CHAPTER V
TEMPERATURE DEPENDENT ANOMALIES

IN THE GYROTROPIC TENSOR

NEAR A PHASE TRANSITION
Since Joffrin and Levelut and Pines produced observational evidence for acoustic activity in trigonal quartz, rotary activity in crystals has been investigated by several authors (Pine and Dresselhane 1969, Portigal et al. 1968, Joffrin et al. 1980, Abraham et al. 1979, Lin et al. 1985, Kumaraswamy and Krishnamurthy, 1982). Rotary activity in crystals arises from spatial dispersion, which makes the compliance matrix nonsymmetric. In the absence of spatial dispersion, shear waves propagating along the z axis in trigonal quartz are degenerate but spatial dispersion splits the degeneracy leading to nondegenerate circularly polarized waves propagating with different velocities along the z axis. As a result of spatial dispersion, the elastic constant matrix has the structure

\[ C_{ij}(\omega, k) = C_{ij}(\omega) + i d_{ijl}(\omega, k) k_l \] (1)

As stated in chapter II, the gyrotropic constants \(d_{ijl}\) are components of a fifth rank tensor. The number of distinct constants for different classes of crystal symmetry have been worked out by Kumaraswamy and Krishnamurthy (1980). For the orthorhombic class, the number
is 11 for $C_{2v}$ and 12 for $D_2$ point groups; and for the cubic class, the number is 9 for $O$ and $T_d$ point groups and 12 for $T$ class of crystals.

Gyrotropic effects can manifest themselves indirectly during a phase transition if there is a coupling between the order parameter and the derivatives of the strain. The coupling of the order parameter with the strains and their spatial derivatives could simulate gyrotropic effects.

We show in this chapter that anomalous changes can take place in the gyrotropic coefficients during a phase transition and these are strongly temperature dependent. We have given the expression for the anomalies in the components of the gyrotropic tensor near the phase transition both above and below the transition temperature for crystals having a linear coupling of the order parameter with the strain, as in the case of KDP. It is seen that the anomalies are strongly temperature dependent diverging as $(T - T_c)^{-1}$ in the high temperature region and have terms depending on temperature as $(T_c - T)^{-1}$ as well as $(T_c - T)^{-1/2}$ in the low temperature phase. We have also given expressions for the polarization angle of the circularly polarized wave propagating along the $c$ axis of the crystal. It is shown that
by studying the rotary activity below the transition temperature the coupling constant in the free energy of the crystal can be evaluated easily.

5.2 The Free Energy Of The Crystal

The free energy of the crystal may be written as a sum of three terms namely, the elastic energy $F_{el}$, the Landau energy $F_Q$ in the order parameter and the coupling energy $F_C$ between the order parameter and the strain and its derivative. We write

$$F = F_{el} + F_Q + F_C$$

The elastic energy $F_{el}$ consists of the second order terms in the strain variables. We shall neglect the third and higher order terms in the elastic energy. If the spatial dispersion is present in the system, the deformation energy will consist of an additional term involving the derivatives of the strain coefficients. We therefore write for $F_{el}$ for a tetragonal crystal:
\[ F_{el} = \frac{1}{2} \int \left( C_{11} \left( \gamma_1^2 + \gamma_2^2 \right) + C_{33} \gamma_3^2 + 2 C_{12} \gamma_1 \gamma_2 + \right. \]
\[ \left. C_{44} \left( \gamma_4^2 + \gamma_5^2 \right) + C_{66} \gamma_6^2 \right) + \]
\[ \frac{\varepsilon}{\bar{\varepsilon}} \frac{\varepsilon}{\bar{\varepsilon}} \int d \vec{k} m \left( \frac{d \gamma_k}{d x_m} \right) \gamma_l, \quad (3) \]

where \( \gamma_1 (\gamma_1) \), \( \gamma_2 (\gamma_2) \), \( \gamma_3 (\gamma_3) \) are the pure strain coefficients; whereas \( \gamma_4 (\gamma_4) \), \( \gamma_5 (\gamma_5) \), \( \gamma_6 (\gamma_6) \) are the shearing strain variables. Near the phase transition, the new parameter \( Q \) emerges. We shall consider a coupling in which the dominant term is the linear coupling between \( Q \) and \( \gamma_6 \).

