarding the mean multiplicity, angular and momentum distributions are similar in the cases of the above said selection criteria and lend some support to the inter-nuclear cascade model.

§ 1. Introduction

With the help of nuclear research emulsion, we can investigate the mechanism of pion production at high energies with target nuclei widely separated in the periodic table. Leaving apart free hydrogen, the emulsion contains mainly two groups of nuclei, viz. CNO and AgBr with average atomic weights 14 and 94 respectively. A study of interactions of high energy pions and protons with emulsion nuclei, therefore, can provide a detailed knowledge about the production mechanism of mesons in light and heavy nuclei.

The problem of separating target nuclei in CNO and AgBr groups is quite formidable and in general unambiguous separation is difficult to achieve. Two different methods have been commonly employed by various authors. One is due to Friedlander\(^1\) and depends entirely upon the number of heavy prongs ($N_h$) emitted in an interaction. The other has been given by Lohrmann \textit{et al.}\(^2\) and depends upon the existence of Coulomb barrier in the case of low excited AgBr nuclei as a result of which particles having energy less than a certain minimum energy are not likely to be emitted, whereas there is no bar on the emission of such particles from the CNO group of nuclei. We have preferred to adopt the latter criterion because it is somewhat less arbitrary. However, by assuming that events with $N_h \geq 7$ definitely belong to AgBr group and that the overall numbers of events belonging to AgBr and CNO groups are governed by the interaction cross-section with emulsion nuclei being geometrical, we have obtained results which are not different from those obtained on Lohrmann’s selection criterion.

The results obtained on pion production in complex nuclei have been sought to be explained on the basis of (1) Inter-nuclear cascade and (2) Tube models of meson production. In the cascade model it is assumed that there are two or more successive collisions in the target nucleus, whereas the tube model\(^3,4\) is based on the assumption that there is a single collision between the incident particle and a tube of nucleons acting as a single target. Friedlander\(^5\) and Barbaro-Galtier \textit{et al.}\(^6\) obtained results on the interactions of protons with emulsion nuclei and explained them on the basis of tube model. The apparent support that these results give to the tube model may be attributed to their different selection criterion which is entirely based upon $N_h$ values. As discussed in § 3 this selection criterion is not satisfactory and has to be modified. The qualitative disagreement with this model has, however, been shown to exist by Barashenkov \textit{et al.}\(^7\) in their studies of the proton-nucleus interactions.

We also find that our results show better agreement with the cascade model than with the tube model. Wherever possible we have compared our results with 24-25 Gev proton-nucleus interactions,\(^8\) in which case the energy available in centre of mass is almost equal to that in our experiment. The two types of interactions are, therefore, likely to show similar behaviour, which indeed has been found to be the case.

§ 2. Experimental Details

A stack of Ilford G. 5. emulsions of the size 23 cm $\times$ 15 cm $\times$ 600 $\mu$m was exposed to (17.2 $\pm$ 0.02) Gev $\pi^{-}$ beam at CERN. The average flux was $4 \times 10^5$ cm$^{-2}$ and the angular spread in the vertical plane was $\pm 5$ mr. The beam tracks were followed until they interacted or left the plate.
The total magnification used for scanning was 1000 X and the scanning rate was 25 cm/hr. The total track length followed was 257.7 meters and 631 inelastic interactions were picked up.

The angular and energy measurements were performed upon Koristka B-4 microscope. The space angles have been measured for all the shower and grey tracks to an accuracy of 20'. The energy measurements have been made upon only those tracks which had a minimum projected track length of 5 mm in case of showers and 2 mm in the case of greys in the scanned plate. A geometrical factor was attached to each accepted track in order to compensate the loss of unscattered tracks due to finite thickness of the plates. The maximum statistical error in energy measurement is 25%.

§ 2. Selection Criterion

To know exactly what nucleus has disintegrated in a particular interaction in complex nuclear emulsion is almost an impossible task. At the most we can separate the target nuclei in the two groups viz. CNO and AgBr. The experimental result which greatly helps in this respect is the number of heavy prongs \( N_h \) which are emitted from the target nucleus. These interactions which have \( N_h \geq 7 \) almost definitely belong to AgBr group, for it has been experimentally verified that for CNO group maximum value of \( N_h \) greater than six is rather improbable. But the interactions for which \( N_h \leq 6 \), the target nuclei may also belong to AgBr group if the collisions are non-centric and the excitation energy supplied to the nucleus is small. It is for such interactions in which heavy prongs emitted are up to six, that a reasonable selection criterion is needed to separate events in the two groups.

Friedlander has adopted a criterion which is based on the following ad hoc assumptions:
1. Target nuclei belong to CNO group if \( 2 \leq N_h \leq 4 \).
2. and they belong to AgBr group if \( N_h \geq 7 \).

This criterion does not take into account the events with \( N_h = 5 \) or \( 6 \) which may belong to both the groups and the fact that about equal percentage of events with \( N_h = 2 \) to \( 4 \) may belong to AgBr group. Because of above reasons this selection criterion has been criticised by Barashenkov et al., Meyer et al., and Tolstov. The second selection criterion is due to Lohrmann et al. and is based upon the existence of Coulomb barrier in the case of not strongly excited heavy nuclei which prevents the emission of low energy particles from them. As discussed in (2) the existence of Coulomb barrier in the case of AgBr nuclei having \( N_h \leq 6 \) inhibits the emission of particles with track lengths \(<65 \mu \) which corresponds to 2.7 Mev protons and 10.7 Mev \( \alpha \)-particles. On the other hand there is no appreciable barrier effect present in the case of CNO nuclei.

The criterion due to Lohrmann et al. thus assumes

i) events with \( N_h \leq 6 \) with at least one prong \(<65 \mu \) belong to CNO group,

ii) events with \( N_h \leq 6 \) with no prong \(<65 \mu \) and all those events with \( N_h \geq 7 \) belong to AgBr group.

As a check of the assumption (ii) above, Dostrovsky et al. calculated the low energy protons and \( \alpha \) particles coming out of proton collisions with Ag nuclei at an energy of 190 Mev. They have estimated that the fraction of events which have a proton or \( \alpha \) particle of track length \(<65 \mu \) is 20%. To this extent, therefore, the selection of events according to assumption (ii) is in error. Lohrmann et al. has also concluded from a study of black prongs in a sample of showers that the magnitude of this error is of the same order. The assumption (i) is also not very rigid and may contribute an error upto 50%.

In the present investigation a total of 631 interactions have been analysed. On the Lohrmann et al. criterion 422 events were found to belong AgBr group and 124 to CNO group. The rest of 85 events are of the type \( 0+1+n_\alpha \) and \( 0+0+n_\alpha \) and have not been included in this investigation.

<table>
<thead>
<tr>
<th>Target Nucleus</th>
<th>Present Investigation</th>
<th>Meyer et al. results</th>
<th>Based upon geometrical cross-sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>14</td>
<td>18</td>
<td>5</td>
</tr>
<tr>
<td>CNO</td>
<td>20</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>AgBr</td>
<td>66</td>
<td>66</td>
<td>65</td>
</tr>
</tbody>
</table>
as they are mainly due to pion-free and pion-bound nucleon interactions. The results in terms of percentage of collisions with various target nuclei are displayed in table I. For comparison the following results have also been included.

(a) Results obtained by Meyer et al.\(^6\) on 25 Gev proton collisions in emulsion nuclei.

(b) Results expected on the basis that cross section for star production is geometrical i.e., it is proportional to \(A^{2/3}\).

While investigating the behaviour of the events with \(N_h \leq 6\) as a separate group, we found that overall properties of the events were similar irrespective of the nature of the nucleus attributed to each event on the basis of above selection criterion. This behaviour was of course quite different from the events with \(N_h \geq 7\) i.e. genuine AgBr events. It was, therefore, fair to assume that the events with \(N_h \leq 6\) show the typical behaviour of the CNO group. On the other hand to obtain unbiased results of AgBr group, the results obtained only on the basis of events with \(N_h > 7\) must be modified because of some of the AgBr events belong to \(N_h \leq 6\) group (even though they show the same multiplicity, angular distribution etc. as the CNO group). This modification may be done assuming that the overall number of events must be equal to the number expected on the basis of interaction cross-section with emulsion nuclei being geometrical. On this assumption 65% of the total number of events i.e. 410 events should belong to AgBr group. Thus to obtain unbiased results on the overall sample, the results obtained from 224 AgBr events with \(N_h \geq 7\) must be modified with 186 (410-224) AgBr events with \(N_h \leq 6\), though having the same properties as the CNO group. Hereinafter we mention the results derived on the above basis as those obtained from "geometrical cross-section". Employing this criterion, we avoid the necessity of invoking some empirical selection criterion like that of Friedlander or Lohrmann et al.

### § 4. Experimental results

#### 4.1. Mean multiplicity

The mean multiplicity for light and heavy nuclei are reported in table II. The results based upon cascade and tube models have also been displayed. These theoretical values have been computed at 25 Gev \(p\)-nucleus interactions by Barashenkov et al.\(^7\).

Table II shows that the mean multiplicities in the case of 24 Gev \(p\)-nucleus interactions is same as those obtained in the present investigation. The tube model predicts larger values of mean multiplicities. Relatively, the cascade model is in better agreement with the experimental results.

The results obtained by Lohrmann's selection criterion and those based upon geometrical cross-section are in good agreement. The dependence of the average number of shower particles on the atomic number \(A\) in the present investigation is given by

\[
\langle n_o \rangle = 3.4 A^{0.14 \pm 0.02},
\]

and is in good agreement with that of 24 Gev \(p\)-nucleus results of Meyer et al.\(^6\) represented by the power law

\[
\langle n_o \rangle = 3.4 A^{0.14 \pm 0.02},
\]

#### 4.2. Variation of \(\langle N_h \rangle\) with \(n_o\)

Figure 1 shows the dependence of \(\langle N_h \rangle\) on \(n_o\) for the light and heavy groups of nuclei. In the case of CNO, \(\langle N_h \rangle\) is independent of number of showers produced in an interaction. The value of correlation co-efficient \(r\) between \(\langle N_h \rangle\) and \(n_o\) for the light nuclei is .07. In case of CNO events obtained on the basis of geometrical cross-section, the value of \(r\) is .05.

