Chapter 2

Dissipative Drift Instability in Dusty Plasma

An investigation has been done on the very low-frequency electrostatic drift waves in a collisional dusty plasma. The dust density gradient is taken perpendicular to the magnetic field $B_0$, which causes the drift wave. In this case, low-frequency drift instabilities can be driven by $E_1 \times B_0$ and diamagnetic drifts, where $E_1$ is the perturbed electric field. Dust charge fluctuation is also taken into consideration for our study. The dust-neutral and ion-neutral collision terms have been included in equations of motion. It is seen that the low-frequency drift instability gets damped in such a system. Both dust charging and collision of plasma particles with the neutrals may be responsible for the damping of the wave. Both analytical and numerical techniques have been used while developing the theory.

2.1 Introduction:

The presence of micron sized dust particles adds a new dimension to two-component plasma. Various interesting phenomena occur in addition to normal wave modes and instabilities, due to the presence of dust particles. Dust-acoustic wave, Dust lattice...
wave are few such examples. The dust particles are relatively massive compared to electrons and ions and they may acquire a large negative charge and these two properties make them very special.

In the near past, the study of various low-frequency phenomena in dusty plasma has received much attention. The dust dynamics plays the key role in supporting these modes. Chu and I reported about a very low-frequency \((\approx 12 \, \text{Hz})\) mode with a wavelength of about 0.5 cm, which was detected in the motion of their strongly coupled charged grains. A study done by Praburam and Goree revealed the existence of a pair of very low-frequency modes, termed as filamentary and great void modes. Rao et al. predicted for the first time the existence of Dust Acoustic Wave (DAW) in dusty plasma. The DAW propagates as the normal mode when the phase speed is much larger than the dust thermal speed. Interpretations of DAW excitation in experiments have typically relied on collisionless inverse Landau damping mechanism. However, a closer examination of experimental conditions revealed that waves are often excited when there is a significant background pressure of neutrals. It is therefore important to study the wave excitation theories into the collisional regime. In magnetized, homogeneous dusty plasma, the presence of charged dust can affect the electrostatic waves such as acoustic wave, electrostatic ion cyclotron waves and lower-hybrid waves.

The effect of charged dust on the propagation of low-frequency electromagnetic waves has been investigated by Pilipp et al. It was seen that the dispersion relation of such waves is significantly altered only near the dust gyro-frequency at which frequency, the waves are in resonance with the grains and hence can be damped by them. Rosenberg investigated ion-acoustic and dust-acoustic instabilities using Vlasov theory for electrons, ions and dust of uniform mass and charge.

The study of drift instability is of great interest in plasma physics. Rosenberg and Krall have done detailed study on low frequency drift instability in a dusty
magnetized plasma both in presence of electron density gradient and dust density gradient opposite to each other. Possible applications of their results to dusty space plasmas are also discussed. Their study has motivated the present workers to study low-frequency drift instabilities taking into account the effect of collision of dust and neutral particles. Kaw et al.\textsuperscript{151} have investigated instabilities of dust acoustic waves in a plasma with a significant background pressure of neutrals. They have demonstrated that the recombination of the background electrons and ions on the surfaces of dust particles and the momentum loss of ions to neutrals in the presence of a relative ion-dust drift act as new processes promoting the excitation of dust-acoustic instability in a collisional plasma. Ivlev et al.\textsuperscript{152} have made study on acoustic modes in a collisional dusty plasma taking into account the influence of ion-neutral collisions, ion drag, and neutral friction. They have assumed that the frequency of ion-neutral collisions is greater than the frequency of ion dust collisions and have derived some interesting results.