For KH\textsubscript{2}PO\textsubscript{4} and similar crystals, \( P_z \) and \( \gamma \) belong to the same irreducible representation and the coupling is linear. The part of the free energy \( F_Q \) corresponding to this is a power series in \( Q^2 \). We shall stop here with the fourth power term in \( Q \) and write

\[ F_Q = \frac{1}{2} \mathcal{L} Q^2 + \frac{1}{4} \beta Q^4 \quad (4) \]

where \( \mathcal{L} = \mathcal{L} (T - T_c) \) \( (5) \)

The coupling energy \( F_C \) consists of terms that couple the order parameter with the strain coefficients as well as their
derivatives. We consider a coupling energy of the following form:

\[ F_c = B \left( \gamma_1 + \gamma_2 + \gamma_3 \right) \omega^2 + \epsilon \omega \gamma_6 + \]

\[ h \omega \left( \frac{d \gamma_1}{dx_3} + \frac{d \gamma_2}{dx_3} + \frac{d \gamma_3}{dx_3} \right) \]

\[ + k \omega \left( \frac{d \gamma_4}{dx_2} + \frac{d \gamma_5}{dx_1} \right) - \int h \left( \gamma_1 + \gamma_2 + \gamma_3 \right) \frac{d \omega}{dx_3} + k \left( \gamma_4 \frac{d \omega}{dx_2} + \gamma_5 \frac{d \omega}{dx_1} \right) \]  \hspace{1cm} (6)

As explained in the chapter III the above expression is a Taylor expansion of the coupling energy in terms of the strain coefficients, derivatives of the strain coefficients and the order parameter. Besides, it is chosen such that it is the sum of invariants under the symmetry group of the tetragonal class. It can serve as a model for certain ferroelectric phase transitions for crystals like KDP. For these, the order parameter \( Q \) is the electrical polarization \( P_z \); the coupling energy is bilinear in \( Q \) as well as in \( \gamma_6 \) and has the form (Lemanov, 1982)

\[ f_c = A P_z \gamma_6 \]  \hspace{1cm} (7)
The terms

\[ h \otimes \left( \frac{d\eta_1}{dx_3} + \frac{d\eta_2}{dx_3} + \frac{d\eta_3}{dx_3} \right) \text{ and } k \otimes \left( \frac{d\eta_4}{dx_2} + \frac{d\eta_5}{dx_1} \right) \]

have been chosen so that they are invariant under some typical crystallographic groups \( \mathbf{4}, \mathbf{42m} \) of tetragonal class or the orthorhombic group \( \mathbf{222} \). One can verify that the expressions (2) and (6) are invariant under the following groups, for which the generator matrices are also quoted (Auld, 1973).

\[ \begin{array}{l}
\text{a) Tetragonal } \mathbf{4} \\
\mathbf{4} \rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}
\end{array} \]

\[ \begin{array}{l}
\text{b) Tetragonal } \mathbf{42m} \\
\mathbf{42m} \rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}
\end{array} \]

\[ \begin{array}{l}
\text{Orthorhombic } \mathbf{222} \\
\mathbf{222} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}
\end{array} \]
5.3 The Gyrotropic Tensor

For a cubic crystal, it has been shown by Kumarswamy and Krishnamurthy (1980) that there are two nonvanishing independent gyrotropic constants for $T_d$ and $O$ point group operations and four constants for $T$ point group operations. These are $d_{453}$ and $d_{152}$; $d_{141}$ and $d_{152}$, $d_{141}$, $d_{163}$ and $d_{453}$ respectively for $O$, $T_d$ and $T$ symmetry class of crystals. For tetragonal crystals, there are 11 for $C_4$, 12 for $S_4$, 5 for $C_{4v}$ and 6 each for $D_{2d}$ and $D_4$ point group operations. The nonvanishing constants satisfy the following symmetry relations