One may expect a constant value of \(\langle N_h \rangle\) irrespective of \(n_o\) in the case of CNO nuclei on the assumption that the atomic masses of such nuclei

\[
\langle n_o \rangle = \frac{1}{N} \sum x^2 - \bar{x} \bar{y} / \sigma_x \sigma_y
\]

where \(x\) and \(y\) are two variables having average values \(\bar{x}\) and \(\bar{y}\) and standard deviations \(\sigma_x\) and \(\sigma_y\) respectively.

**Table II. Mean multiplicities for various target nuclei**

<table>
<thead>
<tr>
<th>Target nucleus</th>
<th>Present investigation</th>
<th>Meyer et al. results</th>
<th>Cascade model</th>
<th>Tube model</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNO</td>
<td>4.70 ± 0.42</td>
<td>4.80 ± 0.20</td>
<td>4.9</td>
<td>6.4</td>
</tr>
<tr>
<td>AgBr</td>
<td>5.06 ± 0.30</td>
<td>5.39 ± 0.30</td>
<td>6.8</td>
<td>8.0</td>
</tr>
</tbody>
</table>

\(\langle n_o \rangle\) is independent of number of showers produced in an interaction. The value of correlation co-efficient \(r\) between \(\langle n_o \rangle\) and \(n_o\) for the light nuclei is .07. In case of CNO events obtained on the basis of geometrical cross-section, the value of \(r\) is .05.

One may expect a constant value of \(\langle n_o \rangle\) irrespective of \(n_o\) in the case of CNO nuclei on the assumption that the atomic masses of such nuclei

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\langle n_o \rangle = \frac{1}{N} \sum x^2 - \bar{x} \bar{y} / \sigma_x \sigma_y
\]

where \(x\) and \(y\) are two variables having average values \(\bar{x}\) and \(\bar{y}\) and standard deviations \(\sigma_x\) and \(\sigma_y\) respectively.
are small (12-16). These nuclei can completely disintegrate with small energy supplied to them. So in these cases, the effect of multi-nucleon interactions does not matter, because the first pion-nucleon collision is sufficient to disintegrate the nucleus. Figure 1 also shows the linear rise in \( \langle N_r \rangle \) with the increase in number of shower particles produced in AgBr group. The correlation co-efficient between \( \langle N_r \rangle \) and \( n \) is +0.42. This rise in the mean value of \( \langle N_r \rangle \) with \( n \) may be explained on the basis of increase of secondary interactions due to inter-nuclear cascade produced in the heavy nucleus. Since the atomic weight \( (A=94) \) for AgBr nuclei is quite large, there is an increase in the value of \( N_h \) (excitation) with the increase in number of collisions taking part in the nucleus \( (n) \). Similar results were reported at 25 Gev \( p \)-nucleus interactions by Meyer et al.

4.3. Angular distribution in lab. system

The values of median angles \( (\theta_{1/2}) \) for AgBr and CNO groups of nuclei are reported below.

\[
\theta_{1/2}^{\text{AgBr}} = (14.2 \pm 1.3)°, \\
\theta_{1/2}^{\text{CNO}} = (22.0 \pm 1.1)°.
\]

The increase in the value of \( \theta_{1/2} \) for heavy nuclei may be attributed to the presence of multi-nucleon interactions taking place in them. The values obtained on the basis of geometrical cross-section are also same within statistical errors.

Figure 2(a) and 2(b) show the lab. angular distribution for shower particles produced in the case of light and heavy nuclei respectively. The experimental results based upon Lohrmann's selection criterion and those based upon geometrical cross-section have been compared with those obtained in the case of \( p \)-nucleus interactions at 25 Gev and also with the predictions of cascade model.

The results obtained show a marked correspondence between the experimental and theoretical results for both light and heavy nuclei.

4.4. Energy and transverse momentum distributions

Figure 3 shows the total energy distribution of pions emitted from CNO and AgBr groups of nuclei in lab. system. Both the distributions behave almost in the same manner within the
statistical errors. The average values of total energy carried by pions are a bit different. These values are

\[ \langle E \rangle_{\text{CNO}} = (2.20 \pm 0.31) \text{ GeV} \]

and

\[ \langle E \rangle_{\text{AgBr}} = (1.60 \pm 0.10) \text{ GeV} \]

Figure 4 shows the transverse momentum distributions of pions for light and heavy groups of nuclei. The average transverse momenta for these groups are

\[ \langle p_t \rangle_{\text{CNO}} = (362 \pm 52) \text{ MeV/c} \]

\[ \langle p_t \rangle_{\text{AgBr}} = (372 \pm 23) \text{ MeV/c} \]

§ 5. Discussion

The experimental results presented in the last section show how the disintegrations produced in CNO groups differ from those of AgBr group. Obviously this difference in the behaviour of these two groups of nuclei arises from their widely different atomic weights. Whereas the targets belonging to CNO group are likely to be completely disintegrated by the very first interaction, there might occur more than one collision in the case of heavier AgBr nuclei. Thus there are greater chances of inter-nuclear cascade being built up in AgBr nuclei than in the nuclei of CNO group. This picture explains qualitatively why \( \langle n_c \rangle \) is higher, \( \langle \theta_{\text{cis}} \rangle \) is larger and \( \langle N_0 \rangle \) increases linearly with \( n_t \) in the case of interactions taking place with AgBr nuclei.

Then the question arises if on the basis of these results we can distinguish between the two models of meson production viz. the hydrodynamical or tube model and the cascade model, which have been proposed to explain the above results. Though a positive answer to the above question is difficult to give, the totality of results seem to be in better accordance with the cascade model than with the tube model. Observed mean multiplicities \( \langle n_0 \rangle \) (Table II) are close to the values expected on the basis of the cascade model than with the predictions of the tube model. Also the variation of \( \langle n_0 \rangle \) with the atomic weight \( A \) is found to be \( \langle n_0 \rangle = 3.4 A^{0.19} \pm 0.01 \), whereas on the basis of tube model the expected variation comes out to be \( \langle n_0 \rangle = K A^0 \). The angular distribution also follows the predictions of cascade model.

Notwithstanding the above results (which are more or less tentative in nature) one should be very careful in drawing any rigid conclusions because of two reasons. Firstly, the separation of events in the two categories is not devoid of ambiguity. As discussed in § 3, the AgBr events which have been selected on the basis of absence of minimum track length (65 \( \mu \)) may be in error upto 20%. Similarly about 50% of CNO events may also belong to AgBr group. In fact it has been shown that the behaviour of events separated on the basis of geometrical cross-section (§ 3) in AgBr group and CNO group behave exactly in the same way as AgBr and CNO interactions separated on the basis of Lohrmann selection criterion.

Secondly, at these high energies the inter-nuclear cascade is expected to be confined to a very narrow cone effectively of the same dimensions as the tube in the tube mechanism. The only difference in the two models is that in the case of tube model the interaction takes place with the part of the nucleus in the tube forming a Coherent excited system as a whole, whereas in the cascade mechanism the successive interactions occur with the individual nucleons in the tube. At such high energies the parameters of the outgoing particles are not likely to be very sensitive to the nature of mechanism giving rise to them. Finally, it must be mentioned that the tube model is supposed to be a good approximation at extremely high energy region but it has been discussed in this paper whether it can be applied to the present energy region or not.
Acknowledgements

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References

CASCADE MODEL ANALYSIS
OF THE INTERACTIONS OF 17.2 GeV MESONS
WITH HEAVY EMULSION NUCLEI

J. M. KOHLI, I. S. MITTRA and M. B. SINGH
Department of Physics, Panjab University,
Chandigarh-14, India

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Abstract: The comparison of the theoretical results obtained in ref. [1] on the basis of internuclear cascade initiated in AgBr nuclei by 17 GeV mesons with our experimental results shows that the theoretical results are in good agreement only with events having \( N_h \geq 8 \) but not with an overall sample of AgBr events.

Artykov et al. [1] have recently shown that the internuclear cascade model can nicely explain the various experimental results obtained by Hoffmann et al. [2] and Kohli et al. [3] regarding the inelastic interactions of 17 GeV mesons with photo-emulsion heavy nuclei. An attempt has been made here to show that the theoretical results are in good agreement with the results obtained from events having \( N_h \geq 8 \). The overall sample of AgBr events does not agree with the predictions of the internuclear cascade model.

In the present investigation we have a sample of 631 inelastic interactions picked up by along-the-track scanning. The various measurement techniques regarding angle and energy of secondaries have been discussed in ref. [3].

These events have been separated into \( \pi \)-AgBr and \( \pi \)-CNO interactions on the basis of the selection criterion of ref. [3]. We have a sample of 422 AgBr events out of which there are 178 events having \( N_h \geq 8 \).

The various theoretical results obtained in ref. [1] have been compared with those obtained in the case of present investigation based upon \( \pi \)-AgBr events and with events having \( N_h \geq 8 \) in table 1. The results obtained by Hoffmann et al. at 17 GeV \( \pi \)-nucleus interaction having \( N_h \geq 8 \) have also been included in table 1.

Figs. 1, 2 and 3 show the angular, transverse momentum and momentum distributions of shower particles produced by \( \pi \)-AgBr interactions along with the sample of 178 events having \( N_h \geq 8 \).

