**INTRODUCTION**

1.1 Newtonian and Non-Newtonian Fluids

A fluid may be defined as a substance that deforms under the slightest shearing stress. Fluids are constantly encountered in everyday life. The atmosphere over the surface of the earth consists mainly of two fluids, air and water. Their movement controls the weather.

In utilizing the earthly materials and forces of nature, the engineers and scientists deal with fluid behaviour. Most of the industrial systems involve the motion of fluids. Fluids are used in cooling and heating processes, in lubrication and in transmission of power. The subject of 'Fluid Mechanics' deals with the behaviour and properties of both stationary as well as moving fluids.

In the year 1738, Daniel Bournoulli [1] gave the simplest equation of state

\[
\tau_{i,j} = -p \delta_{i,j}
\]  \hspace{1cm} (1.1.1)

where \( \tau_{i,j} \) is the stress tensor, \( \delta_{i,j} \) is the Kronecker delta and \( p \) is the hydrostatic pressure. The fluids which obey this equation are called perfect fluids. Such fluids are frictionless and incompressible. Because there is no internal friction in such fluids, the fluid force acting on any small plane surface in the fluid consists of only the normal pressure, which is isotropic. Although no such fluid actually exists in nature, yet this concept of ideal fluid greatly simplifies the mathematical treatment of flow cases.
The governing equations of a viscous incompressible liquid are the constitutive equations and the field equations. The latter consist of equations of mass, momentum, moment of momentum and energy. If we eliminate stress tensor out of the constitutive equations and the equations of motion, we arrive at Navier-Stokes equations, which together with equation of continuity form the basic working equations. The Equation of moment of momentum is identically satisfied because the stress tensor is taken as symmetric, couple stresses having been assumed to be zero. Equation of energy is considered to give heat flux.

The motion of viscous incompressible fluid is governed by the equation of continuity and Navier [2], Stokes [3] equations. They are

\[ \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j^2} \]

where comma denotes covariant differentiation and

\[ U_i \left[ \Xi \left( U_1, U_2, U_3 \right) \right] \] is the velocity field of the fluid element at the point \( x \left[ \Xi \left( x_1, x_2, x_3 \right) \right] \)
Thus in order to study a general flow problem we have to solve a set of four non-linear partial differential equations under prescribed boundary conditions and initial conditions. These equations (1.1.2) and (1.1.3) involve four unknowns namely three velocity components and pressure $p$.

To discuss the constitutive character of a fluid, in 1845, Stokes [3] enunciated the hypothesis "the stress tensor at a point $\mathbf{x}$, at time $t$, is a function of the rate of deformation $\mathbf{d}_{ij}$, at $\mathbf{x}$ and at time $t$, where 2 $\mathbf{d}_{ij} = \mathbf{\dot{u}}_{ij} + \mathbf{\ddot{u}}_{ij}$." Here $\mathbf{d}_{ij}$ (summation convention not used) represents the rate of distortion in the direction of $x_i$ coordinate while $\mathbf{\dot{u}}_{ij}$ represents the rate of shear in the $(x_i, x_j)$-plane.

Newton, in his paper, "On the Attrition of liquids," proposed that stress depends linearly on the rate of deformation of the fluid in motion and satisfies the equation

$$\mathbf{\tau}_{ij} = -\mu \delta_{ij} + 2 \mu \mathbf{d}_{ij}, \quad (1.1.4)$$

where $\mu$ is the constant of proportionality between the shearing stress and shearing strain-rate and is called the coefficient of viscosity. The fluids which conform to this equation of state are classed as Newtonian fluids. The main characteristics of the Newtonian fluids are:

(i) Newtonian fluids at rest can take up only the isotropic pressure $p$. 


(ii) Application of the slightest stress results in motion.

(iii) Newtonian fluids are isotropic, having no preferred direction with respect to response to the applied stress.

(iv) As soon as the strain is removed, the fluid returns to its original hydrostatic state.

(v) Viscosity coefficient $\mu$ is the ratio of the applied shearing stress to the shearing strain-rate produced. Thus for the Newtonian fluids the shearing stress and shearing strain-rate are linearly related to each other.

In order to explain certain experimental results like Coulomb's Experiment (1789) and D'Alembert's paradox, it was felt that more general constitutive equations are necessary than those that describe the perfect fluid. In fact Coulomb found that when a liquid is placed between two coaxial cylinders and the inner cylinder is rotated, a couple must be applied to the outer cylinder in order to keep it at rest. This means that the effect of the disturbance created in the fluid by the rotation is propagated up to the outer cylinder and that the fluid exerts a tangential force on it. This evidently could not arise just from pressure which is a normal force.