It has been shown that the properties of the dissipative drift wave can be modified in the presence of charged dust grains\textsuperscript{153} owing to the modification of the equilibrium quasi-neutrality condition and the dust particle dynamics. Dust charge fluctuations can also modify the properties of electrostatic drift waves. Using a kinetic approach, Benkadda et al.\textsuperscript{154} have estimated the growth rate of a new type of drift instability, driven by the process of dust charging. From their study they have found that the growth rate is comparable to or much larger than the usual drift instability, even for a low dust density. By continuing this study using a hydrodynamics approach they have seen that a high rate of dissipation due to the charging of dust particles take place\textsuperscript{155}, which produces a high nonadiabaticity in the longitudinal electron motion which increases the range of unstable drift waves. Very recently, Shukla et al.\textsuperscript{156} have investigated the existence of the drift dissipative instability in a non-uniform magnetoplasma whose constituents are the electrons, positive ions, negative ions and
negatively charged dust grains that are stationary and non-uniformly distributed. They have given a fluid theory description. They have reported that the presence of the negatively charged, massive dust grains affects both the drift wave frequency and the growth rate of the drift dissipative instability. The dust particles may be spatially localized in laboratory dusty plasma. There may exist gradients in density of charged plasma species at the edge of the dust cloud in order to maintain overall charge-neutrality. In presence of a magnetic field, such plasma may lead to drift instability.

In this chapter, low-frequency dissipative drift instability is investigated in a plasma system consisting of electrons, ions, dust particles and neutrals on the basis of fluid theory. For the completeness of the theory, self-consistent fluctuation of charge on dust particles is also incorporated. The dust particles are like sinks of plasma particles. The dissipative effect arises due to charging of dust grains by plasma currents. The inhomogeneity in dust density in a direction perpendicular to the magnetic field $\mathbf{B}_0$ gives rise to drift wave. Electron density gradient is also taken into consideration along x-direction. If there is a feedback mechanism, then a drift instability occurs and the plasma becomes unstable. On the other hand, in presence of collision and other dissipative effects, the wave damping might occur. The attempt has been made to study the effect of dust density gradient and ion-neutral collision on the damping of drift mode for low frequency regimes, where characteristic time scale is governed by dust inertia. The wave dynamics is governed by the relatively heavy dust grains. The theory is tested in a typical laboratory dusty plasma device. It is assumed that the perturbed electric field is constrained so as to produce negligible magnetic perturbation, so that the drift wave may be assumed to be electrostatic.

Many astrophysical plasma and plasma processing situations have significant amount of neutral pressure. In such cases collisionless theory is no longer valid. Experimental observations also reveal that the waves are often excited when there is high neutral
pressure. It is therefore necessary to consider full equation of motion for plasma species including the collisional terms instead of considering them as Boltzmann fluid. In this study we have considered full equation of motion for plasma species including dust taking into account ion-neutral collisional term. Effect of ion-neutral collision frequency is investigated on low-frequency drift wave.

The theoretical model and equations are discussed in section 2.2 of this chapter. Section 2.3 deals with the dispersion relation. In section 2.3, analysis of the general results are given along with discussion. The conclusions are summarized in section 2.4.

2.2 Theoretical model and equations:

We are interested in studying the low frequency drift wave in a dusty plasma consisting of electrons, ions, neutral particles and negatively charged dust particles both in presence of electron and dust particle inhomogeneity and collision between ions with neutral particles and dusts with neutral particles. The negatively charged dust particles have density gradient along $-x$ direction (Fig.2.1). Since electrons are more mobile than the ions, more electrons are absorbed on the dust surface than ions. Hence an electron depletion takes place which results in an electron density gradient along $+x$ direction. Ion density inhomogeneity may be neglected in this case. Overall charge neutrality implies that

$$\frac{dn_e}{dx} = -Z_d \frac{dn_d}{dx}$$

Where, it is assumed that the variation of $Z_d$ with $n_d$ (and therefore with $x$) is small compared with the variation of $n_d$ with $x$. Ion number density is assumed to be uniform. An electric field $E_0 \hat{x}$ arising due to an external field from the electrode configuration confines the negatively charged dust.
The governing equations for dust particles are

$$m_d n_d \left( \frac{\partial \vec{v}_d}{\partial t} + \vec{v}_d \cdot \nabla \vec{v}_d \right) = n_d Q_d \left( \vec{E} + \vec{v}_d \times \vec{B}_0 \right) - \nabla p_d - (\vec{v}_d - \vec{v}_n) m_d n_d \nu_{dn}$$