The experimental results are in good agreement with theory if it were assumed that the internuclear cascade is developed in the whole of the nucleus, i.e. the nucleus is highly excited \( (N_h \geq 8) \). In order to accommo-
Table 1

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Theory</th>
<th>Present investigation</th>
<th>Hoffmann et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_h &gt; 8$</td>
<td>$\pi$-AgBr</td>
<td>$N_h &gt; 8$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>7.1 ± 0.5</td>
<td>7.0 ± 0.5</td>
<td>7.1 ± 0.2</td>
</tr>
<tr>
<td>$n_{sp}$</td>
<td>0.83 ± 0.08</td>
<td>0.82 ± 0.7</td>
<td>0.9 ± 0.3</td>
</tr>
<tr>
<td>$n_{gs}$</td>
<td>0.87 ± 0.09</td>
<td>0.76 ± 0.19</td>
<td>0.6 ± 0.2</td>
</tr>
<tr>
<td>$n_{gp}$</td>
<td>3.1 ± 0.3</td>
<td>3.0 ± 0.4</td>
<td>3.0 ± 0.4</td>
</tr>
<tr>
<td>$n_g$</td>
<td>4.0 ± 0.4</td>
<td>4.2 ± 0.3</td>
<td>4.5 ± 0.4</td>
</tr>
<tr>
<td>$p_{gs}$ (GeV/c)</td>
<td>0.39 ± 0.04</td>
<td>0.37 ± 0.04</td>
<td>0.59 ± 0.02</td>
</tr>
<tr>
<td>$p_{gp}$ (GeV/c)</td>
<td>1.35 ± 0.14</td>
<td>1.59 ± 0.14</td>
<td>1.44 ± 0.20</td>
</tr>
<tr>
<td>$p_g$ (GeV/c)</td>
<td>0.59 ± 0.06</td>
<td>0.61 ± 0.09</td>
<td>0.53 ± 0.10</td>
</tr>
<tr>
<td>$\theta_{gs}$ (deg)</td>
<td>24.05 ± 20.3</td>
<td>25.5 ± 1.8</td>
<td>22.0 ± 1.1</td>
</tr>
</tbody>
</table>

$n$ is the average multiplicity, $p$ and $p_t$ are the mean total and mean transverse momenta of secondary particles produced. $\theta_{gs}$ (deg) is the median angle. The indices $s$ and $g$ denote the corresponding quantities of shower and grey particles.

Fig. 1. The angular distribution of charged shower particles. The solid line denotes events having $N_h > 8$, dot-and-dash denotes AgBr events and the dashed line denotes theoretical histogram.
Fig. 2. The transverse momentum distribution of shower particles. The notations are the same as used in fig. 1.

Fig. 3. The momentum distribution of shower particles. The notations are the same as in fig. 1.

date the overall sample of $\pi$-AgBr interactions at 17 GeV, the calculations of ref. [1] require further modifications.

The authors are thankful to Prof. B. M. Anand for discussion of results.

REFERENCES

SOME ASPECTS OF TRANSVERSE MOMENTUM IN 17.2 GeV PION-NUCLEON INTERACTIONS

J. M. KOHLI

Physics Department, Panjab University, Chandigarh

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Abstract. On the basis of 169 pure pion-nucleon interactions, the behaviour of transverse momentum distribution has been studied. The average value of \( p_t \) is (344 ± 26) MeV/c. The dependence of \( p_t \) on multiplicity and emission angles in the c.m. system has also been investigated. The experimental results can be well understood on the assumption that most of the pions come from both nucleon and boson resonant states. The definite structure of the nucleon also helps in explaining quite a few aspects of the transverse momentum.

1. INTRODUCTION

Various authors have investigated the behaviour of transverse momentum \( (p_t) \) of secondaries from N-N and \( \pi \)-N collisions at both accelerator and cosmic ray energies. It has been found that the transverse momentum distribution is quite regular in its behaviour and the angles and energies of the secondary particles are correlated in such a way as to make the average value of \( p_t \) almost constant (300 to 400 MeV/c) independently of primary energies.

In the present investigation, the experimental results have been sought to be explained on the basis of the isobar model. The definite structure of the nucleon has also been incorporated.

2. EXPERIMENTAL DETAILS

A stack of Ilford G.5 emulsions of the size 23 cm \( \times \) 15 cm and of thickness 600 \( \mu \) was exposed to (17.2 ± 0.2) GeV \( \pi^- \) beam at CERN. The total track length followed was 470.7 meters and 1166 interactions were located. The total magnification used for scanning was 1000 \( \times \) and the scanning rate was 25 cm/h.

169 pure pion-nucleon inelastic interactions were sorted out of these 1166 events which were of the type 0+0-\( n_N \) and 0+1-\( n_N \) with (effective target mass) \( M_T < 1 M_N \).
3. TRANSVERSE MOMENTUM DISTRIBUTION OF PIONS

The average value of $p_t$ obtained in the present investigation along with those obtained at various incident pion energies have been listed in table 1.

Table 1

<table>
<thead>
<tr>
<th>Incident energy (GeV)</th>
<th>Detector</th>
<th>$\langle p_t \rangle$ (MeV/c)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.2</td>
<td>Emulsion</td>
<td>344 ± 26</td>
<td>Present work</td>
</tr>
<tr>
<td>18.0</td>
<td>Heavy Liq. B.C.</td>
<td>360 ± 10</td>
<td>Bellini et al. [1]</td>
</tr>
<tr>
<td>16.0</td>
<td>Hydrogen B.C.</td>
<td>360 ± 10</td>
<td>Goldsack et al. [2]</td>
</tr>
<tr>
<td>8.0</td>
<td>Emulsion</td>
<td>290 ± 20</td>
<td>Dubey and Kohli [3]</td>
</tr>
<tr>
<td>7.5</td>
<td>Emulsion</td>
<td>286 ± 18</td>
<td>Grote et al. [4]</td>
</tr>
<tr>
<td>6.7</td>
<td>Emulsion</td>
<td>310 ± 20</td>
<td>Belyakov et al. [5]</td>
</tr>
<tr>
<td>5.9</td>
<td>Heavy Liq. B.C.</td>
<td>303 ± 13</td>
<td>Bellini et al. [1]</td>
</tr>
<tr>
<td>4.4</td>
<td>Emulsion</td>
<td>300 ± 23</td>
<td>Malhotra [6]</td>
</tr>
</tbody>
</table>

The average transverse momentum for protons is $(445 ± 111)$ MeV/c.

The statistical errors shown in the present investigation are based upon the actual number of tracks upon which the scattering measurements have been carried out, whereas the average value is given on the basis of the total number of tracks after taking geometrical factors into consideration.

Fig. 1 shows the transverse momentum distribution for pions. The smooth curves are due to following theoretical distributions.

(A) The linear exponential distribution (LD) given by

$$L(p_t) \, dp_t = \frac{1}{\rho_0} \, p_t \, \exp \left( -\frac{p_t}{\rho_0} \right) \, dp_t ,$$  \hspace{1cm} (1)

(B) The Boltzmann distribution (BD) given by

$$B(p_t) \, dp_t = \frac{2}{\sigma^2} \, p_t \, \exp \left( -\frac{p_t^2}{\sigma^2} \right) \, dp_t .$$ \hspace{1cm} (2)

Where $\rho_0$ and $\sigma$ are constants and are connected with $\langle p_t \rangle$ by the relation

$$\langle p_t \rangle = 2\rho_0 \left( \frac{\rho_0}{\sigma} \right)^{1/2} = \sqrt{\frac{2}{\pi}} \alpha .$$ \hspace{1cm} (3)

In a paper on $p_t$ distribution of pions from cosmic ray jets, Imaeda et al. [7] concluded that (LD) is in better agreement with the experimental results. Aly et al. [8] on the other hand applied (B.D.) and found that this distribution gives satisfactory results in the primary energy interval of 16 to 1000 GeV. Fig. 1 shows that our experimental distribution is in better agreement with the linear exponential distribution.
Fig. 1. Transverse momentum distribution of charged pions. The smooth curves represent linear-exponential (solid curve) and Boltzmann (dashed curve) distributions.

Fig. 2. Average transverse momentum of pions plotted against $\theta'$, the angle of emission in the c.m. system.
4. DEPENDENCE OF $\langle p_t \rangle$ ON $n_s$

The various values of $\langle p_t \rangle$ at different $n_s$ have been reported in table 2.

<table>
<thead>
<tr>
<th>$n_s$</th>
<th>$\langle p_t \rangle$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>195 ± 43</td>
</tr>
<tr>
<td>2</td>
<td>184 ± 41</td>
</tr>
<tr>
<td>3</td>
<td>322 ± 47</td>
</tr>
<tr>
<td>4-9</td>
<td>382 ± 38</td>
</tr>
</tbody>
</table>

The values of $\langle p_t \rangle$ for $n_s < 2$ are comparatively smaller as compared with those for $n_s \geq 3$. At 4.4 GeV [6] no noticeable dependence of $\langle p_t \rangle$ on $n_s$ was observed.

5. VARIATION OF $\rho_t$ WITH $\theta^*$

Fig. 2 shows the variation of $\langle p_t \rangle$ of pions at various intervals of $\theta^*$, the emission angle in $\pi$-N c.m. system. There is a tendency of $\langle p_t \rangle$ to increase rapidly with increasing angle $\theta^*$ up to 20°. Beyond this angle, the values of $\langle p_t \rangle$ are almost saturated right up to 135°. At still larger angles, there again appears a fall in the values of $\langle p_t \rangle$. The rise in the value of $\rho_t$ with $\theta^*$ has also been reported at 7 GeV by Grote et al. [4].

6. TRANSVERSE MOMENTUM OF PIONS FOR $\pi$-$\pi$ AND $\pi$-CORE EVENTS

One can separate the pure pion-nucleon interactions into two groups i.e. $\pi$-$\pi$ and $\pi$-core interactions on the basis of effective target mass. The $\pi$-$\pi$ events have $M_T < 1M_T$. Here one assumes that the incident pion interacts with a pion of the pion cloud surrounding the dense nucleon core. $\pi$-core events are characterised by the direct interaction of the incident pion with the core itself. Out of 169 pure pion-nucleon interactions, there are 102 events belonging to $\pi$-$\pi$ interactions and 67 to $\pi$-core interactions.

The average values of $\rho_t$ for $\pi$-$\pi$ and $\pi$-core events are:

$\langle p_t \rangle_{\pi-\pi} = 195 \pm 21$ MeV/c and $\langle p_t \rangle_{\pi-core} = 412 \pm 45$ MeV/c.

Grote et al. [4] at 7 GeV have also obtained almost similar values of $\langle p_t \rangle$ for $\pi$-$\pi$ and $\pi$-core events. Fig. 3 shows the normalised $p_t$ distribution for $\pi$-$\pi$ and $\pi$-core events. The $p_t$ distribution of pions due to $\pi$-$\pi$ events are mainly confined in a narrow range, whereas for $\pi$-core events, the distribution extends up to 1 GeV. The smooth curves are due to eq. (1) and they fit better individually in the cases of $\pi$-$\pi$ and $\pi$-core events than in total sample of pure $\pi$-nucleon interactions (fig. 1).
Fig. 3. Transverse momentum distribution of pions associated with π-π (dashed curve) and π-core (solid curve) events. The smooth curves represent the linear exponential distribution plotted in the individual cases of π-π and π-core interactions.