For cubic crystals,

a) $T$ point group:

\[
\begin{align*}
    d_{141} &= d_{252} = d_{363} \\
    d_{152} &= d_{263} = d_{341} \\
    d_{163} &= d_{241} = d_{352} \\
    d_{453} &= d_{561} = -d_{462}
\end{align*}
\]

b) $T_d$ point group:

\[
\begin{align*}
    d_{141} &= d_{252} = d_{363} \\
    d_{152} &= d_{263} = d_{341} \\
    d_{252} &= d_{263} = d_{341} \\
    = d_{163} &= d_{241} = d_{352}
\end{align*}
\]
c) 0 point group:

\[ d_{453} = d_{561} = -d_{462} \]

\[ d_{152} = d_{263} = d_{341} \]

\[ = -d_{163} = -d_{241} = -d_{352} \]

Besides the \( d_{ijk} \) satisfy the following antisymmetric relation with respect to the interchange of indices \( i \) and \( j \).

\[ d_{ij1} = -d_{j1i} \]

For crystals belonging to tetragonal class the relations are:

**C\(_4\) point groups:**

\[ d_{133} = d_{233} \]

\[ d_{141} = -d_{252} \]

\[ d_{142} = d_{251} \]
\[ d_{151} = d_{242} \]
\[ d_{152} = - d_{241} \]
\[ d_{163} = - d_{263} \]
\[ d_{341} = - d_{352} \]
\[ d_{342} = d_{351} ; d_{453} \]
\[ d_{461} = - d_{562} \]
\[ d_{561} = d_{462} \]

b) \text{S}_4 \text{ point group:} \\
-----------------
\[ d_{123} ; d_{133} = - d_{233} \]
\[ d_{141} = d_{252} ; \]
\[ d_{142} = - d_{251} \]
\[ d_{151} = -d_{242} \]
\[ d_{152} = d_{241} \]
\[ d_{163} = d_{263} \]
\[ d_{341} = d_{352} \]
\[ d_{342} = -d_{351} \]
\[ d_{363}; \quad d_{461} = -d_{562} \]
\[ d_{462} = d_{561} \]

c) \quad C_{4v} \text{ point group: }

\[ d_{133} = d_{233} \]
\[ d_{142} = d_{251} \]
\[ d_{151} = d_{242} \]
\[ d_{342} = d_{351} \]
d) $D_{2d}$ point group:

\[ d_{141} = d_{252} \]

\[ d_{152} = d_{241} \]

\[ d_{163} = d_{263} \]

\[ d_{341} = \]

\[ d_{363}, \quad d_{462} = d_{561} \]

e) $D_4$ Point group:

\[ d_{141} = d_{252} \]

\[ d_{152} = -d_{241} \]
\[ d_{163} = -d_{263} \]

\[ d_{341} = -d_{352} \]

\[ d_{363} = d_{462} = -d_{561} \]

Let us denote by \( d_{ikm}^\ast \) the modified gyrotropic constants arising from the coupling of the order parameter with the strain and its derivatives.

Then

\[ d_{ikm}^\ast = \frac{d}{d\mathbf{m}^\ast} \left( \frac{\partial F}{\partial \mathbf{m}^\ast / \partial x_m} \right) \]  

(13)

If \( \Delta d_{ikm}^\ast \) is the gyrotropic anomaly near the phase transition, then

\[ \Delta d_{ikm}^\ast = d_{ikm}^\ast - d_{ikm} \]  

(14)

5.4 The Landau-Khalatnikov Equation

The Landau-Khalatnikov equation is

\[ \dot{\mu} \dot{\alpha} = - \frac{\partial F}{\partial \alpha} \]  

(15)
where $\mu$ is the viscosity coefficient. Now $F$ is a function of $Q$, the strain variables and the derivatives of strain.

We shall write as before

$$Q^* = Q_0 + \delta Q$$

$$\gamma^*_f = \gamma_{f0} + \delta \gamma_f$$

so that $Q^*$ and $\gamma^*_f$ denote the fluctuations of the order parameter and the strain variables from their equilibrium values.

