Chapter 1

Introduction

1.1 Introduction

Software reliability has become an important area of research since software is an essential part of many industrial and commercial systems. Software reliability measure is a tool to evaluate software reliability engineering since there is no complete, scientific and quantitative measure to assess software. An important phase of software life cycle is the testing phase because a great deal of effort is put into this phase and majority of the cost is associated with this phase. One of the most challenging problems faced in the software industry is the development of reliable software, and software reliability is the most significant component of continuous application availability. Also, the important way of measuring the quality of the software is counting the number of remaining faults after the release, and many models are proposed for the estimation of this quantity. Software reliability is the
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probability that the software will be functioning without failure during a specified
period of time, under the given environmental conditions. Even for two identical
copies of the same software, the reliability may be different under different envi-
enments. The reliability may also vary from one machine to another, from one
time point to another and also from one software to another. The following are the
differences identified when considering software and hardware systems: (1) software
has no ageing property, (2) once a fault is removed from the software, there is no
chance of the occurrence of the failure again and (3) the execution of two copies from
a software program gives exactly the same result. Software reliability models play
an important role in developing software systems and enhancing the performance of
computer software. In general, software reliability models can be classified into two
types, depending on the operating domain. The most popular category of models
depends on failure time, which uses the concepts such as mean time between failures
and failure intensity function. The second category of software models measures re-
liability as the ratio of successful runs to the total number of runs. The intensity (or
failure rate) function plays a pivotal role in modelling software failure time data.

1.1.1 Existing models

Throughout the literature on failure time of software systems, certain parametric
models have been used such as the Rayleigh model introduced by [Schick and Wolverton, 1973],
exponential model introduced by [Moranda, 1975] and [Musa, 1985], and power
model introduced by [Crow, 1986]. These distributions have closed form expressions for tail area probability and simple formula for intensity functions. For more details on various parametric models, one could refer to [Lyu, 1996]. There are other approaches based on Markov process, non-homogeneous Poisson process, Bayesian techniques and software metric approach for modelling and analysis of software failure time data. The model classification by [Musa and Okumoto, 1983] allows the relationships to be established for models within the same classification groups and shows where model development has occurred. They classify models according to the following attributes (a) wall clock versus execution time, (b) the total number of failures experienced may be finite or infinite, (c) the two important types of distribution of the number of failure experienced by time \( t \) are Poison and Binomial, (d) functional form of the failure intensity experienced in terms of time and (e) functional form of the failure intensity function expressed in terms of the expected number of failures experienced. Exponential failure time class of models are Jelinski-Moranda de-eutrophication model, non homogeneous Poison process (NHPP) model, Shneidewind’s model, Musa’s basic execution model and Hyper-exponential model. Weibull model and S-shaped reliability growth model are the Weibull and Gamma failure time class of models. Infinite failure category models are Duane’s model, Geometric model, Musa-Okumoto logarithmic Poison model etc.. For various models using the above approaches, one could refer to [Xie, 1991]. In the distribution function approach, reliability measures such as hazard rate, mean
residual function and other higher moments of residual life are used for understanding the generating mechanism of the lifetime data and for distinguishing between various models through ageing properties. These measures have been widely used in the fields of reliability, income analysis, survival analysis and insurance. Some other models deal mainly with the inference problems based on the failure data and these models include Bayesian models and other statistical methods. There are several static models which do not take the dynamic aspect of the failure process into consideration, such as input-domain based models, seeding and tagging models and software complexity models. Recently [Huang et al., 2007], [Ahmad et al., 2008], [Ahmad et al., 2010] and [Kapur et al., 2011] have discussed the software reliability growth models.

The models described above are based on the distribution function of failure time and reliability measures derived from it. An alternative and equivalent approach for modelling statistical data is to use quantile function. Even though both the functions convey the same information about the distribution, the methodologies and concepts based on distribution function are more popular in practice. One of the reasons for the popularity of the distribution function approach is the inferential procedure due to maximum likelihood estimation, but for the inference purposes, quantile based estimates are more robust under censoring which is very common in reliability theory. There are many simple quantile functions for empirical model building where distribution functions are not effective. In such situations, conventional methods of analysis using distribution functions are not appropriate.
Random numbers from any distribution can be generated using appropriate quantile functions. The characteristics derived from quantile function are more applicable in modelling and data analysis. One reason for this is that quantile based studies are carried out mostly when the traditional approach fails to give results of the desired quality. In most of the cases of modelling and analysis, there has been no systematic and parallel development for replacing distribution functions by quantile functions. For the quantile function of order statistics, there are explicit general distribution forms.