. VARIATION OF $\langle p_t \rangle$ WITH EMISSION ANGLE FOR π-π AND π-CORE EVENTS

The variation of average $p_t$ with emission angle in lab. system for π-π and π-core events have been reported in table 3.

The average $p_t$ behaves in entirely different ways for the two said classes of events. $\langle p_t \rangle$ remains constant irrespective of the emission angle in case of π-π events, whereas it increases rapidly with the increase in emission angle in case of π-core events.

. DISCUSSION

One can explain the various results regarding transverse momentum
Table 3
Variation of $\langle P_T \rangle$ with emission angle.

<table>
<thead>
<tr>
<th>Emission angles in lab system</th>
<th>$0^\circ$-$5^\circ$</th>
<th>$5^\circ$-$15^\circ$</th>
<th>$15^\circ$-$30^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle P_T \rangle_{\pi-N}$ $(\text{MeV} / c)$</td>
<td>$180 \pm 23$</td>
<td>$200 \pm 48$</td>
<td>-</td>
</tr>
<tr>
<td>$\langle P_T \rangle_{\pi-core}$ $(\text{MeV} / c)$</td>
<td>$206 \pm 29$</td>
<td>$323 \pm 63$</td>
<td>$561 \pm 187$</td>
</tr>
</tbody>
</table>

qualitatively on the basis of Isobar model. In $\pi$-$N$ collisions, both boson resonant states ($\eta(548)$, $\omega(782)$, $\rho(750)$ etc.) and nucleon states ($N^{*}_{3/2}(1238)$, $N^{*}_{1/2}(1512)$, $N^{*}_{1/2}(1668)$, $N^{*}_{1/2}(1920)$ etc.) may be excited and in the c.m. system, the pion isobar moves in the forward direction and heavy isobar in the backward direction.

Malhotra [6] has calculated the various values of $\langle P_T \rangle$ of pions in the rest system of the isobars so far established for their various assumed decays. Here it is assumed that the emission of the pions is isotropic in the rest system of the isobar. Various chain decays of isobars ($N^{*}_{3/2} \rightarrow N^{*}_{1/2} + \pi - \rightarrow N^{*}_{1/2} + 2\pi - \rightarrow N^{*}_{1/2} + 3\pi -$ etc.) have also been taken into consideration. The main results may be summed up as:

(a) The threshold energy of the incident pion in $\pi$-$N$ collisions for the excitation of the highest nucleon isobars is 2.36 GeV. For the excitation of pion resonant states ($\eta$, $\omega$ and $\rho$), it is less than 1 GeV.

(b) The average transverse momentum of pions from nucleon isobars ranges from 61 MeV/c to 77 MeV/c. In case of pion resonances, the average transverse momentum varies from 92 to 273 MeV/c. So the expected values of $\langle P_T \rangle$ in case of heavy isobars is much larger than for boson states.

According to above conclusions one can say that at an incident energy as high as 17 GeV in the present investigation, all the well known nucleon and pion isobars are excited. There is enough experimental evidence [9] also that at these high energies, the pion isobars play quite a prominent role in the production of pions in the case of $\pi$-$N$ collisions.

The general behaviour of transverse momentum along with its average value may be explained by taking suitable combinations of expected values of $\langle P_T \rangle$ for various assumed decay schemes of both light and heavy isobars.

It may be possible now to explain the behaviour of $\langle P_T \rangle$ with emission angle $\theta^*$ (fig. 2). At smaller angles, the contribution to $\langle P_T \rangle$ comes from the boson states which move in forward direction giving smaller $\langle P_T \rangle$. As $\theta^*$ in creases, the pions from heavier isobars start contributing to average transverse momentum. Our experimental observations again show a tendency of $\langle P_T \rangle$ to decrease in the extreme backward direction. It may be due to a possible decay of $N^*$ going to nucleon and boson state (e.g. $N^{*} \rightarrow N + \pi^*$). Here $\pi^*$ shall move relatively in the backward direction yielding smaller values of $\langle P_T \rangle$ at extreme values of $\theta^*$.

The average transverse momentum for $\pi$-$\pi$ events is $(195 \pm 21)$ MeV/c and for $\pi$-core events, it is $(412 \pm 45)$ MeV/c. Such a behaviour of $P_T$ may
explained on the basis that in \( \pi\pi \) events, the nucleon does not go to an excited state and simply recoils and only boson states contribute. The average transverse momentum in the case of \( \pi\pi \)-core events is the net contribution from light and heavy isobars. The other consequence of this picture is the lack of dependence of \( \langle p_T \rangle \) on emission angle in lab. system in case of \( \pi\pi \) events and linear rise of \( \langle p_T \rangle \) with increase in angle in case of \( \pi\pi \)-core events. Actually such a behaviour has experimentally been observed in the present investigation (table 3).

With the aid of the uncertainty relation, one can find the lower limit to the radii of the regions in which the \( \pi\pi \) and \( \pi\pi \)-core events are limited. Asking the average values of transverse momentum for \( \pi\pi \) and \( \pi\pi \)-core events, we have

\[
\bar{\Delta}r_{\pi\pi} \geq 1.0 \text{ fm},
\]

and

\[
\bar{\Delta}r_{\pi\pi \text{-core}} \geq 0.4 \text{ fm}.
\]

This obvious result further supports that in general there are two types of events present in \( \pi\pi \)-N interactions. The incident pion may interact with the virtual pion cloud surrounding the dense nucleon core, or it may interact with the core itself. The dimensions of the core are far less than those of the surrounding meson cloud.

Another important fact is the dependence of \( \langle p_T \rangle \) on \( n_q \). Table 2 shows that \( \langle p_T \rangle \) for \( n_q \leq 2 \) is much smaller than for \( n_q > 2 \). One may explain such behaviour of \( \langle p_T \rangle \) on the basis of \( \pi\pi \) and \( \pi\pi \)-core events. The average multiplicity for \( \pi\pi \) events is \( (2.66 \pm 0.27) \) and in case of \( \pi\pi \)-core events, it is \( 4.69 \pm 0.58 \). So most of events with \( n_q \leq 2 \) belong to \( \pi\pi \) interactions giving smaller values of \( \langle p_T \rangle \). At larger multiplicities, the contribution from \( \pi\pi \)-core events becomes dominant giving higher values of \( \langle p_T \rangle \). At lower incident energies, it has been found that \( \langle p_T \rangle \) is independent of \( n_q \). Here the average multiplicity is quite small \( (2.18 \pm 0.29 \text{ at } 4.4 \text{ GeV}) \) so as not to produce any definite resolution in the average multiplicities of \( \pi\pi \) and \( \pi\pi \)-core events.

The present investigations support the predictions of Isobar model incorporated with a definite structure of the nucleon.

REFERENCES

Abstract. The saturation of $U(3) \otimes U(3)$ algebra is examined in a special mixing scheme. It is shown that the model gives the right value not only for the proton-neutron axial-vector coupling parameters, but for the nucleon electromagnetic form factors and their first derivative at $q^2 = 0$ as well.

It is well known that the saturation of the $SU(6) \text{W}$ or $U(3) \otimes U(3)$ (chiral or collinear) algebra of currents by members of a $SU(6)$ multiplet yields the familiar $SU(6)$ results [1]. On the other hand, the obtained coupling constants do not always agree with experimental results (compare e.g. the $SU(6)$ prediction $G_A / G_V = -1/2$ and the experimental value $G_A / G_V = -1.18$). The inclusion of intermediate states of higher masses seems to be necessary to achieve a better saturation of commutation relations.

If we want to have a non-vanishing matrix element of a generator between a member of our basic multiplet and a higher state we are forced to put out particles into mixtures of irreducible representations rather than into pure representations of the group in question [2-4].

Moreover, if we want to have non-vanishing magnetic moments from a $U(3) \otimes U(3)$ algebra the particles must have non-singlet transformation properties under spatial rotations independently of the purely "internal" spin quantum number [5].

In addition, there are some restrictions also on the electric dipole operators [5,6] which are used generally to calculate the anomalous magnetic moments:

(i) These operators - unlike charge operators - contain parts which transform under $U(3) \otimes U(3)$ like $(3,3)_g + (3,3)_g$ in addition to the "normal" part $(8,1) + (1,8)$ (ref. [7]).

(ii) The dipole operators have parts which transform under $O(3)$ (spatial rotations) like a vector [5, 3, 9].

In the present note we discuss a special mixing model which satisfies the above-mentioned criteria. The model that was mentioned in a paper of Cabibbo and Ruegg [3] has the following properties:

The model works in the framework of the collinear $U(3) \otimes U(3) \otimes U(1)_L$. 
CENTRAL AND PERIPHERAL INTERACTIONS
IN PION-NUCLEON INELASTIC EVENTS AT 17.2 GeV

J. M. KOHLI
Department of Physics, Panjab University, Chandigarh, India

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Abstract: The present investigation is based upon 169 pure pion-nucleon interactions at 17.2 GeV. The experimental results based upon various parameters like effective target mass distribution, mean multiplicity, angular and energy distributions may be explained on the basis of a definite structure of nucleon. The incident pion may interact with the central "core" of the nucleon or with the pion of the virtual meson cloud surrounding the core itself.

1. INTRODUCTION

With the advent of high-energy accelerators, it has become possible to probe deeper into the structure of the nucleon. Several investigators [1-4] have confirmed that the nucleon consists of a dense "core" surrounded by a virtual meson cloud.

A study of effective target mass $M_T$ in case of pure pion-nucleon interactions (sect. 3) reveals the existence of two types of interactions i.e. the incident pion may interact with the central core of the nucleon or with the pion of the virtual meson cloud surrounding the core. The former type of events have been termed as $\pi$-core interactions, while the latter are called $\pi$-$\pi$ interactions. In the present investigation we have obtained in detail the distinctive features of these two types of events. The behaviour of various parameters like mean multiplicity angular and energy distributions in lab and c.m. system have been studied for the two said types of events respectively. The behaviour of the transverse momentum also suggests the existence of a dense core of nucleon surrounded by a virtual meson cloud [5].