Expanding $\partial F/\partial \Theta$ as a power series around their equilibrium values, we find that

$$\dot{\alpha} + \frac{1}{\mu} \left( \frac{\partial^2 F}{\partial \alpha^2} \right)_0 \alpha^* = -\frac{1}{\mu} \frac{\xi}{\epsilon} \left( \frac{\partial^3 F}{\partial \alpha \partial \gamma \partial \gamma} \right)_0 \gamma^*$$

$$-\frac{1}{\mu} \frac{\xi}{m} \frac{\xi}{m} \left( \frac{\partial^2 F}{\partial \gamma \partial \gamma} \right)_0 \gamma^*$$

$$\left( \partial \gamma / \partial \gamma \right)_0$$

We assume that the terms coupling the order parameter with the derivatives of the strain in $F_C$ are much smaller than the terms coupling the order parameter with the strain, so we neglect them in the first order. This is because the
gyrotropic anomalies are smaller than the elastic anomalies.

Hence we have

\[
\dot{\gamma}^* + \frac{Q^x}{\tau} = -\frac{1}{\mu} \frac{\varepsilon}{\varepsilon} \left( \frac{\partial^2 F}{\partial \alpha \partial \alpha} \right)_0 \gamma^x
\]

(18)

where \( \tau \) is the relaxation time and is given by

\[
\tau = \mu \left( \frac{\partial^2 F}{\partial \alpha^2} \right)_0^{-1}
\]

\[
= \mu / \xi
\]

where \( \xi = \left( \frac{\partial^2 F}{\partial \alpha^2} \right)_0 \)

(19)

In (16), we have expanded \( \partial F / \partial \alpha \) about the equilibrium values of \( Q_0 \) and of the order parameter and the strains. In the high temperature phase, these are all zero, but in the low temperature, low symmetry phase, the quantities have values given by equations (14) to (18) of the appendix 1 of chapter III. Since gyrotropic tensors are small, the first approximation to the equilibrium values of \( Q \) and \( \gamma \) can be obtained by neglecting spatial dispersion effects. The expression given in the appendix 1 in chapter III for \( Q \) and \( \gamma \) in the phase transition and low temperature phases will be a
first approximation, independent of \( k \), for these quantities. Assuming that \( Q \) and \( \gamma \) vary as \( \exp(i \omega t) \), where \( \omega \) is the acoustic frequency, we get

\[
\alpha^* = - \frac{P/\xi}{\omega} \left( \frac{\partial^2 F}{\partial \omega \partial \gamma^*} \right). 
\]

where \( P = 1/(1 + i \omega \tau) \)

It follows immediately that

\[
\left( \frac{\partial Q^*}{\partial \gamma^*} \right)_0 = - \frac{P/\xi}{\omega} \left( \frac{\partial^2 F}{\partial \omega \partial \gamma^*} \right)_0. \tag{22}
\]

In the high temperature phase, all the \( \gamma^* \) have zero values. We have further

\[
\left( \frac{\partial^2 F}{\partial \omega \partial \gamma^*} \right)_0 = \alpha 
\]

while \( \left( \frac{\partial^2 F}{\partial \omega \partial \gamma^*} \right)_0 = 0 \) if \( i \neq 6 \) \tag{23}

Hence in this limit, we have

\[
\left( \frac{\partial Q}{\partial \gamma^*} \right)_0 = - \frac{P/\xi}{\omega} \alpha_{6i} \tag{24}
\]
5.5 Gyrotropic Anomalies in the High Temperature Phase

In order to evaluate the gyrotropic anomalies we shall denote the last two terms in $F_c$ by $T_1$ and $T_2$ respectively, so that

$$T_1 = \Phi \left[ h \left( \frac{d\gamma_1}{dx_3} + \frac{d\gamma_2}{dx_3} + \frac{d\gamma_3}{dx_3} \right) + k \left( \frac{d\gamma_4}{dx_2} + \frac{d\gamma_5}{dx_1} \right) \right]$$

(25)

and

$$T_2 = -h \left[ (\gamma_1 + \gamma_2 + \gamma_3) \frac{d\omega}{dx_3} \right] - k \left( \gamma_4 \frac{d\omega}{dx_2} + \gamma_5 \frac{d\omega}{dx_1} \right)$$