1.1.2 Reliability and survival analysis

The concept of ageing has an important role in reliability analysis and in identifying life distributions. Ageing describes how a system improves or deteriorates with age. Many classes of life distributions are categorized or defined according to their ageing properties. An important aspect of such classifications is that the exponential distribution is a member of each class. The stochastic ageing plays an important role in any reliability analysis and many test statistics have been developed for testing exponentiality against different ageing alternatives. Ageing represents the phenomenon by which the residual life of a unit is affected by its age in some probabilistic sense. Most of the ageing concepts existing in the literature are described on the basis of measures defined in terms of the distribution function. For the lifetime data analysis, many quantile functions can be utilized. The existing definitions based on distribution function are not adequate to analyse the
ageing properties. In such situations, quantile based analysis plays an important role. A detailed study on quantile based analysis on ageing concepts is discussed in [Nair and Vineshkumar, 2011]. Various ageing concepts based on the reliability characteristics such as failure rate function, survival function and the mean residual life function are recorded in literature. The definitions and the properties of the basic ageing classes using the distribution function have been taken from [Lai and Xie, 2006].

1.1.3 Software reliability growth models

There are essentially two types of software reliability models. The first type of models attempt to predict software reliability from design parameters and are called defect density models. These models use code characteristics such as lines of code, nesting loops and external references etc., to estimate the number of defects in the software. The second type of models attempt to predict software reliability from test data. These models are called software reliability growth models, which attempt to statistically correlate defect detection data with known functions. If the correlation is good, the known function can be used to predict future behaviour of the system. The parameters of the software reliability growth models are related to the total number of defects contained in a set of code. If the parameter is known and the current number of defects is discovered, the residual defects can be identified, which helps us to decide about the shipment of the software and also the cost of supporting the document.
1.1.3.1 Software reliability data

There are two types of data for modelling software reliability growth models. The first one uses the failure time and the second one uses the number of failures. The different categories for measuring test time data are calendar time, number of tests runs and execution time. When the testing machines run continuously, test time can be measured as calendar time. The number of tests run would be a good measure if all tests had a similar probability of detecting a defect, but often this does not happen. However, the test effort is often asynchronous; execution time or number of tests run is normally used instead of calendar time.

1.1.3.2 Software reliability growth model types

Generally, there are two classes of software reliability growth models: one concave and the other S-shaped. Both these models have the same asymptotic behaviour. In other words, defect detection rate decreases as the number of defects detected and repaired increases, and the total number of defects detected asymptotically approaches a finite value. One of the reasons for asymptotic behaviour is that a finite number of codes would have a finite number of defects. Repair and new functionality tests may introduce new defects, which increase the original number of defects. Some models account these defects while others neglect them. Also, it is assumed that defect detection rate is proportional to the number of defects in the code and when a defect is repaired, the total number of defects in the code
decreases, and so defect detection rate decreases as the number of defects detected and repaired increases. The concave model strictly follows this pattern, but in the S-shaped model, it is assumed that the early testing is not as effective as the later testing. So, there is a ramp-up period during which the defect detection rate increases.

1.1.4 Quantile based reliability analysis

Even before the nineteenth century, researchers had used the quantile based measures in various applications of statistics. The Belgian scientist [Quetelet, 1846] initiated the use of inter-quartile range as a quantile based measure for statistical analysis. After that, researchers focused on different applications based on quantiles such as representation of distribution by quantile functions, estimation of parameters, use of different measures etc., [Galton, 1883], [Galton, 1889] and [Hastings et al., 1947] introduced a family of distributions by a quantile function, which led to the development of many quantile based families of distributions in the later period. [Parzen, 1979] emphasized the representation of a distribution in terms of quantile function and its role in data analysis and modelling. These were followed by [Parzen, 1991], [Parzen, 1996] and [Parzen, 2004] in different areas. [Gilchrist, 2000] systematically presented various properties of quantile function and its use in statistical modelling.
For a non-negative continuous random variable $X$ with right continuous distribution function $F(x)$, the quantile function of $X$ is defined as

$$Q(u) = F^{-1}(u) = \inf\{x : F(x) \geq u\}, \quad 0 \leq u \leq 1.$$  \hspace{1cm} (1.1)