2. EXPERIMENTAL DETAILS

A stack of Ilford G.5 emulsions of the size 23 cm x 15 cm and of thickness 600 $\mu$m was exposed to $(17.2 \pm 0.2)$ GeV $\pi^-$ beam at CERN. The total track length followed was 470.7 meters and 1166 interactions were located. There were 221 events of the type $0 + 0 + n_N$ and $0 + 1 + n_N$. Multiple Coulomb scattering measurement was performed upon all the secondary shower tracks emitted from these 221 events having projected length $\geq 0.5$ mm in the scanned
LIKE PION SPECTRA

ate. For greys, the restriction on minimum projected length was reduced to 2 mm. All the secondaries thus selected were identified from p/3-ionisation on plot. The space angles of all the secondary tracks were measured up to an accuracy of 1°.

. SELECTION OF CENTRAL AND PERIPHERAL INTERACTIONS

One can estimate the mass of the target particle purely from the conservation laws of energy and momentum. From these laws one can define target mass $M_t$ as

$$M_t = \sum_i (E_i - \rho_i \cos \theta_i) - (E_0 - \rho_0),$$  \hspace{1cm} (1)

where $E_i, \rho_i$ and $\theta_i$ denote the total energy, momentum and angle of emission in the lab system with respect to the direction of the incident particle expectedly of the $i$th particle given out in the interaction, and $E_0, \rho_0$ represent the energy and momentum of the incident particle. The target mass $M_t$ does not depend upon any model and its concept is entirely kinematical in nature. For very high incident energy $E_0 - \rho_0 \ll 1$, we have

$$M_t = \sum_i (E_i - \rho_i \cos \theta_i).$$  \hspace{1cm} (2)

In case of pion-nucleon collisions, we have

$$M_N = \sum_j (E_j - \rho_j \cos \theta_j) + (E_p - \rho_p \cos \theta_p),$$  \hspace{1cm} (3)

here $M_N$ denotes the mass of the nucleon, subscript $p$ denotes the recoil target nucleon and $\Sigma_j$ denotes the summation of over-all secondaries except the recoil nucleon.

The effective target mass, $M_T$, is defined as

$$M_T = \sum_j (E_j - \rho_j \cos \theta_j).$$  \hspace{1cm} (4)

Thus $M_T$ denotes the mass of the nucleon which actually participates in the production of secondaries, mainly pions.

The experimental difficulties in the direct determination of $M_T$ are two-fold. Firstly the angles of emission and the momenta carried by neutral particles are unknown and secondly all the emitted charged shower particles are not amenable to scattering and ionisation measurements. In view of these difficulties we have followed after Lim [3], an empirical method to determine $M_T$.

According to this method a quantity $\delta_j$, defined as
\[ \delta_j = \frac{1}{m_j} (E_j - p_j \cos \theta_j), \]

is computed for all the secondary shower tracks coming from an interaction only on the basis of emission angle \( \theta_j \) in the lab system. The summation of \( \delta_j \), is connected with \( M_T \) by a constant conversion factor \( K = M_T / \Sigma \delta_j \).

Fig. 1 shows the \( \Sigma \delta \) distribution for events belonging to the \( 0+0+n_s \) and \( 0+1+n_s \) types with \( n_s > 2 \). The value of the conversion factor \( K = 0.45 \).

There are two peaks in the \( \Sigma \delta \) distribution. One occurs at \( M_T = M_N \) and the other at \( M_T = M_T^* \). This structure in the \( M_T \) distribution of pure pion-nucleon events has also been observed by several authors [1-4], while studying \( \pi-N \) and \( p-N \) interactions at accelerator and cosmic ray energies. Considering the accepted picture of the nucleon as consisting of dense core surrounded by a virtual pion cloud, the observed distribution in \( M_T \), it can be thought that we are dealing with two different types of interactions. Those events which have \( M_T < M_N \) are those in which the interaction takes place.

Fig. 1. Differential distribution of \( \Sigma \delta \). The arrow heads show various expected values of effective target mass in terms of nucleon mass.
with a pion of the pion cloud and those with $M_T > M_\pi$ (but less than or equal $M_N$) represent interactions with the core of the nucleon. We have termed these two types of events as $\pi-\pi$ or peripheral and $\pi$-core or central events separately.

Out of 221 events, we have 169 events having $M_T \leq M_N$. The remaining 22 events are due to pion-multinucleon interactions. In all there are 102 $\pi$ events and 67 $\pi$-core events. All the $n_\pi=1$ events having shower track in forward direction have been considered as $\pi-\pi$ interactions.

The cross section for the inelastic 102 $\pi-\pi$ interactions is 14.5 mb. This value of cross section may be compared with 11 mb reported by Gainotti et al. [6] at 16.2 GeV $\pi$-p interactions.

MEAN MULTIPLICITY OF SECONDARY PARTICLES

Fig. 2 shows the $n_\pi$ distribution for $\pi-\pi$ and $\pi$-core types of events. The $\pi$-core histogram is shifted towards higher values of $n_\pi$. Many authors have taken $\pi-\pi$ and $\pi$-core events entirely on the basis of $n_\pi$, i.e. events with $n_\pi \leq 2$ or 3 as due to $\pi-\pi$ interactions, and rest due to $\pi$-core events. Here we find that $\pi-\pi$ events have a distribution in $n_\pi$ which extends up to $n_\pi = 5$ and at the same time there is quite a good proportion (22%) of $\pi$-core events with $n_\pi \geq 3$.

The values of mean multiplicity for $\pi-\pi$ and $\pi$-core events are

\[ \langle n_\pi \rangle_{\pi-\pi} = 2.66 \pm 0.27 \quad \text{and} \quad \langle n_\pi \rangle_{\pi \text{-core}} = 4.69 \pm 0.58. \]

Fig. 2. Multiplicity distribution of $\pi-\pi$ (dashed curve) and $\pi$-core (solid curve) events.
The mean multiplicity for $\pi-\pi$ events at 17.2 GeV may be compared with the values obtained at 6.2 GeV $p$-nucleon and 4.4 GeV $\pi$-nucleon interactions because the energy available in the c.m. system for all these cases is almost same. The mean multiplicity in case of 6.2 GeV $p$-nucleon interactions is $(2.18 \pm 0.29)$ and at 4.4 GeV $\pi$-nucleon interactions is $(2.12 \pm 0.21)$. These values are slightly less than $\pi-\pi$ multiplicity in the present investigation. It may be due to the presence of $\pi-\pi$ events in the case of 6.2 GeV and 4.4 GeV interactions, which tends to lower the average energy available in the c.m. system for these events.

5. ANGULAR DISTRIBUTION OF SECONDARIES IN THE LAB SYSTEM

The median angles of $\pi-\pi$ and $\pi$-core events are

$\theta_{1,\pi-\pi} = (6.2 \pm 0.6)^\circ$ and $\theta_{1,\pi\text{-core}} = (12.5 \pm 1.6)^\circ$,

respectively.

The secondaries from $\pi-\pi$ interactions are confined in much smaller $\theta_c$ than those from $\pi$-core interactions.

Another way to study the angular distribution is to plot the differential distribution in logtg $\theta$. It can be shown [8] that if the particles are emitted isotropically in the c.m. system and $\beta_c/\beta^* = 1$, i.e. the ratio of the velocity of the c.m. system to that of the particles in the c.m. system is equal to unity, the logtg $\theta$ distribution will be a Gaussian centred around $-\log \gamma_c$, where $\gamma_{c.m.}$ is the Lorentz factor of the c.m. system.

Fig. 3 shows the logtg $\theta$ distribution in the case of $\pi-\pi$ events (solid line) and total 169 pion-nucleon events (dotted line). The two Gaussians a and b are given by

$$\frac{dN}{d\theta} = \frac{1}{\sqrt{2\pi}} \left( \frac{1}{\gamma_{c.m.}} \right)^2 e^{-\frac{1}{2} \left( \frac{x - \log \gamma_{c.m.}}{\gamma_{c.m.}} \right)^2},$$

where $x = \log tg \theta$. These two Gaussians are centred around $-\log \gamma_{c.m.} = -\log 7.9$ and $-\log 3.15$ for $\pi-\pi$ and $\pi$-core events, respectively. By adding the relative contributions of these Gaussians a and b at various values of x, the curve c is drawn.

On matching the curve c with the dotted histogram due to pion-nucleon events, we observe 66 tracks which are not accommodated by either $\pi-\pi$ or $\pi$-core events. These pions may be due to "persistence of pions". Similar results have been reported at 16.2 GeV [6] and 4.4 GeV [7], $\pi$-nucleon interactions. These tracks are mostly coming at very small angles and carry large energies. The average number of such persisting pion per collision 0.39 and is slightly higher than the value 0.24 obtained at 4.4 GeV [7] $\pi-N$ collisions. The increase in incident energy may increase the degree of persistence and secondly the persisting pions are emitted relatively at small angles.
Fig. 3. The log $\theta$ distributions of shower particles emitted from $\pi-N$ (solid curve) and the total 169 events (dashed curve). Curves a and b are due to eq. (6) for $\pi-N$ and $\pi$-core events. The curve c is the sum of both these curves (a and b).

. ANGULAR DISTRIBUTION IN THE c.m. SYSTEM

The overall sample of 169 pure-pion nucleon events in the $\pi-N$ c.m. system are highly asymmetric. The value of asymmetry defined as the ratio of number of forward to backward going pions from these events is 2.0. Figs. 4 and 5 show the angular distribution for $\pi-N$ and $\pi$-core events respectively. The values of asymmetry in the case of $\pi-N$ and $\pi$-core events are 0.98 and 0.40 respectively. The presence of asymmetry in the case of $\pi$-core events may be attributed to the presence of persisting pions which move in the lab system at relatively smaller angles and carry most of the incident energy. These tracks shall be collimated in the forward direction in the c.m. system. If the contribution of these 66 persisting pions is also excluded from the forward moving pions in the case of $\pi$-core interactions, the value of asymmetry reduces from 1.40 to 0.90 which corresponds to almost a symmetric distribution.