(2.2.1)

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \vec{v}_d) = 0$$

(2.2.2)

where, $m_d, n_d, \vec{v}_d$ are the mass, density and velocity of dust particles, $p_d$ is the dust pressure, whereas $\nu_{dn}$ is the collision frequency of dust particles of radius $r_d$ with the neutrals of mass $m_n$, density $n_n$ and velocity $\vec{v}_n$ given as

$$\nu_{dn} = \frac{4 \pi}{3} r_d^2 v_n n_n \left( m_n / m_d \right)$$

The density gradient of dust particles in presence of the magnetic field gives rise to diamagnetic drift. The expression for drift velocity can be obtained as

$$\vec{v}_{dD} = -\frac{1}{B_0 Q_d m_d} \frac{T_d}{\partial_t} \frac{\partial n_d}{\partial x} \hat{y}$$
where, $T_d$ is the temperature of the dust species. Due to the external field $E_0 \times B_0$, the drift is $\vec{u}_d = - \left( \frac{E_0}{B_0} \hat{y} \right)$.

Theory of dust charge fluctuation is taken into consideration. The charge $Q_d$ on dust is treated as dynamical variable responding to electron and ion currents driven by self-consistent fields. The charging equation for dust particles is written as

$$\frac{dQ_d}{dt} = I_e + I_i \quad (2.2.3)$$

where

$$I_i = \pi \tau_d^2 e \sqrt{\frac{8 T_i}{\pi m_i}} n_i \left[ 1 - \frac{e(\Phi_{f0} - \Phi_i)}{K_B T_i} \right]$$

$$I_e = -\pi \tau_d^2 e \sqrt{\frac{8 T_e}{\pi m_e}} n_e \exp \left( \frac{e(\Phi_{f0} - \Phi_i)}{K_B T_e} \right)$$

and $\Phi_{f0}$ is floating potential, $\Phi$ is plasma surface potential. $\Phi_{f0}$ is determined by the condition that in equilibrium

$$I_{e0} + I_{i0} = 0$$

Equation of motion for ions is given as

$$m_i n_i \left( \frac{\partial \vec{v}_i}{\partial t} + \vec{v}_i \cdot \nabla \vec{v}_i \right) = n_e \left( \vec{E} + \vec{v}_i \times \vec{B}_0 \right) - (\vec{v}_i - \vec{v}_n) m_i n_i \nu_n \quad (2.2.4)$$

where $\nu_n = n_n \sigma_{n} c_i$ is ion-neutral collision frequency, $n_n$ is the neutral density and $\sigma_{n}$ is the ion-neutral collisional cross-section and $c_i^2 = \frac{K_B T_i}{m_i}$.

Continuity equation for ion is

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{v}_i) = 0 \quad (2.2.5)$$

Equations of motion for electrons are

$$m_e n_e \left( \frac{\partial \vec{v}_e}{\partial t} + \vec{v}_e \cdot \nabla \vec{v}_e \right) = -n_e e \left( \vec{E} + \vec{v}_e \times \vec{B}_0 \right) - \vec{\nabla} p_e \quad (2.2.6)$$

where, $\vec{\nabla} p_e = -T_e \nabla n_e$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}_e) = 0 \quad (2.2.7)$$
Here, $m_e, n_e$ and $v_e$ are the mass, density and velocity of the electrons respectively.

