Chapter 3

A software reliability model using quantile function

3.1 Introduction

Software reliability models play an important role in developing software systems and enhancing the performance of computer software. An introduction and a brief history to the software reliability are given in Chapter 1. The models described in the introductory chapter are based on distribution function of failure time of the software system. An alternative and equivalent approach for modelling statistical data is to use quantile function. Even though both the functions convey the same information about the distribution, the methodologies and concepts based on distribution function are more popular in practice. However, quantile functions have

\footnote{Part of the contents of this chapter has been published in Journal of Probability and Statistics, Hindwai Publishing Corporation by Bijamma Thomas, Midhu N.N. and P.G. Sankaran (2014).}
several properties that are not shared by distributions, which makes it more convenient for analysis. There are explicit general distribution form for the quantile function of order statistics. There are many simple quantile functions which are very good in empirical model building where distribution functions are not effective. In such situations conventional methods of analysis using distribution functions are not appropriate.

For various properties and applications of quantile functions, we refer to [Parzen, 1979], [Gilchrist, 2000], [Sarabia, 2008] and [Sarabia et al., 2010]. Recently [Nair and Sankaran, 2009b] introduced the basic concepts in reliability theory in terms of quantile functions. The existing quantile models like generalized lambda distribution [Ramberg and Schmeiser, 1974], generalized Tukey lambda family [Freimer et al., 1988] and the five parameter version in [Tarsitano, 2010] contain at least four parameters. Because of the high flexibility and difficulty in estimating the parameters, these models are not practical in real life applications. There are many distributions that are proposed as bathtub hazard rate data, the form for distribution functions have at least three parameters to estimate. The Jones distribution has only two parameters and the form of the quantile function make applications and inference easier. Also the study of reliability properties and analysis becomes more practical. For more properties and applications of quantile functions in reliability analysis one could refer to [Nair et al., 2008], [Nair and Vineshkumar, 2010], [Nair and Vineshkumar, 2011], [Midhu et al., 2013], [Midhu et al., 2014] and [Nair et al., 2013]. Thus there is a case for adopting quantile functions as models of lifetime and base their analysis with the aid of functions.
A software reliability model using quantile function

derived from them. Motivated by this facts, in the present work, we introduce a
class of software reliability models using quantile function.

The rest of the chapter is organized as follows. In Section 3.2 we present model and
the properties of the model. The proposed class of quantile functions have several
desirable distributional properties. The existing well known lifetime models are
members of the class as special cases or through approximations. Various reliability
characteristics of the model are discussed in Section 3.3. The proposed class is a
family of flexible lifetime models as it can be used for modelling and analysis of
lifetime data having different ageing criteria by choosing different combinations of
parametric values. Approximation to some well known distributions are carried out
in Section 3.4. The inference on parameters of the model is discussed in Section
3.5. We also apply the model to a software failure time data. Section 3.6 provides
brief conclusion of the study.

3.2 The model and properties

Let $X$ be a non-negative continuous random variable representing the failure time
of a software with right continuous distribution function $F(x)$. Then the quantile
function of $X$ is defined as

$$Q(u) = F^{-1}(u) = \inf\{x : F(x) \geq u\}, 0 \leq u \leq 1. \quad (3.1)$$
For every $0 < x < \infty$ and $0 \leq u \leq 1$ we have $F(x) \geq u$ if and only if $Q(u) \leq x$. Thus if there exists an $x$ such that $F(x) = u$ then $F(Q(u)) = u$ and we have $Q(u)$ is the smallest value of $x$ for which $F(x) = u$. Further if $F(x) = u$ is continuous and strictly increasing $Q(u)$ is the unique value of $x$ such that $F(x) = u$. Then we can find $x$ in terms of $u$ which is the quantile function of $X$.

If $f(x)$ is the probability density function of $X$, then $f(Q(u))$ is called the density quantile function. The derivative of $Q(u)$ is expressed as $Q'(u) = q(u)$ which is known as the quantile density function of $X$. Differentiating $F(Q(u)) = u$ we have

$$f(Q(u))q(u) = 1$$

(3.2)

where $f(x)$ is the density of $X$. Now we consider a class of distributions defined by the quantile density function

$$q(u) = ku^a(1 - u)^b.$$  

(3.3)