. ENERGY OF PIONS IN THE $\pi-N$ c.m. SYSTEM

Fig. 6 shows the total energy distribution of pions in the case of $\pi-N$ events in the $\pi-N$ c.m. system. The average value of total energy for these ions is $(325 \pm 23)$ MeV. The value of average energy in the case of overall sample of 169 pion-nucleon interactions is $(491 \pm 32)$ MeV.
8. DISCUSSION

The various results regarding peripheral (π-π) and central (π-core) interactions have been dealt in the last sections. The behaviour of these two types of interactions differs from each other in almost every respect and it seems legitimate to assume that entirely different physical processes are involved in their production. One possible interpretation is that in π-π events incident pion interacts effectively with the virtual pion surrounding the core of the nucleon and in π-core events the effective target is the core of the nucleon itself.

Kaplon and Shen [9] have discussed how far it is justified to treat event with effective target mass \( M_T \approx M_T \) as π-π events. According to these authors the fact that in certain cases \( M_T \) turns out to be equal to the pion mass may not necessarily mean that the effective collision is only with a virtual pion. For they argue that \( M_T = M_T \pm M_T \) may arise because of the fact that four momentum transfer in such collisions is of the order of one nucleon mass and \( (E_p - p_p \cos \theta_p) \) [see eq. (3)] is of the order of half a nucleon mass.

In this connection we have studied the distribution of \( (M_N - (E_p - p_p \cos \theta_p)) \) for all the 169 events wherever the outcoming proton could be isolated. The average value of this quantity turns out to be half a nucleon mass, but otherwise it shows a similar distribution to \( M_T \) (fig. 1). Fig. 7 shows the existence of two peaks in the distribution of \( (M_N - (E_p - p_p \cos \theta_p)) \) at the ex-
Fig. 6. Total energy distribution of pions in case of $\pi-\pi$ events in the $\pi-\pi$ c.m. system.

Fig. 7. Differential distribution in $M_{NN} - (E_p - P_y \cos \theta_p)$ for protons isolated in the recent investigation (shaded portion). The total histogram is due to protons picked from present investigation along with those from the interactions at 8 and 7.5 GeV pion-nucleon interactions.
pected values of target masses corresponding to a pion mass and a nucleon mass. In order to increase the statistics, the final histogram is due to protons from 500 $\pi^-N$ interactions at 8 GeV (ref. [10]) and 7.5 GeV (ref. [1]) along with the present work. It may also be noted that if the interaction is assumed to be with a free pion, the kinematical considerations [eq. (2)] require that $\Sigma^0$ should be equal to 0.66. Fig. 1 shows a peak in the interval 0 - 0.5, which qualitatively agrees with the above result on the assumption that nucleon also share some energy being loosely bound with the interacting pion. Even the four-momentum transfer in $\pi^-\pi$ events is quite less. Kaplon and Shen also observe that if the collision is readily with a target pion, the value of $\gamma_{c.m.}$ the Lorentz factor for centre of mass should be centred around $\gamma_{c.m.} = 9$ and they find it is not so. We, however, find that for our sample of $\pi^-\pi$ events, the peak is around $-\log \gamma_{c.m.}$ as can be seen from logtg $\theta$ distribution (fig. 3). The absence of peak at $\gamma_{c.m.} = 9$ in the case of Kaplon and Shen may be because of the fact that in their method the histogram is plotted only for events with $n_\pi \geq 4$, whereas $\pi^-\pi$ events are mainly confined in the lower values of multiplicity (fig. 2).

Further, the pure pion-nucleon interactions are highly asymmetrical in the $\pi^-N$ c.m. system. This asymmetry has been shown to be due to the presence of tracks due to $\pi^-\pi$ events and also due to persisting pions (sect. 6). The $\pi^-\pi$ and $\pi^-\pi$ core events are symmetrical in their own c.m. systems. The logtg $\theta$ distribution suggests that these persisting pions are not create pions and are emitted at relatively smaller angles ($< 8^\circ$) and carry most the incident energy. The degree of persistence in the present case is 0.39 and this value increases with increase in incident energy. In view of the above considerations it seems quite justified to consider that the nucleon has a definite structure. The incident pion may interact with the central core of the nucleon or with the virtual pion surrounding the central core itself. The behaviour of the transverse momentum which has been presented in a separate paper [5] also suggests the same results.

The author is thankful to Dr. I. S. Mittra for discussion of results.

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COHERENT PRODUCTION OF PIONS 17.2 GeV PION-NUCLEUS INTERACTIONS IN EMULSION NUCLEI

By
D. P. Dubey, J. M. Kohli and M. B. Singh
COHERENT PRODUCTION OF PIONS 17.2 GeV
PION–NUCLEUS INTERACTIONS IN
EMULSION NUCLEI

BY D. P. DUBEY, J. M. KOHLI AND M. B. SINGH
(Physics Department, Panjab University, Chandigarh)

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ABSTRACT

The coherent production of pions at 17.2 GeV in pion interactions with nuclei has been studied using Nuclear Emulsion Technique. The kinematical selection of such events was made out of three prong events in which all the prongs were identified as pions. The mean free path of coherent events was 65 meters. It was found that diffraction dissociation takes place with lighter nuclei, whereas the Coulomb dissociation plays a major role in the case of heavier emulsion nuclei.

1. INTRODUCTION

The inelastic interaction of high energy pions with nucleus can be broadly divided into two groups. One, in which the pion interacts with individual nucleons in the nucleus, as the wavelength of high energy pions is of the order of the size of the nucleon. This results in pion production, excitation and subsequent evaporation of the nucleus. The other kind of interaction is that in which interaction takes place with the nucleus as a whole. The later process is known as the coherent interaction. It was suggested by Feinberg and Pomeranchuk,1 and Good and Walker2, 3 that coherent interaction of pions with nucleus can result in emission of pions. The inelastic coherent interaction with nuclear field is called diffraction dissociation whereas the interaction with Coulomb field is termed as Coulomb dissociation. Detailed theoretical analysis of these interactions have been reported by Drell,4 Mathews and Salam,5 Zhizhin et al.6 and Belenkii.7 It has been suggested that the cross-section for production of these events is a function of the charge of the target nucleus. Nuclear emulsion has an advantage of providing targets of high charge and is quite a convenient technique for the study of these coherent events. The production of pions by the diffraction and Coulomb dissociation of pions at energies ranging between 14–18 GeV has been studied in Nuclear Emulsion8–11 and heavy liquid Bubble Chamber.12–13
In this paper we report the analysis of 3 prong events identified as coherent interaction selected from the events of the type

\[ \pi^- + \text{Nucleus} \rightarrow \pi^- + \pi^+ + \pi^- + n\pi^0 + \text{Nucleus}. \]

The results obtained have further been compared with the values reported by various workers using Nuclear Emulsion and Bubble Chamber techniques.

2. EXPERIMENTAL DETAILS

A stack of Ilford G.5 emulsions of thickness 600 \( \mu \)m and of the size 23 cm. \( \times \) 15 cm. was exposed to 17.2 GeV\( \pi^- \) beam at CERN. The average flux was \( 4 \times 10^4 \text{cm}^{-2} \cdot \text{sec}^{-1} \). 471 meters of track length was followed and 1166 events were picked up. The total magnification used for line scanning was 1000 \( \times \) and the scanning rate was 25 cm./hr.

There were 27 events which satisfied the following criteria:

(a) no heavy prong \( (N_h = n_b + n_g) \) associated with the event,

(b) the number of charged relativistic secondaries was three,

(c) no electron track or recoil associated with the event.

These 27 events have been termed as 'clean' events. There is another sample of 45 events with \( N_z < 3 \) and \( n_s = 3 \). These events do not include the 'clean' events and are called 'dirty' events. These events are essentially not due to coherent production of pions.

The angular and energy measurements have been carried on Koristka R-4 microscope. The tracks having minimum projected length equal to 5 mm. in the scanned plate have been accepted for scattering measurements. The remaining tracks were assigned energies from the consideration of emission angles alone. The number of such tracks was, however, less than 20% of all the charged secondaries.

3. SEARCH FOR COHERENT EVENTS

3.1. \( \Sigma \sin \alpha_1 \) distribution.—Since the nucleus remains in the ground state before and after the interaction, the momentum transfer to it, is small and also the point of momentum transfer is localised outside the nucleus. This condition roughly puts an upper limit to the longitudinal component of the momentum transfer \( (q_{l1}) \). From uncertainty principle one finds that the value of \( q_{l1} \) should be less than or equal to \( R^{-1} = (m_{\pi}/A)^{1/2} \), where \( R \) is
the radius of the nucleus and $A$ its mass number. For carbon nucleus $q_{11}$ should be less than or equal to 60 MeV/c and its value is still smaller in case of heavier nuclei ($A_2B_r$).

It can be shown that this condition can be satisfied only by the events which undergo the following conditions:

$$\sum_{i=1}^{n} \sin a_i \leq \frac{1}{m_{\pi}} q_{11} \text{ max.}$$

(1)

where $a_i$ is the angle between the direction of the incident pion and the $i$-th particle in the laboratory system.

The Coulomb dissociation is more prominent when the charge of the target nucleus is very high. So the main contribution to the cross-section for such events is due to the collisions with heavier nuclei ($A_2B_r$) for which $q_{11} \approx 30$ MeV/c. Thus, for Coulomb dissociation, we expect

$$\sum_{i=1}^{n} \sin a_i \leq 0.22.$$  

(2)

On the other hand the diffraction dissociation is expected to take place with semi-transparent nuclei (CNO). In this case $q_{11} \text{ max.} = 60$ MeV/c and hence the geometrical condition for such events is

$$\sum_{i=1}^{n} \sin a_i \leq 0.43$$

(3)

These conditions are necessary but not sufficient.

Figure 1 shows the $\sum_{i=1}^{n} \sin a_i$ normalised distribution for 'clean' and 'dirty' events. There lies a marked difference in the two distributions. The 'clean' events are concentrated towards the lower values of $\sum_{i=1}^{n} \sin a_i$, whereas the 'dirty' events show an all-round spread. Out of 27 clean events, there are 19 events with $\sum_{i=1}^{n} \sin a_i \leq 0.45$, whereas out of 45 dirty events, there are 14 events which satisfy this condition.