Also, if we calculate the drag on a sphere placed in a uniform flow of a fluid according to the theory of perfect fluid, the total drag is zero. This is contrary to experimental results and this is called D'Alembert's paradox.
There are some fluids which do not conform to the equation (1.1.4) and exhibit significant deviation from the prediction of Newton, when subjected to certain experiments. All such fluids are classed as Non-Newtonian Fluids, characterized by a nonlinear relation between the applied shearing stress and the shearing strain-rate produced.

(i) A highly viscous fluid, like castor oil issuing in the form of a free jet out of a circular pipe under constant pressure gradient shows swelling at the end. This phenomenon called Merrington [4] effect could not be explained on the basis of the Newtonian Constitutive equations.

(ii) A liquid when placed between two coaxial circular cylinders and the outer cylinder be rotated with a constant speed, exhibits what is called Weissenberg [5] effect. If the fluid is water, it shows a slight depression at the inner cylinder and a slight rise at the outer one. When Weissenberg used a highly viscous fluid in his experiment, he found that the fluid is drawn inward against the action of a centrifugal force and shows a rise at the inner cylinder and a depression at the walls of the rotating cylinder.

(iii) Some fluids are capable of sustaining a certain amount of finite stress before flow begins. This is contrary to the property of Newtonian fluids. This effect is due to the plasticity of the fluid.
All these observations cannot be explained on the basis of the Newtonian constitutive equations. There exist in literature, a large number of constitutive equations, suggested to explain the behaviour of non-Newtonian fluids. At a given temperature and pressure, the viscosity is constant for each Newtonian fluid and in case of a non-Newtonian fluid the viscosity at a given temperature and pressure is a function of the velocity gradient. Emulsions, colloidal suspensions and gels are included in this class of non-Newtonian fluids.

Non-Newtonian fluids may be further classified according to the manner in which the viscosity varies with the rate of shear. Bingham material, sometimes called ideal plastic, can withstand a certain amount of shearing stress before it flows. When the shearing stress reaches a certain yield value, the material deforms. Sewage sludge is a common example of a Bingham plastic. In most real plastics the viscosity does not become constant until fairly high rates of shear are attained. Suspension of clay in water behaves like real plastics and is used as drilling mud in the petroleum industry.

Those materials in which the viscosity decreases with the rate of shear but the material deforms as a shearing stress is applied, are called Pseudoplastic materials. Organic solvents like polystyren and metallic soaps in gasoline are examples of this class.
Non-Newtonian fluids may be thixotropic or non-thixotropic. If the fluid possesses some sort of structure which is broken down when it is subjected to shear and on removal of the shearing stress the viscosity instead of being the same at zero rate of shear, will change with time as the fluid builds up the structure it had. If a thixotropic fluid is tested in an apparatus in which the rate of shear can be increased and then decreased the relationship between the stress and the rate of shear will be found to be different when the stress is decreasing. Among the various non-Newtonian fluids, anisotropic fluids, subfluids, microfluids and Rivlin-Erickson fluids have been given more attention in recent years.

1.2 Visco-elastic Fluid Models:

There exist in literature, constitutive equations, characterizing various types of fluids. Most of these conform to a class of viscous fluids which have some elasticity and are called visco-elastic fluids. While they are fluid-like in their capacity to sustain unlimited deformation, if the applied forces are suddenly removed, they show some tendency to recoil. High polymers, thick oils, paints, animal blood etc., all belong to this class of fluids. While in motion, some energy is stored up in such fluids called the strain energy and some energy is dissipated in order to overcome the viscous forces. For such fluids, however small the rate of strain may be, it is responsible for
the reverse flow that follows on removal of stress and the fluid attains the original state. During flow, the natural state of the fluid changes continually and always tries to attain the instantaneous state of the deformed state, but it never succeeds completely. This is the measure of the elasticity of the fluid. To incorporate this time lag, some relaxation time is introduced in the equation of state.