(26)
Sustituting for \( Q \) from (20) we find that

\[
T_1 = -\hbar \frac{P}{\xi} \left[ \xi \left( \frac{\partial^2 F}{\partial \omega \partial \eta} \right)_0 \eta^* \right] \times \\
\left[ d\eta_1^*/dx_3 + d\eta_2^*/dx_3 + d\eta_3^*/dx_3 \right]

- \kappa \frac{P}{\xi} \left[ \xi \left( \frac{\partial^2 F}{\partial \omega \partial \eta} \right)_0 \eta^* \right] \times \\
\left( d\eta_4^*/dx_2 + d\eta_5^*/dx_1 \right)
\]

while

\[
T_2 = \frac{\hbar P}{\xi} \left( \eta_1^* + \eta_2^* + \eta_3^* \right) \left[ \xi \left( \frac{\partial^2 F}{\partial \omega \partial \eta} \right)_0 \right] \times \\
\left( d\eta_4^*/dx_2 + d\eta_5^*/dx_1 \right)
\]

(27)

In the high temperature phase, the relations (23) are satisfied, so that we obtain

\[
\Delta d_{163}^* = \Delta d_{263}^* = \Delta d_{363}^*
\]

\[
= \hbar \frac{p_c}{\xi} = \frac{\hbar p c}{\alpha^4 (T - T_c)}
\]

(28)

and

\[
\Delta d_{163}^* = \Delta d_{263}^* = \Delta d_{363}^*
\]

(29)
\[ \Delta d_{462}^* = - \Delta d_{561}^* \]

\[ = \frac{k PC}{\xi} = \frac{k PC}{\alpha'(T - T_C)} \]  

(30)

The other constants are zero. It can be verified that the relation

\[ \Delta_{dij}^* = - \Delta_{dijk}^* \]  

(31)

is satisfied.

The expression (29) and (30) show that the gyrotropic anomalies are temperature dependent and vary inversely as \((T - T_C)\). They have a singularity at \(T = T_C\).

5.6 Gyrotropic Anomalies In The Low Temperature Phase

In the low temperature phase, the equilibrium values of \(Q_0, \eta_\nu, \eta_\ell, \eta_\omega, \eta_\zeta, \xi_\zeta\) as well as \(\xi\) are given by equations (14) to (18) of the appendix 1 in chapter III, so that from (20) we obtain
\[ \alpha = - P \varepsilon \frac{\gamma_6}{\xi} - \frac{2 \gamma_0}{\xi} (\gamma_1 + \gamma_2 + \gamma_3) \]  

(32)

Hence the term coupling the order parameter with the strain in the free energy is given by

\[ C \frac{\partial}{\partial \xi} \gamma_6 = - C \frac{\partial}{\partial \xi} \left[ P \varepsilon \frac{\gamma_6}{\xi} + 2 \gamma_0 \right] \left( \gamma_1 + \gamma_2 + \gamma_3 \right) \]  

(33)

Substituting the expression for \( Q \) in (20) in \( T_1 \) and \( T_2 \), we find that

\[ T_1 + T_2 = - h \frac{P \varepsilon}{\xi} \left( \frac{\partial \gamma_1^*}{\partial x} + \frac{\partial \gamma_2^*}{\partial x} + \frac{\partial \gamma_3^*}{\partial x} \right) \]

\[- \left[ k \frac{P \varepsilon}{\xi} + \frac{2 k P B \gamma_0}{\xi} (\gamma_1 + \gamma_2 + \gamma_3) \right] \left( \frac{\partial \gamma_1^*}{\partial x} + \frac{\partial \gamma_2^*}{\partial x} + \frac{\partial \gamma_3^*}{\partial x} \right) + h \frac{P \varepsilon}{\xi} \left( \gamma_1 + \gamma_2 + \gamma_3 \right) \left( \frac{\partial \gamma_1^*}{\partial x} + \frac{\partial \gamma_2^*}{\partial x} + \frac{\partial \gamma_3^*}{\partial x} \right) \]