The value of $\gamma$ defined as the ratio of the number of events with $\sum_{i=1}^{n} \sin a_i \leq 0.45$ to the events with $\sum_{i=1}^{n} \sin a_i > 0.45$ for clean and dirty events are 2.38 and 0.45 respectively.
3.2.—Total energy carried by secondary pions.—As the energy transfer to the target nucleus is very small, the total energy of the secondaries other than the recoiling target is almost equal to the energy of the primary pion.

Figure 2 shows $\sum_{i=1}^{n} E_i$ plotted against $\sum_{i=1}^{n} \sin \alpha_i$ for all the 'clean' and 'dirty' events.

![Figure 1](image1.png)

**Fig. 1.** Normalised $\sum_{i=1}^{n} \sin \alpha_i$ distribution in the case of 'clean' (---) and dirty (----) events.

![Figure 2](image2.png)

**Fig. 2.** $\sum_{i=1}^{n} E_i$ plotted against $\sum_{i=1}^{n} \sin \alpha_i$ for both 'clean' and 'dirty' events. The points marked (□) and (○) belong to 'clean' and 'dirty' events respectively.
events. There are 9 ‘clean’ and 4 ‘dirty’ events which satisfied both the conditions simultaneously, i.e., (i) $\sum_{i=1}^{n} E_i > 10$ GeV and (ii) $\sum_{i=1}^{n} \sin a_i < 0.45$. Some authors$^{10,13}$ have used the energy limit $\sum_{i=1}^{n} E_i > 0.75 E_0$, where $E_0$ is the energy of the incident pion. We have put the energy limit of 10 GeV due to errors involved in the energy measurements. The events with energy balance in case of clean sample is 33% and is in good agreement with the value 36% obtained in bubble chambers.$^{13}$

3.3. The distribution in transverse momentum transfer.—Figure 3 shows the $q_\perp$ distribution for all the 9 observed ‘clean’ events. The total histogram is due to combined data of Caforio$^{10}$ and that of the present work. The smooth curve is due to Belenkin$^{7}$ and is given by

$$f(q_\perp) dq_{\perp} = q_{\perp} \exp \left( -\frac{q_{\perp} R}{2} \right) dq_{\perp}. \quad (4)$$

Here the relative contribution of all the emulsion nuclei in proportion to their geometrical cross-sections have been considered.

For carbon nuclei the most probable value of $q_\perp$ is 85 MeV/c and 95% of all the events should have $q_\perp$ less than 210 MeV/c. One may stretch the upper limit to 300 MeV/c in order to take into account the errors in the momentum measurements,
There is one event out of 9 ‘clean’ events with energy balance which has $q_x > 300$ MeV/c. Out of 4 ‘dirty’ events, there are two events which have $q_x > 300$ MeV/c. After excluding these events one is left with 8 ‘clean’ and 2 ‘dirty’ events which satisfy the above-mentioned condition for the production of coherent events.

3.4. Invariant mass distribution.—Figure 4 shows the invariant mass distribution $M^*$ for all the eight most probable coherent events along with the combined results of Caforio et al. and the present investigation. All the values of $M^*$ lie between 0.5 and 1.5 GeV.

![Invariant mass distribution](image)

Fig. 4. Invariant mass distribution for the eight coherent events observed in the present investigation (shaded portion) along with the combined histogram due to Caforio et al. and present investigation.

The maximum value of $M^*$ produced by diffraction or Coulomb dissociation by a nucleus of mass number $A$ and an incident momentum $p_0$ is given by

$$M^*_{\text{max.}} = m_\pi \left( \frac{2p_0}{m_\pi A^2} + 1 \right) \approx 1.5 \text{ GeV.}$$

(5)

The lower limit of the invariant mass may be simply the rest mass of the three pions (420 MeV) or it may be the rest mass of $\rho$ meson and a pion combined (900 MeV).

3.5. The distribution in longitudinal momentum transfer.—Purely from the consideration of conservation laws of energy and momentum, the value of $q_{11}$ is given by

$$q_{11} = \left( \frac{M_A + p_0}{2M_A p_0} \right) q_x^2 + \frac{M^*^2 - m_\pi^2}{2p_0} + T_{\text{exc}}.$$  

(6)
where $T_{\text{exc}}$ is the excitation energy of the target nucleus and is quite small. $M^*$ is the invariant mass of the emitted pions.

Figure 5 shows the $q_{11}$ differential distribution of all the eight most probable coherent events. Combined distribution of these events along with the results of Caforio et al. has also been plotted. There lies a general agreement with both the results and all the values of $q_{11}$ are confined between 10 and 80 MeV/c.

4. DISCUSSION OF RESULTS

Out of 471 meters of track length followed, there are eight ‘clean’ events and two ‘dirty’ events, which satisfy all the accepted conditions for coherent production. Since out of 45 ‘dirty’ events, there are two events which satisfy all the conditions for coherent events, one may expect that out of these eight events, there is a small contribution (1-2%) of incoherent events. For the maximum 6% contamination due to electrons and muons in the incident beam, we have the total mean free path equal to 65 meters. This value of mean free path is comparable with $(51 - 11)^{18}$ meters reported by Caforio et al. at almost the same incident energy.

If it were assumed that all these coherent events are due to Coulomb and diffraction dissociation on CNO nuclei only, the cross-section for such process is $(61 \pm 2.3) \text{ mb}$. Further if it were assumed that all the emulsions...
nuclei contributed equally, the cross-section is \( (3.4 \pm 1.3) \text{ mb} \) and finally if it were assumed that the cross-section is proportional to \( A^{4} \), its value would be \( (1.4 \pm 0.5) \text{ mb} \). The cross-section obtained in the case of coherent events produced on carbon nuclei with bubble chamber \(^{13}\) is \( (1.7 \pm 0.5) \text{ mb} \) and in nuclear emulsions obtained by Caforio et al. \(^{6}\) is \( (1.7 \pm 0.5) \text{ mb} \) on the assumption that cross-section is proportional to \( A^{4} \). Therefore, it follows that a majority of these events is due to \( AgBr \) nuclei. It has been discussed in Sec. 3.1, that all the events having \( \Sigma \sin \alpha_i \leq 0.22 \) are mainly attributed to Coulomb dissociation. There are six such events. The remaining two events may be due to diffraction dissociation. The cross-section based upon these events for the diffraction dissociation on CNO nuclei is \( 1.8 \text{ mb} \) and is in good agreement with the value \( 1.7 \text{ mb} \) reported by C. Bellini on carbon nuclei. So one may conclude that the diffraction dissociation takes place on lighter semi-transparent nuclei, whereas the Coulomb dissociation plays a major role in the case of heavier \( AgBr \) nuclei.

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ON THE INTERACTIONS OF 17.2 GEV $\pi^-$ MESONS WITH EMULSION NUCLEI

J. M. KOHLI
Department of Physics, Panjab University,
Chandigarh-14, India

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ON THE INTERACTIONS OF 17.2 GeV \( \pi^+ \) MESONS WITH EMULSION NUCLEI

J. M. KOHLI
Department of Physics, Panjab University, Chandigarh-14, India

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Abstract: 260 m of track length was followed and 631 interactions were picked up. On the basis of the effective target mass \( M_T \), these events were divided into pure pion-nucleon and pion-multinucleon interactions. The behaviour of various parameters like mean multiplicity, median angle, transverse momentum, energy in lab and c.m. system and coefficient of inelasticity has been studied for these two types of events. The study of heavy tracks is useful in understanding the mechanism of nuclear disintegration. The original picture of the internuclear cascade model requires some further modifications in the light of an effect called 'trailing'.

1. INTRODUCTION

With the help of nuclear research emulsion, one can investigate the mechanism of pion production at high energies in pion-nucleon and pion-multinucleon interactions. It is interesting to note that the behaviour of pure pion-nucleon events of the type \( 0 + 0 + n_\pi \), \( 0 + 1 + n_\pi \) with effective target mass \( M_T \) less than one nucleon mass is exactly similar to that of events with \( M_T < M_N \) irrespective of \( N_\pi \) values. One can understand this on the assumption that the incident primary is responsible for the production of fast pions constituting the shower tracks, the subsequent break up of the nucleus by the recoils being almost an independent process.

Recently we have pointed out that the theoretical results of Artykov et al. [1] based upon the internuclear cascade model do not agree with our experimental results [2] concerning inelastic interactions of 17.2 GeV pions with heavy emulsion nuclei. An effect called 'trailing' must be incorporated with the calculations of internuclear cascade developed within the nucleus.

The experimental results based upon the present investigation agree well with the results obtained from proton-nucleus interactions at 22.5 GeV. The energy available in the c.m. system in both cases is almost equal.

A stack of Ilford G.5 emulsions of the size 23 cm \( \times \) 15 cm \( \times \) 600 \( \mu \)m was exposed to the 17.2 \( \pm \) 0.2 GeV \( \pi^+ \) meson beam at CERN. The average flux was \( 4 \times 10^4 \) cm\(^{-2} \) and the angular spread in the vertical plane was \( \pm 5 \) mr. The beam tracks were followed until they interacted or left the plate. The
total magnification used for scanning was 1000 X and the scanning rate was 25 cm/h. The total track length followed was 257.7 m and 631 inelastic interactions were picked up. The energy and angular measurements of shower and grey tracks have been discussed elsewhere [4].

2. EXPERIMENTAL RESULTS

2.1. Average prong multiplicities

The average multiplicities of heavy, grey, black and shower tracks for these 631 interactions have been reported in table 1. For comparison the following results have also been included: (a) results obtained by Winzeler et al. [3, 5] at 22.5 GeV and 6.2 GeV p-nucleus interactions; (b) results obtained at 3500 GeV by Barkow et al. [6].