This distribution satisfied the general properties like symmetry, modality, tail behaviour, order statistics, shape properties based on the mode, L-moments and transformations between members of the family. This class of distribution in (3.3) is same as in [Jones, 2007].
Quantile function $Q(u)$ is defined in terms of quantile density function as

$$Q(u) = \int_{0}^{u} q(u) du.$$  

Quantile function for the class of distributions in (3.3) can be obtained as

$$Q(u) = \int_{0}^{u} ku^a(1-u)^b du.$$  

$$Q(u) = k\beta(u, a + 1, b + 1)$$  \hspace{1cm} (3.4)

$k > 0$, $a$ and $b$ are real, $0 \leq u \leq 1$. where $\beta(u, a + 1, b + 1)$ is the incomplete beta function with parameters $a$ and $b$. The support of the distribution (3.4) is $(Q(0), Q(1)) = (0, \beta(1, a + 1, b + 1))$.

The derivative of (3.3) gives

$$q'(u) = k[u^a b(1-u)^{b-1} -1 + (1-u)^b au^{a-1}].$$

Equating $q(u) = 0$ and solving for $u$ we get $u = \frac{a}{a+b}$. Thus the members of the family has either uni modal density with mode(anti mode) at $u = \frac{a}{a+b}$ or monotone density when $u \neq \frac{a}{a+b}$. Figure 3.1 gives the quantile density function for different values of parameters.
The quantile function defined in (3.4) is become exponential when \( a = 0 \) and \( b = -1 \), that is, here,

\[
Q(u) = \int_0^u \frac{k}{1-u} du
\]

or

\[
Q(u) = -\log(1-u)
\]

, exponential. and linear hazard quantile distribution, when \( a = 0 \) and \( b = -2 \), that is, here from (2.19)

\[
H(u) = \frac{1-u}{k}
\]

, which is a linear function in \( u \). Similarly when \( a = -1 \) and \( b = -1 \) the quantile function defined in (3.4) become linear hazard quantile distribution, see [Midhu et al., 2014].
The distributional characteristics such as location, dispersion, skewness and kurtosis can be expressed through quantile terms. For the class of distributions in (3.4) the quantile based measure of location is the median defined by

$$M = Q\left(\frac{1}{2}\right) = \beta(1,a+1,b+1).$$

Dispersion is measured by the inter quartile range which is expressed as

$$IQR = Q\left(\frac{3}{4}\right) - Q\left(\frac{1}{4}\right) = \beta\left(\frac{3}{4},a+1,b+1\right) - \beta\left(\frac{1}{4},a+1,b+1\right).$$

The Galton’s coefficient of skewness is obtained from (3.4) as

$$S = \frac{\left[Q\left(\frac{3}{4}\right) + Q\left(\frac{1}{4}\right) - 2M\right]}{IQR}$$

which can also be expressed as

$$S = \frac{\beta\left(\frac{3}{4},a+1,b+1\right) + \beta\left(\frac{1}{4},a+1,b+1\right) - 2\beta\left(\frac{1}{2},a+1,b+1\right)}{\beta\left(\frac{3}{4},a+1,b+1\right) - \beta\left(\frac{1}{4},a+1,b+1\right)}.$$

In the case of extreme positive skewness $Q\left(\frac{1}{4}\right) \to M$ and in the extreme negative skewness $Q\left(\frac{3}{4}\right) \to M$ so that $S$ lies between $-1$ and $+1$. When the distribution is symmetric $M = \frac{Q\left(\frac{1}{4}\right) + Q\left(\frac{3}{4}\right)}{2}$ and hence $S=0$. The measure of kurtosis can be defined as
A software reliability model using quantile function

\[
T = \frac{[Q(\frac{3}{5}) - Q(\frac{5}{8}) + Q(\frac{3}{8}) - Q(\frac{1}{8})]}{\text{IQR}}.
\]

For the family of distributions in (3.4) \( T \) is given by

\[
T = \frac{\beta(\frac{7}{8}, a + 1, b + 1) - \beta(\frac{5}{8}, a + 1, b + 1) + \beta(\frac{3}{8}, a + 1, b + 1) - \beta(\frac{1}{8}, a + 1, b + 1)}{\beta(\frac{3}{4}, a + 1, b + 1) - \beta(\frac{1}{4}, a + 1, b + 1)}.
\]

The L-moments are often found to be more desirable than the conventional moments in describing the characteristics of the distributions as well as for inference. The L-moments exists whenever \( E(X) \) is finite, where as for many distributions additional restrictions are required for the conventional moments to be finite. L-moments generally score over the usual moments in providing smaller sampling variance, robustness against outliers and easier characterization of distributional characteristics, especially for quantile functions that have no tractable distribution functions, for more details one could refer to [Nair and Sankaran, 2009b]. The L-moments have generally lower sampling variances and robust against outliers. See [Hosking, 1996] and [Sankarasubramanian and Srinivasan, 1999] for details. The first L-moment is the mean which is given by

\[
L_1 = \int_0^1 (1 - u)q(u)du
\]
\[
\begin{align*}
\int_0^1 ku^a(1-u)^{b+1}du &= k\beta(a + 1, b + 2),
\end{align*}
\]
where \(\beta(\alpha, \beta)\) is the beta function with parameters \(\alpha > 0\) and \(\beta > 0\).