The governing equations for neutrals are

$$m_n n_n \left( \frac{\partial \vec{u}_n}{\partial t} + \vec{u}_n \cdot \nabla \vec{u}_n \right) = - \left( \vec{v}_n - \vec{v}_d \right) m_n n_n v_d n_n + \left( \vec{v}_n - \vec{v}_i \right) m_n n_i v_i$$  \hspace{1cm} (2.2.8)

$$\frac{\partial n_n}{\partial t} + \nabla \cdot \left( n_n \vec{u}_n \right) = 0$$  \hspace{1cm} (2.2.9)

The equations may be finally closed by Poisson’s equation given as

$$\nabla^2 \Phi = 4\pi \left[ e n_e + Q_d n_d - e n_i \right]$$  \hspace{1cm} (2.2.10)

### 2.3 Dispersion Relation:

The normal mode analysis is used to solve the equations (2.2.1)–(2.2.10). We consider that the low-frequency wave is propagating in the $yz$-plane such that the perturbed quantities vary as $\exp \left( ik_y y + ik_z z - i\omega t \right)$, where, $k_z \ll k_y$. Linearizing equations (2.2.1)–(2.2.10) and taking Fourier transformation, we get the perturbed parameters as

$$n_{d1} = \frac{Q_{D2} E_{1y} + Q_{D3} E_{1z} + Q_{D4} Q_{d1}}{Q_{D1}}$$  \hspace{1cm} (2.3.1)

$$n_{e1} = \frac{1}{\omega} \left[ Q_{v1} n_{d1} + Q_{v2} E_{1y} + Q_{v3} E_{1z} + Q_{v4} Q_{d1} \right]$$  \hspace{1cm} (2.3.2)

$$n_{e1} = \frac{1}{A_{e1}} \left[ C_{e1} E_{1y} + D_{e1} E_{1z} \right]$$  \hspace{1cm} (2.3.3)

$$Q_{d1} = \frac{1}{X} \left[ A \left( 1 + a_0 Q_{d0} \right) n_{e1} \right] - \frac{B}{X} \left( 1 - b_0 Q_{d0} \right) n_{1}$$  \hspace{1cm} (2.3.4)

Substituting these first order perturbed quantities into the linearized Poisson equation

$$\nabla^2 \Phi_1 = 4\pi \left[ e n_{e1} + Q_{d0} n_{d1} + Q_{d1} n_{d0} - e n_{i1} \right],$$

we get the dispersion relation as

$$1 + \chi_e + \chi_d + \chi_i = 0$$  \hspace{1cm} (2.3.5)
Here,

\[ \chi_s = -\frac{4\pi e}{k^2 A_{el}} \left( k_y C_{el} + k_z D_{el} \right) \]

\[ \chi_d = -\frac{4\pi I}{k^2} \left[ \frac{Q_d 0}{(\omega - Q_1)} \left( k_y Q_{dy} + k_z Q_{dz} + n_{d0} \left( \frac{k_y \alpha_y}{\alpha_d} + \frac{k_z \alpha_z}{\alpha_d} \right) \right) \right] \]

\[ \chi_\lambda = \frac{4\pi e}{k^2} \left( k_y Q_{\lambda y} + k_z Q_{\lambda z} \right) \]

Where, the terms appearing in equations (2.3.1)-(2.3.5) are described in Appendix A.

### 2.4 Analysis of the Results and Discussions:

We consider the limit \( \omega < \nu_m \) and \( \omega < \nu_{dm} \). Then equation (2.3.5) reduces to

\[ C_1 \omega^5 + C_2 \omega^4 + C_3 \omega^3 + C_4 \omega^2 + C_5 \omega + C_6 = 0 \quad (2.4.1) \]

Where, the coefficients \( C_i \) \( (i = 1, 2, 3, 4, 5, 6) \) are described in Appendix A. Assuming \( \omega = \omega_r + i \gamma \), equation (2.4.1) is numerically solved for following values of the parameters obtained in typical dusty plasma laboratories:

\[ m_d = 4 \times 10^{13} m_p, \quad n_{e0} = 5 \times 10^{16} m^{-3}, \quad n_{d0} = 1 \times 10^{14} m^{-3}, \quad n_{n0} = 10^{22} m^{-3}, \]

\[ n_{e0} = 2.0 \times 10^{14} m^{-3}, \quad T_e \sim 0.7 \text{ eV}, \quad T_i \sim 3 \text{ eV}, \quad T_d \sim .03 \text{ eV}, \quad \text{radius of the dust } r_d \sim 10 \mu m, \quad B_0 = 2 \text{ Tesla}, \quad Z_d = 5 \times 10^3 \text{ and } \sigma_m \sim 10^{-14} \text{ cm}^2. \]