<table>
<thead>
<tr>
<th>Primary energy (GeV)</th>
<th>Type of interaction</th>
<th>$\langle N_h \rangle$</th>
<th>$\langle N_g \rangle$</th>
<th>$\langle N_b \rangle$</th>
<th>$n_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.2</td>
<td>$\pi$-nucleus</td>
<td>7.8 ± 0.3</td>
<td>2.10 ± 0.09</td>
<td>5.69 ± 0.25</td>
<td>5.35 ± 0.27</td>
</tr>
<tr>
<td>22.5</td>
<td>p-nucleus</td>
<td>6.60 ± 0.25</td>
<td>3.38 ± 0.14</td>
<td>5.22 ± 0.29</td>
<td>6.5 ± 0.3</td>
</tr>
<tr>
<td>6.2</td>
<td>p-nucleus</td>
<td>5.25 ± 0.18</td>
<td>3.38 ± 0.11</td>
<td>5.68 ± 0.21</td>
<td>3.2 ± 0.1</td>
</tr>
<tr>
<td>3500</td>
<td>p-nucleus</td>
<td>6.80 ± 1.0</td>
<td>2.06 ± 0.5</td>
<td>-</td>
<td>22.0</td>
</tr>
</tbody>
</table>

The average multiplicity of grey tracks is higher in p-nucleus interactions than in $\pi$-nucleus interactions. The incident proton in p-nucleus interaction may emerge out as a grey particle after suffering a number of collisions within the nucleus.

2.2. Correlation between $\langle N_h \rangle$ and $n_s$

Fig. 1 shows the $\langle N_h \rangle$ plotted against $n_s$. There exists a linear relationship between $\langle N_h \rangle$ and $n_s$. Similar variations at 22.5 GeV and 3500 GeV p-nucleus interactions have also been shown in the same figure.

The slope for 22.5 GeV p-nucleus interactions and that in the present investigation are almost the same. There is a general increase in the slope with decrease in incident energy.

2.3. Comparison of single nucleon and multinucleon interactions

On the basis of the effective target mass [7, 8] the events under investigation have been classified as follows:

Class A: This class is comprised of events which have $M_T \leq M_N$ irrespective of the $N_h$ values. There are 229 such events. These are essentially events in which the incident pion interacts with a single nucleon.

Class B: All the events which have $M_T > M_N$. The total number of such events is 311.
Class C: A sample of 174 pure pion-nucleon events picked up by 470 m of line scanning. These events are of the type $0^+0^+n_s$ and $0^+1^+n_s$ with $M_T \leq M_N$.

Table 2 shows the comparative study of various parameters for the three types of events enlisted above.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Class A</th>
<th>Class B</th>
<th>Class C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_s$</td>
<td>$3.76 \pm 0.24$</td>
<td>$7.56 \pm 0.44$</td>
<td>$3.70 \pm 0.27$</td>
</tr>
<tr>
<td>$E_T$(GeV)</td>
<td>$2.20 \pm 0.15$</td>
<td>$1.60 \pm 0.10$</td>
<td>$2.22 \pm 0.17$</td>
</tr>
<tr>
<td>$P_{3T}$(MeV/c)</td>
<td>$347 \pm 24$</td>
<td>$354 \pm 25$</td>
<td>$344 \pm 26$</td>
</tr>
<tr>
<td>$E_\pi$</td>
<td>$485 \pm 30$</td>
<td>$513 \pm 30$</td>
<td>$491 \pm 32$</td>
</tr>
<tr>
<td>$\eta_f$</td>
<td>$0.40$</td>
<td>$-0.14$</td>
<td>$0.33$</td>
</tr>
<tr>
<td>$\eta_B$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$(\theta_\frac{1}{2})_B$</td>
<td>$(9.1 \pm 0.6)^0$</td>
<td>$(27.5 \pm 1.3)^0$</td>
<td>$(8.1 \pm 0.6)^0$</td>
</tr>
<tr>
<td>Inelasticity</td>
<td>$0.94 \pm 0.02$</td>
<td>$-1$</td>
<td>$0.91 \pm 0.03$</td>
</tr>
</tbody>
</table>

$n$ is the average multiplicity, $E$ and $P_T$ are the mean total energy and mean transverse momentum of secondary particles. $E_T$ denotes the total energy of pions in $\pi - N$ C-system and $\eta$ denotes the asymmetry parameter defined as $(F-B)/(F+B)$, where $F$ and $B$ are the number of particles moving in forward and backward direction in the c.m. system. $\theta_\frac{1}{2}$ is the median angle of shower particles.
Fig. 2. Variation of the mean multiplicity of showers (○) and greys (▲) with effective target mass $M_T$.

Fig. 3. Variation of median angles of showers with $M_T$. 
It is interesting to note that the behaviour of various parameters is similar in case of class A and class C events, whereas class B events behave quite differently.

2.4. Variation of effective target mass with $\langle n_q \rangle$, $\langle n_g \rangle$, and $\theta_2$

Fig. 2 shows the behaviour of $\langle n_q \rangle$ and $\langle n_g \rangle$ versus $M_T$ in case of the 631 pion-nucleus interactions under investigation. There is a linear rise of $\langle n_q \rangle$ and $\langle n_g \rangle$ with increase in the effective target mass. Fig. 3 shows the behaviour of the median angle $\theta_2$ versus effective target mass. There again is a linear rise of $M_T$.

2.5. Results regarding grey tracks

The angular distribution of greys from all stars is highly asymmetric in the lab system. The forward to backward ratio of these tracks in the lab system is 2.1. This forward backward asymmetry has been observed in case of proton-nucleus interactions at 22.5 GeV as well. The average momentum and transverse momentum carried by secondary protons (grey tracks) are $(590 \pm 93)$ and $(438 \pm 61)$ MeV/c respectively.

3. DISCUSSION OF RESULTS

In the preceding sections we have compared the results based upon pure pion-nucleon events (class C) with the events having $M_T < M_N$ irrespective of $N_h$ values (class A). It is interesting to note that the results regarding mean multiplicity, median emission angles of secondaries, transverse momentum, energy and coefficient of inelasticity behave in a similar manner in the above said classes of events. Results based upon multinucleon events are entirely different. One can explain the similar behaviour of class A and class C events on the assumption that the production of mesons and evaporation of the nucleus are two independent processes. The incoming pion interacts with a nucleon of the target nucleus, and moves with the ejected nucleon in an excited compound state which results in the emission of mesons. This process is separated in time from the evaporation of the nucleus which happens at a relatively later stage. Hence the number of mesons and slow prongs boiled off from the residual nucleus is uncorrelated and the behaviour of various parameters listed in table 2 should remain independent of the number of heavy prongs $N_h$ emitted in an interaction, as long as only one nucleon takes part in the interaction. Furthermore, the average multiplicity for such events having $M_T < 1M_N$ should remain constant irrespective of $N_h$ values. Fig. 4 shows the variation of $\langle n_g \rangle$ with $N_h$ for all the events having $M_T < 1M_N$. It is evident that the average multiplicity remains constant independent of $N_h$. The general behaviour of $\langle n_g \rangle$ with $N_h$ for all the 631 events has also been reported.

Various authors [9, 10] have used events with $N_h < 6$ with only one grey prong due to pure pion-nucleon interactions. We have stressed that the emission of heavy prongs has nothing to do with single pion-nucleon interaction. More important is that the effective target mass should not be more...
than one nucleon mass. However, it is true that the contribution of events with $M_T < M_N$ continues to decrease with increasing $N_h$ values. For events with $N_h < 1$, the number of events with $M_T < M_N$ is 88%, whereas for events with $N_h = 6$ its values is 71% and for events with $N_h > 7$, it is not more than 25%.

Recently, we have shown [2] that the calculations of Artykov et al. [1] based upon the internuclear cascade model do not agree with the experimental results based upon inelastic interactions of 17 GeV pions with heavy emulsion nuclei. The theoretical results are in good agreement with a restricted sample of interactions having $N_h \geq 8$. The theoretical and experimental results regarding mean multiplicity of shower and grey tracks, average momentum, transverse momentum and median angle of showers have been given in table 3.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle n_s \rangle$</td>
<td>7.1 ± 0.5</td>
<td>5.89 ± 0.3</td>
</tr>
<tr>
<td>$\langle n_g \rangle$</td>
<td>4.0 ± 0.4</td>
<td>2.3 ± 0.1</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>1.35 ± 0.14</td>
<td>1.81 ± 0.11</td>
</tr>
<tr>
<td>$\beta_{log}$</td>
<td>24.05 ± 20.3</td>
<td>22.9 ± 1.1</td>
</tr>
<tr>
<td>$P_{1x}$</td>
<td>0.39 ± 0.14</td>
<td>0.37 ± 0.02</td>
</tr>
</tbody>
</table>
The experimental values of \( \langle n_0 \rangle \), \( \langle n_\pi \rangle \) and \( \theta_1 \) are consistently lower than the expected theoretical values, whereas the predicted value of average momentum carried by shower tracks is smaller than the experimental value. The experimental and theoretical values of average transverse momentum are in good agreement with one another.

Figs. 2 and 3 show the dependence of \( \langle n_0 \rangle \), \( \langle n_\pi \rangle \) and \( \theta_1 \) with the effective target mass \( M_T \). The experimental values of \( \langle n_0 \rangle \), \( \langle n_\pi \rangle \) and \( \theta_1 \) given in table 3 correspond to an average effective target mass equal to 3.1 GeV (3.3 \( M_T \)). The theoretical values correspond to an average \( M_T \) which varies from 4.4 to 6.0 GeV. In order to modify the theoretical results of Artykov et al. and to match them with the experimental results, one should take into consideration an effect called 'trailing'. After the primary interaction, the fast moving secondaries leave a 'trail' of less dense nucleonic matter behind them. The time in which one produced fast secondary following behind the other is small compared with the diffusion time of the nucleons to come in the path of these secondaries. At 17 GeV, the median angle of pure pion-nucleon interaction is as small as 8° shows that a few shower particles are sufficient to clear up the cone and 'trailing' begins to play a dominant role resulting a smaller number of multinucleon interactions within the nucleus. The decrease of \( \langle n_\pi \rangle \) with increasing incident energy (table 1) may also be due to 'trailing'. Since the energy carried by the secondaries is quite large and the median angles are fairly small at these high energies, the internuclear cascade is expected to be confined to a very narrow cone and does not develop throughout the nucleus. It is suggested that, if the above said effects are taken into consideration, the theoretical results based upon internuclear cascade may agree with the experimental results.

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