The second L-moment can be expressed as

\[
L_2 = \int_0^1 (u - u^2)q(u)du
\]

\[
= \int_0^1 (u - u^2)ku^a(1-u)^{b}du
\]

\[
= \int_0^1 ku^{a+1}(1-u)^{b+1}du
\]

\[
= k\beta(a + 2, b + 2).
\]

The third L-moment is obtained as

\[
L_3 = \int_0^1 (3u^2 - 2u^3 - u)q(u)du
\]
A software reliability model using quantile function

\[
k \int_0^1 u(1-u)(2u-1)u^a(1-u)^b \, du
\]

\[
= k \int_0^1 (2u-1)u^{a+1}(1-u)^{b+1} \, du
\]

\[
= k \left( 2 \int_0^1 u^{a+2}(1-u)^{b+1} \, du - \int_0^1 u^{a+1}(1-u)^{b+1} \, du \right)
\]

\[
= 2k\beta(a+3, b+2) - k\beta(a+2, b+2)
\]

and the fourth L-moment is given by

\[
L_4 = \int_0^1 (u - 6u^2 + 10u^3 - 5u^4)q(u)\,du
\]

\[
= \int_0^1 u(1-u) (5u^2 - 5u + 1) \, q(u)\,du
\]

\[
= k \int_0^1 (5u^2 - 5u + 1) u^{a+1}(1-u)^{b+1} \, du
\]

\[
= k \int_0^1 (1 - 5u(1-u))u^{a+1}(1-u)^{b+1} \, du
\]
A software reliability model using quantile function

\[
\begin{align*}
&\quad = k \int_0^1 u^{a+1}(1-u)^{b+1} du - 5k \int_0^1 u^{a+2}(1-u)^{b+2} du \\
&\quad = k\beta(a+2, b+2) - 5k\beta(a+3, b+3).
\end{align*}
\]

L-coefficient of variation, analogous to the coefficient of variation based on ordinary moments is obtained as

\[
\tau_2 = \frac{L_2}{L_1} = \frac{k\beta(a+2, b+2)}{k\beta(a+1, b+2)}
\]

\[
= \frac{\Gamma a + 2\Gamma b + 2/\Gamma a + b + 4}{\Gamma a + 1\Gamma b + 2/\Gamma a + b + 3} = \frac{a + 1}{a + b + 3}.
\]

The L-coefficient of variation is 0 when \(a = -1\) and it has the upper bound 1 when \(b = -2\) and \(a\), any real number. The L-coefficient of skewness is measured as

\[
\tau_3 = \frac{L_3}{L_2} = \frac{2k\beta(a+3, b+2) - k\beta(a+2, b+2)}{k\beta(a+2, b+2)}
\]

\[
= \frac{2\Gamma a + 3\Gamma b + 2/\Gamma a + b + 5}{\Gamma a + 2\Gamma b + 2/\Gamma a + b + 4} - 1.
\]
The L-coefficient of skewness lies between $(-1, 1)$, that is, $\tau_3 = -1$ when $a = -2$ and $\tau_3 = 1$ when $b = -2$. The L-coefficient of kurtosis is given by

$$\tau_4 = \frac{L_4}{L_2} = \frac{k \beta(a + 2, b + 2) - 5k \beta(a + 3, b + 3)}{k \beta(a + 2, b + 2)}$$

$$= 1 - \frac{5 \Gamma a + 3 \Gamma b + 3 / \Gamma a + 6}{\Gamma a + 2 \Gamma b + 2 / \Gamma a + 4}$$

$$= 1 - \frac{5(a + 2)(b + 2)}{(a + b + 5)(a + b + 4)}.$$ 

The L-coefficient of kurtosis attains the upper bound 1 when $a = -2$ or $b = -2$ and has the lower bound $-1/4$.