\[ \frac{\partial \gamma_6^*}{\partial x} + k \gamma_6^* \left[ \frac{P \varepsilon}{\xi} \frac{\partial \gamma_6^*}{\partial x} + \frac{2 P B \gamma_0}{\xi} \left( \frac{\partial \gamma_1^*}{\partial x} + \frac{\partial \gamma_2^*}{\partial x} + \frac{\partial \gamma_3^*}{\partial x} \right) \right] \]

\[ k \gamma_6^* \left[ \frac{P \varepsilon}{\xi} \frac{\partial \gamma_6^*}{\partial x} + \frac{2 P B \gamma_0}{\xi} \left( \frac{\partial \gamma_1^*}{\partial x} + \frac{\partial \gamma_2^*}{\partial x} + \frac{\partial \gamma_3^*}{\partial x} \right) \right] \]  

(34)

Substituting this expression for the free energy we obtain
immediately the gyrotropic anomalies in the low symmetry phase. These are given by

\[ \Delta d_{613}^* = \Delta d_{623}^* = \Delta d_{633}^* \]

\[ = - \frac{h \rho c \xi}{\xi_0} \quad \text{(35)} \]

\[ \Delta d_{642}^* = \Delta d_{651}^* \]

\[ = \frac{\kappa \rho c \xi}{\xi_0} \quad \text{(36)} \]

In the low temperature phase, the gyrotropic constants depend inversely on \( \xi_0 \) with singularity at \( T_0 \). (given in appendix 1 of chapter III).

Besides these, there are other nonvanishing gyrotropic constants which normally vanish in the cubic phase. These are

\[ \Delta d_{142}^* = \Delta d_{151}^* = \Delta d_{242}^* = \Delta d_{251}^* = \Delta d_{342}^* \]

\[ = \Delta d_{351}^* = -\frac{2 \rho \kappa \mathbf{B} \xi_0}{\xi_0} \quad \text{(37)} \]

It can be verified that the antisymmetric relation (31) is satisfied in this region too. The above equations show that the gyrotropic anomalies for \( \Delta d_{613}^* \), \( \Delta d_{623}^* \), \( \Delta d_{633}^* \),
$\Delta d_{642}^*$ and $\Delta d_{651}^*$ depend on temperature as $(T-T_0)^{-1}$ whereas the the anomalies for $\Delta d_{142}^*, \Delta d_{151}^*, \Delta d_{242}^*, \Delta d_{251}^*$, etc depends on temperature as $Q_0/\xi^{-}$. Again it follows that if a coupling develops during the phase transition between the order parameter and the derivatives of the strain, these can simulate gyrotrropic effects even in the case of crystals which do not ordinarily display gyrotrropic effects.

5.7 Rotary Activity

As stated in the introduction, spatial dispersion splits the degeneracy in the velocities of the two transverse acoustic modes propagating along the principal axis (001). In the presence of spatial dispersion, the velocities of the two shear modes propagating along the (001) direction are given by

$$\rho v_+^2 = C_{66} + \Delta d_{442}$$

$$\rho v_-^2 = C_{66} - \Delta d_{442}$$

The splitting of the degeneracy in the two shear
modes produces a difference in the velocity of the two circularly polarized modes traveling in the (001) direction, resulting in the rotation of the plane of polarization. The polarization angle is given by (Auld, 1973)

$$\theta(\gamma) = \frac{1}{2} \omega \left( \frac{1}{v_-} - \frac{1}{v_+} \right) l$$

(40)

Where $\omega$ is the frequency of the ultrasonic wave and $l$ is the distance which it travels in the crystal. Before substituting (39) and (40) in (41), we shall first examine whether rotary activity is at all possible, as waves get normally damped during a phase transition. Let us first evaluate the damping of the shear waves propagating along the (001) direction for a cubic crystal. During a phase transition, anomalies appear in the elastic constants and as shown in section 6, the elastic constant $C_{66}$ changes into $C_{66}' = C_{66} + \Delta C_{66}'$, where

$$\Delta C_{66}' = - PC^2 \left( 2 - \frac{\omega}{\omega_0} \right) \frac{1}{\omega_0}$$

(41)

In the above expression, $C$ is the coupling coefficient, that couples the order parameter with the strain variables, of the order of $10^{17}$ (Lemanov, 1982); $\omega$ is the frequency of the ultrasonic wave, $\tau$ is the relaxation time.
Further, $P = 1/(1 + i\cdot 2\pi)$ and is complex. Lemanov estimates the order of $\omega$ to be $10^{22} - 10^{23}$, so that $4C_{66}^*$ is again of the same order $10^{11} - 10^{12}$ as that of the elastic constants in agreement with experimental observation.