For every $0 < x < \infty$ and $0 \leq u \leq 1$ we have $F(x) \geq u$ if and only if $Q(u) \leq x$. If there exists an $x$ such that $F(x) = u$, then $F(Q(u)) = u$ and $Q(u)$ is the smallest value of $x$ for which $F(x) = u$. Further if $F(x) = u$ is continuous and strictly increasing, $Q(u)$ is the unique value of $x$ such that $F(x) = u$. Random numbers from any distribution can be generated using appropriate quantile functions. One reason for this is that quantile based studies were carried out mostly when the traditional approach failed to give results of the desired quality. In most of the cases of modelling and analysis, there has been no systematic and parallel development for replacing distribution functions by quantile functions. For the quantile function of order statistics, there are explicit general distribution forms. Researchers like [Parzen, 1979] and [Gilchrist, 2000] have pointed out some distinct properties and characteristics of quantile function that are useful in reliability analysis. Recently, [Nair and Sankaran, 2009b] introduced the basic concepts in reliability theory in terms of quantile functions. The characteristics derived from quantile function are more applicable in modelling and data analysis. In reliability, a single long term survivor can have a marked effect on mean life, especially in the case of heavy tailed models which are very common. In such cases, quantile based estimates are generally found to be more precise and robust against outliers. In life testing experiments,
one need not conduct study until the failure of all the items, but only a percentage of them. In such contexts, quantile based approach provides efficient estimates for survival function. This calls for a quantile based approach. The existing quantile models like generalized lambda distribution [Ramberg and Schmeiser, 1974], generalized Tukey lambda family [Freimer et al., 1988] and the five parameter version in [Tarsitano, 2010] contain at least four parameters. Because of the high flexibility and difficulty in estimating the parameters, these models are not practical in real life applications. There are many distributions that are proposed, such as bathtub hazard rate data, in which the form for distribution functions has at least three parameters to estimate. The Jones distribution has only two parameters and the form of the quantile function makes applications and inference easier. Also the study of reliability properties and analysis becomes more practical. For more properties and applications of quantile functions in reliability analysis, one could refer to [Nair et al., 2008], [Nair and Vineshkumar, 2010], [Nair and Vineshkumar, 2011], [Midhu et al., 2013], [Nair et al., 2013] and [Midhu et al., 2014].

1.2 Present study

The study of the reliability properties associated along with probability distribution function $F(x)$, density function $f(x)$ and survival function $\bar{F}(x) = 1 - F(x)$ along with various other characteristics such as failure rate, mean, percentiles, higher moments of residual life, etc., is used for understanding how the failure time data arises
in practice. A systematic study on the application of quantile function in reliability studies has been carried out by [Nair and Sankaran, 2009b]. They have discussed commonly used reliability measures in terms of quantile functions and have derived various relationships connecting them. They have also analysed the quantile function model discussed in [Hankin and Lee, 2006] in the context of reliability analysis. Our present work extends these ideas to develop the necessary theoretical framework for modelling and data analysis of software reliability data based on quantile functions. This new approach provides us an alternative methodology and new models that have desirable properties. In this thesis, we study reliability analysis of software data using quantile functions. We introduce quantile functions, those are used in modelling and analysis of software data.

After this introductory chapter, the rest of this thesis is organized in five chapters. In Chapter 2 we give a brief discussion of definitions and properties of quantile functions. We also present reliability measures like hazard function and mean residual life function based on distribution function approach as well as quantile function approach. We discuss L-moments which are alternative to conventional moments and have several advantages over usual moments. Different reliability characteristics based on distribution function and their equivalent quantile based functions are also discussed. The concept of total time on test transform (TTT) and its relationship with other reliability measures are also presented.

In Chapter 3 we introduce a software reliability model using quantile function and study its properties. It is observed that the proposed class has several desirable
properties, and several existing well known distributions are members of the class of distributions as special cases or through approximations. Various reliability characteristics are discussed. Some of the methods of estimation like least squares, maximum likelihood and matching moments often provide estimators and/or their standard errors in terms of moments. Outliers have a significant effect on the estimates so derived. For example, in the case of samples from the normal distribution, all the above methods give the sample mean as the estimate of the population mean, whose values change significantly in the presence of an outlying observation. In this context, the parameters of the model are estimated using L-moments and the model is applied to a real data set. The approximations to two well known distributions, ie, inverse Gaussian distribution and Weibull distribution, are also carried out.

There are several reliability measures in literature to describe the patterns of failure of systems. One of the popular measures is hazard rate function. Many of the lifetime models based on quantile functions do not have explicit expressions for hazard rate function, and so those models cannot be employed for the analysis of lifetime data. Hazard quantile function which is equivalent to hazard rate function is used in such situations. Accordingly in Chapter 4, we present a class of distributions based on the hazard quantile function and study its properties. Several existing well known lifetime distributions are members of the class of distributions as special cases or through approximations. Various reliability characteristics are discussed. The parameters of the model are estimated using the method of L-moments and the model is applied to a real data set. Non parametric estimator of $H(u)$ given in
[Nair and Sankaran, 2009a] can be employed in practice to identify the approximate lifetime model for a given dataset.

We discuss a family of distributions having inverse linear form for mean residual quantile function $M(u)$ and study its properties in Chapter 5. The proposed class has several desirable properties, and various reliability characteristics are discussed. Several existing well known distributions are members of the class of distributions as special cases or through approximations. We also derive useful characterizations connecting identities among mean residual quantile function $M(u)$, hazard quantile function $H(u)$, the variance residual quantile function $V(u)$ and quantile based total time on test transform (TTT) $T(u)$. The parameters of the model are estimated using L-moments and the model is applied to two real data sets.

Finally, we summarize major conclusions of the present work in Chapter 6. We also discuss future work that originates from the present work, to be carried out in this area.