### 3.3 Reliability characteristics

The reliability is the probability that a system or unit will perform the required function under the conditions specified for its operations for a given period of time. The primary concern in reliability theory is to understand the patterns in which failures occur, for different mechanisms and under varying operating environments,
as a function of its age. This is accomplished by identifying the probability distribution of the lifetime represented by a non-negative random variable $X$. Accordingly, several concepts have been developed that help in evaluating the effect of age, based on the distribution function of the lifetime random variable $X$ and the residual life. One of the important concepts in reliability analysis is the hazard function which is defined in Chapter 2. [Nair and Sankaran, 2009b] studied the quantile form of the hazard function termed as hazard quantile function which is given by

$$H(u) = \frac{1}{(1 - u)q(u)}. \quad (3.5)$$

$H(u)$ can be interpreted as the conditional probability of the failure of a unit in the next small interval of time given the survival of the unit at 100(1-u)% point of the distribution.

For the class of distributions (3.3), we have

$$H(u) = \frac{1}{ku^a(1 - u)^{b+1}}. \quad (3.6)$$

Plots of the hazard quantile functions for different values of parameters are presented in Figure 3.2.

The shape of the hazard function is determined by the derivative of $H(u)$. For the hazard quantile function of (5.15) the derivative is given by
Since $k > 0$, $H(u)$ changes sign according to the term $g(u) = [(a + b + 1)u - a]$.

The sign of $g(u)$ changes according to the values of $a$ and $b$. The following are the different cases corresponding to the different values of $a$ and $b$.

Case 1: If $a < 0$ and $b > -(a + 1)$, $g(u)$ is positive and hence $H(u)$ has increasing failure rate.
Case 2: If $a > 0$ and $b < -(a + 1)$ then $g(u)$ is negative and then $H(u)$ has decreasing failure rate.

Case 3: If $a = 0$ and $b < -1$ then $g(u)$ is negative which leads that $H(u)$ has decreasing failure rate.

Case 4: When $a = 0$ and $b > -1$, then $g(u)$ is positive and hence $H(u)$ has increasing failure rate.

Case 5: When $a < 0$ and $-1 \leq b < -(a + 1)$, $g(u)$ is positive and $H(u)$ has increasing failure rate.

Case 6: If $a > 0$ and $(a + 1) < b \leq -1$, $g(u)$ is negative. Thus it follows that $H(u)$ has decreasing failure rate.

Case 7: When $a < 0$ and $b < -1$, $H(u)$ increases up to a maximum at $u = a/(a + b + 1)$ and then decreases. So $H(u)$ is Upside Bathtub here and

Case 8: When $a > 0$ and $b > -1$, $H(u)$ decreases to a minimum at $u = a/(a+b+1)$ and then increases. So $H(u)$ is Bathtub.

Mean residual function is a well known measure, which has been widely used in various fields of reliability and survival analysis. In quantile set up the mean residual quantile function is expressed as

$$M(u) = (1 - u)^{-1} \int_u^1 [Q(p) - Q(u)]dp.$$
The above identity can also be expressed as

\[ M(u) = \frac{1}{1 - u} \int_u^1 (1 - p)q(p)dp. \]

\( M(u) \) is interpreted as the average remaining life beyond the \( 100(1-u)\% \) of the distribution. For our class of distributions \( M(u) \) is obtained as

\[ M(u) = \frac{1}{1 - u} \int_u^1 kp^a(1 - p)^{b+1}dp \]

\[ = \frac{1}{1 - u} \beta(p, a + 1, b + 2). \]

Plots of the mean residual functions for different values of parameters are presented in Figure 3.3.

![Figure 3.3: Plots of \( M(u) \) for different values of parameters](image-url)
3.4 Approximations

Since the class of distributions (3.4) can not be converted to a tractable form for its distribution function, its relationship with other known standard distributions can be assessed only through approximations. When we have a quantile function that provides approximation to many type of distributions, only one functional form for $Q(u)$ and the related inferential aspects are sufficient for modelling and analysis, as the quantile function will adapt automatically to the suitable form. In this section we attempt to fit our distribution to some well-known lifetime distributions like inverse Gaussian distribution and Weibull distribution. The same approach can be used to approximate other distributions. We use the method of L-moments for finding the values of the parameters.