The normalized values of real and imaginary parts of frequency i.e. \( (\omega_r/\omega_{pd}) \) and \( (\gamma/\omega_{pd}) \) are plotted across \( k_y \lambda_{Dd} \) in Figs (2.2 - 2.5), where

\[ \lambda_{Dd} = \frac{\lambda_{De} \lambda_{Di}}{\sqrt{\lambda_{De}^2 + \lambda_{Di}^2}} \]

here, \( \lambda_{De} \) and \( \lambda_{Di} \) are the electron and ion Debye radii respectively. The negative values of \( \gamma \) indicate a dissipative, collisional drift instability in the regime \( \omega_{pd} < \omega << \omega_{pn} \).
Figure 2.2: $\frac{\omega}{\omega_{pd}}$ vs. $k_y\lambda_{Dd}$, for $K\lambda_D = 2.0$. 
Figure 2.3: $\frac{1}{\omega_{pd}}$ vs. $k_y \lambda_{Dd}$ for $K = 1.7/\lambda_{Di}, 2.0/\lambda_{Di}, 2.3/\lambda_{Di}$ respectively.
In order to see the role of density gradient scale-length on the dissipative instability, \((\gamma/\omega_{pd})\) is plotted for different values of dust density gradient \(K = \frac{n_d}{n_{d0}} = 1.7, 2.0\) and 2.3 respectively in the units of \(\lambda_D\) in Fig.2.3. It is seen that with the increase in dust density gradient, damping of the wave decreases. Density inhomogeneity is the driving force for drift instability. However, due to the presence of dust and collisions between plasma particles with neutral, the wave is dissipated. This can be seen from Fig.2.4 where, \((\gamma/\omega_{pd})\) is plotted for different values of ion-neutral collision frequency. This clearly shows that larger is the collision frequency, more is the dissipation rate. Low frequency drift mode gets damped in presence of high neutral pressure.

Fig.2.5 shows the dissipation rate of dusty plasma for different values of \(\left(\frac{\rho}{T_c}\right)\).
Figure 2.5: $\gamma/\omega_{pd}$ vs. $k_y\lambda_{Dd}$, for $T_i/T_e = 0.0125$, 0.0145, 0.0175 respectively.
The charging frequency for dust particles may be given as

\[ \nu_{ch} = \frac{\omega_{p1} r_d}{v_{Ti}\sqrt{2\pi}} (1 + \tau + z) \]

where, \( v_{Ti} = \sqrt{T_i/m_i} \) is the ion thermal velocity, \( \tau = \frac{T_i}{T_e} \), \( z = \frac{Z_de^2}{r_d T_e} \). It is seen from our calculations that larger is the value of \( T_i/T_e \) higher is the charging frequency and higher is the damping. Charging of dust particles leads to dissipation of drift wave in plasma in the low frequency regime.

2.5 Conclusions:

We have presented a theory for low frequency dissipative drift instability in dusty plasma. Drift wave in such plasma arises due to inhomogeneity of dust density in presence of magnetic field. The dispersion relation has been obtained by normal mode analysis. The electron density gradient is in opposite direction to the dust density gradient. The drift wave gets damped due to self-consistent fluctuation of the charge on dust particles. Another wave damping mechanism that is discussed here is through ion-neutral collision. Our analysis clearly reveals that ion-neutral collision plays a major role in the process of the wave getting damped.

The study of dissipative drift wave is important in the ionosphere plasma turbulence, in astrophysical plasma and in edge plasma turbulence of tokamak.