If we now write

$$\rho \nu^2 = C_{66}^* = a + ib$$

then

$$\nu = \nu_{\gamma} + i\nu_i = \nu_{\gamma} (1 + i\nu_i/\nu_{\gamma})$$

$$\sim \left(\frac{a}{\rho}\right)^{1/2} \left(1 + \frac{ib}{2a}\right) \quad (4.2)$$

Let us consider a wave $u = u_0 e^{i(\omega t - k.z)}$ travelling along the $z$ direction. Since the velocity is complex, the wave vector $k$ is complex, and one has in fact

$$u = u_0 e^{-\frac{3}{2} \omega \nu_{\gamma}/\nu_{\nu}} e^{i(\omega t - \omega \nu/\nu_{\gamma})} \quad (4.3)$$

The damping coefficient of the wave is given by $(\omega \nu_i/\nu_{\gamma})^2$.

Choosing $\omega \sim 10^9 - 10^{10}$ Hz; $\nu_{\gamma} \sim 10^6$ and $\nu_i/\nu_{\gamma} \sim 1/10$ to 1, we find that the damping coefficient is of the order of $10^2$ to $10^4$. This means that the wave could travel a distance of the order of $10^4 \lambda^0$ inside the crystal even under unfavourable conditions. This is larger than the propagating length of 1000 A$^0$ found in the experiment of Lin et al, for acoustic activity. Thus, even though elastic waves are damped during a
phase transition, it is possible to observe rotary activity experimentally. Lin also finds that in the case of Bi_{12}GeO_{20}, the faster of the circularly polarised phonon has larger attenuation when it travels along (111). The difference in the attenuation coefficients between the left and right circularly polarised waves become obvious if we note that the attenuation coefficient \( \omega v_i / v_r^2 \) depends on the real part of the velocity, which is different for the two circularly polarised modes.

Substituting from (30) and (37) the expressions for \( d_{642}^* \), we find that

\[
\Theta (y) = - \left( \frac{l}{\omega c_{66}} \right) \left( \frac{\rho}{\rho c_{66}} \right) \frac{k \rho c}{\alpha^2} \left( \frac{1}{T - T_c} \right)
\]

\[\text{for } T > T_c \quad (44)\]

and

\[
\Theta (z) = - \left( \frac{l}{\omega c_{66}} \right) \left( \frac{\rho}{\rho c_{66}} \right) \left( \frac{k \rho c}{\xi^2} \right)
\]

\[\text{for } T < T_c \quad (45)\]

Thus the presence of any term coupling the order parameter with the strain variables and its derivatives could induce rotation of plane of polarization of the transverse acoustic modes. Measurements of the polarization angle above and below the transition temperature will help one to determine product of the coupling constants \( (C, K) \). We have shown in chapter III or equation (34) of this chapter that measurement of the
elastic anomalies for $C_{66}$ could yield the value of the coupling constant $C$. Thus combined with studies of elastic anomalies which could yield the value for $C$, studies on rotary activity could yield the value for the coupling constant $K$.

5.8 Conclusion

The coupling between the order parameter and the derivatives of the strain components, results in anomalous changes in the gyrotrropic coefficients. They become highly dependent on temperature. In the high temperature region the gyrotrropic coefficients vary as $1/(T-T_c)$, but in low temperature phase they vary as $1/\xi$ with a singularity at $T_0$. Again the coupling of the order parameter with the strain and its derivatives induces rotary activity in crystals. Measurement of the polarization angles below and above the phase transition helps one to evaluate the coupling constants in the free energy of the crystal.
REFERENCES