3.4.1 Inverse Gaussian distribution

Probability density function of inverse Gaussian is given by

$$f(x) = \frac{e^{-\frac{(x-\mu)^2}{2\lambda}}}{\sqrt{2\pi x}} \frac{\lambda}{\sqrt{x}}.$$ 

The inverse Gaussian distribution with parameters $\mu$ and $\lambda$ has L-moments $L_1 = \mu$, $L_2 = \frac{\mu^3}{\lambda}$ and $L_3 = 3\sqrt{\frac{\mu}{\lambda}}$. We equate L-moments of inverse Gaussian distribution with the L-moments of the proposed model. There is no explicit closed form expression for $k$, $a$ and $b$ in terms of of $\mu$ and $\lambda$. However, as an illustration we assume
μ = 1 and λ = 3 and the corresponding values of (3.4) is k = 0.2960, a = −0.6683 and b = −1.22. Figure 3.4 gives the probability density function (p.d.f) of the inverse Gaussian distribution and the dotted line represents that of the approximated quantile function. For measuring the closeness between the two models, we find

\[ \sup_{x} |F_1(x) - F_2(x)|, \]

where \( F_1(x) \) is the distribution function of the proposed model and \( F_2(x) \) is the distribution function of the inverse Gaussian distribution. Now we obtain

\[ \sup_{x} |F_1(x) - F_2(x)| = 0.02203 \]

which is very small.

\[ f(x) = \frac{e^{-\left(\frac{x}{\beta}\right)^\alpha}}{\beta^\alpha (\frac{x}{\beta})^{-1+\alpha}}. \]

\[ f(x) \]

**Figure 3.4:** Probability density functions of Inverse Gaussian and it’s approximation

### 3.4.2 Weibull distribution

The probability density function of Weibull distribution is given by
The Weibull distribution with parameters $\alpha$ and $\beta$ has L-moments $L_1 = \beta \Gamma(1 + \frac{1}{\alpha})$, $L_2 = \beta(1 - 2^{-\frac{1}{\alpha}}) \Gamma(1 + \frac{1}{\alpha})$ and $L_3 = \frac{3 - 2(1 - 3^{-\frac{1}{\alpha}})L_2}{(1 - 2^{-\frac{1}{\alpha}})}$. We equate L-moments of Weibull distribution with the L-moments of the proposed model. There is no explicit closed form expression for $k$, $a$ and $b$ in terms of $\alpha$ and $\beta$. However, as an illustration we assume $\alpha = 0.5$ and $\beta = 2$ and the corresponding values of (3.4) is $k = 5.1613$, $a = 1.142$ and $b = -1.2857$. Figure 3.5 gives p.d.f of the Weibull distribution and the dotted line represents the p.d.f of approximated quantile function. For measuring the closeness between the two models, we find $\sup_x |F_1(x) - F_3(x)|$, where $F_1(x)$ is the distribution function of the proposed model and $F_3(x)$ is the distribution function of the Weibull distribution. Now we obtain $\sup_x |F_1(x) - F_3(x)| = 0.0072$ which is also very small.

![Figure 3.5: Probability density functions of Weibull and it’s approximation](image-url)
3.5 Estimation of parameters and data analysis

For estimating the parameter of the distributions, which are expressed in terms of quantile function, there are different methods available (see Gilchrist, (2000)). Among these different methods, the commonly used techniques are method of minimum absolute deviation, method of least squares, method of maximum likelihood and method of L-moments. Recently method of L-moments is widely used as an alternative to the conventional methods in inference problems in view of the robustness in the estimates produced (see Hoskings, (1996)). To estimate the parameters of the function given in (3.4) we use the method of L-moments. The simple algebraic expressions of the L-moments explained in Section 3.2 admit the applicability of the L-moments method for estimating the parameters of the model (3.4). Let \( x_1, x_2, \ldots, x_n \) be a random sample of size \( n \) with quantile function (3.4). Since there are three parameters in the model, we take three sample L-moments \( l_r, r=1,2,3 \) those are given by

\[
l_1 = \left( \binom{n}{1} \right)^{-1} \sum_{k=1}^{n} x(k) \quad (3.7)
\]

\[
l_2 = \frac{1}{2} \left( \binom{n}{2} \right)^{-1} \sum_{k=1}^{n} \left\{ \left( \binom{k-1}{1} - \binom{n-k}{1} \right) \right\} x(k) \quad (3.8)
\]
and

\[
l_3 = \frac{1}{3} \left( \begin{array}{c} n \\ 3 \end{array} \right)^{-1} \sum_{k=1}^{n} \left\{ \left( \begin{array}{c} k - 1 \\ 2 \end{array} \right) - 2 \left( \begin{array}{c} n - k \\ 1 \end{array} \right) \left( \begin{array}{c} k - 1 \\ 1 \end{array} \right) + \left( \begin{array}{c} n - k \\ 2 \end{array} \right) \right\} x_{(k)}
\]

where \( x_k \) is the \( k \)th order statistic. We equate sample L-moments to population L-moments given by

\[
l_r = L_r : r = 1, 2, 3.
\]

Solutions of set of equations (3.10) give the estimates of \( k \), \( a \) and \( b \). The set of equations (3.10) are non-linear in \( k \), \( a \) and \( b \). We use Newton Raphson method to find the values of \( k \), \( a \) and \( b \).

Hosking(1990) has studied asymptotic properties of L-moment estimates. The following theorem provides asymptotic normality of sample L-moments.

**Theorem 3.1** (Hosking(1990)). *Let \( X \) be a real-valued random variable with quantile function \( Q(u, \theta) \), where \( \theta \) is a vector of \( m \) parameters. Assume that variance of \( X \) is finite. Let \( l_r, r = 1, 2, \ldots, m \) be sample L-moments calculated from a random sample of size \( n \) drawn from the distribution of \( X \). Then \( \sqrt{n}(l_r - L_r), r = 1, 2, \ldots, m \), converge in distribution to the multivariate normal \( N(0, \Lambda) \), where the elements...*
$\Lambda_{rs}(r, s = 1, 2, \ldots, m)$ of $\Lambda$ are given by

$$
\Lambda_{rs} = \int_{0}^{1} \int_{0}^{1} \left\{ P_{r-1}^{*}(u)P_{s-1}^{*}(v) + P_{s-1}^{*}(u)P_{r-1}^{*}(v) \right\} u(1-v)q(u)q(v)dudv,
$$

where $P_{r}^{*}(x)$ is the $r$th shifted Legendre polynomial defined by

$$
P_{r}^{*}(x) = \sum_{k=0}^{r} (-1)^{r-k} \binom{r}{k} \binom{r+k}{k} x^{k}.
$$

Since the set of equations (3.10) are non-linear in k, a and b asymptotic distributions of L-moment estimates of the parameters are difficult to obtain. One can use bootstrap method to obtain the asymptotic variance of the estimates.

Now we apply the model (3.4) to a real dataset taken from [Musa, 1980]. The dataset represents failure time in seconds of a command-and-controlling system during in-house testing using a simulation of the real operational environment. The dataset consist of failure time for a sample of size 136. [Musa, 1980] fitted a software reliability growth model to the data. The proposed model (3.4) fitted to this dataset.

We estimated the parameters using the method of L-moments and the estimates are given as $\hat{a} = 0.484752$, $\hat{b} = -1.21617$ and $\hat{k} = 712.679$.

Since the estimate $a > 0$ and $b < 0$, $H(u)$ has decreasing failure rate as shown in Figure 3.6.
The plot of mean residual quantile function $M(u)$ corresponding to the data set is given in Figure 3.7. From the figure it is clear that $M(u)$ is increasing in $u$.

To check the goodness of fit, we use Q-Q plot which is given in Figure 3.8. The Figure 3.8 shows that most of the data points are close to the straight line. This indicates that the quantile function given in (3.4) is a reasonable fit to the data.
We also employed chi-square goodness of fit to check the adequacy of the model. The chi-square statistic value is 8.73 with p-value 0.891. This indicate that the proposed model is a reasonable one for the given data set.

3.6 Conclusion

A probability distribution can be specified either in terms of the distribution function or by quantile function. Although both convey the same information about the distribution, with different interpretations, the concepts and methodologies based on distribution functions are traditionally employed in most forms of statistical theory and practice. One reason for this is that quantile based studies were carried out mostly when the traditional approach fails to provide results of desired
quality. Except in a few isolated areas, there have been no systematic parallel developments aimed at replacing distribution functions in modelling and analysis by quantile functions. However, the feeling that through an appropriate choice of the domain of observations, a better understanding of a chance phenomenon can be achieved, is fast gaining acceptance.

Motivated by this fact, in the present work, we have introduced a class of quantile function models, useful in software reliability analysis. The proposed class has several desirable properties and several existing well known distributions are members of the class of distributions as special cases or through approximations. Various reliability characteristics were discussed. The parameters of the model were estimated using L-moments and the model was applied to a real data set. The method of maximum likelihood can also be employed to find the estimates of the parameters (see [Gilchrist, 2000]). The proposed class of quantile functions is a flexible model in the sense that it has the property of increasing hazard rate and decreasing hazard rate, bathtub hazard rate and upside down hazard rate by changing the parametric values.