Classical mechanics is mostly concerned with materials for which linear constitutive equations are valid. There has been progress in extending this theory in recent times to include materials for which the constitutive equations are non-linear. Developments in this direction are mainly due to Noll [6], Rivlin and Ericksen [7], Cotter and Rivlin [8], Wang [9,10] and Pipkin and Rivlin [11]. They have assumed in their theories that a functional relation exists between the stress, deformation gradients and their material time rates of any prescribed order. The theories of visco-elasticity, given by Maxwell [12], Voigt [13,14] and Zaremba [15,16,17] have lately undergone fast developments. Visco-elastic fluids may be isotropic or anisotropic in their undisturbed state. The former is more usual, but Ericksen [18,19] has developed a theory of anisotropic fluids. This vast development in visco-elasticity is mainly due to the recent extensive industrial demand on plastic and high polymers, which has given new impetus to
engineering techniques, applications and development
of the mathematical and physical foundations. Constitutive
equations for materials possessing continuous memory of past
history have also been given by Noll[20,21], Green and
Rivlin[22,23], Green, Rivlin and Spencer [24] . Some
of these theories, being of general nature, include all
previous theories as special cases. Under the present
section we recapitulate the basic equations representing
various visco-elastic fluid models:

(a) Reiner-Rivlin fluids

Reiner [25] and Rivlin [26] developed a theory
of incompressible, viscous, non-Newtonian fluid with a tensorial
non-linearity and is called Reiner-Rivlin fluid. Such fluids
have a natural elasticity and thus account for rigidity of
such substances as gelatine. We may describe such fluids, by
explicitly constitutive equations

\[ \tau_{ij} = -p I + \alpha_0 I + \alpha_1 d_i j + \alpha_2 d_i k d_k \alpha_j, \tag{1.2.1} \]

where \( p \) is a function of density and coefficients \( \alpha_0, \alpha_1, \)
and \( \alpha_2 \) are functions of density and of the three principal
invariants of the stretch tensor \( d_{ij} \). \( I \) is a unit matrix.

As pointed out by Trues Dell and Noll [27] ,
the two normal stress functions in every steady viscometric
flow of any Reiner-Rivlin fluid are equal. It was, however,
found that the two normal-stresses for the case of
polyisobutylene solution are not equal. This experimental evidence is responsible for the rejection of this model (1.2.1). It has provided a motivation for the research of suitable constitutive equations of greater generality in the past few years.

We mention next some constitutive equations, as the results that we are reporting here are based on them. These constitutive equations are more general and more realistic than Reiner-Rivlin constitutive equations.

(b) Oldroyd's B fluid

In an elastic fluid, the properties of an element at \((x,t)\) may depend on previous rheological state through which that element has passed. Oldroyd[28] made a basic assumption that the state of the material element at any point \(x\) and at time \(t\) is not affected by states of the material elements of the surrounding fluid and it is independent of the motion of the fluid element as a whole in space. Therefore, if we exclude all quantities connected with absolute motion in space, the only quantities present in the equation of state are those which define the relative distance of the relative motion of the parts of arbitrary element of the fluid;

The total deformation for a fluid which undergoes an instantaneous elastic response when stressed and which can also flow, will be the sum of the elastic deformation and the deformation due to flow. If \(\mathbf{T}\) is the stress
applied, the elastic deformation is $\frac{T}{G}$ where $G$ is the shear modulus. The instantaneous rate of flow is given by $\frac{T}{\mu}$, where $\mu$ is the viscosity and thus if $\delta$ is the total deformation, then

$$\frac{d\delta}{dt} = \frac{d\delta_e}{dt} + \frac{d\delta_2}{dt} = \frac{1}{G} \frac{d\tau}{dt} + \frac{T}{\mu}$$  \hspace{1cm} (1.2.2)$$

In generalization of (1.2.2), we have

$$\left(1 + \lambda_1 \frac{d}{dt}\right) \tau = 2 \mu \delta_{ij}$$  \hspace{1cm} (1.2.3)$$

where $\lambda_1 = \frac{\mu}{G}$ is the relaxation time for the stress.

In order to include the retardation response of the deformation, Frolich and Sack [29] generalized it further to the form

$$\left(1 + \lambda_1 \frac{d}{dt}\right) \tau = 2 \mu \left(1 + \lambda_2 \frac{d}{dt}\right) \delta_{ij}$$  \hspace{1cm} (1.2.4)$$

where $\lambda_2$ is the strain retardation time. Equation (1.2.4) is the required Hills's equation of state, containing only three parameters, namely viscosity and two relaxation times.
(c) Walter's fluid

Equation of state for this fluid was obtained by Walter [30]. On integration of (1.2.3) and replacing the stress relaxation time $\lambda$, by $\lambda$ we get

$$T_{ij} = \frac{2\mu}{\lambda} \int_{-\infty}^{t} \exp \left[ 1 - \frac{1}{\lambda} (t-t') \right] d\lambda \delta(T) dt'. \quad (1.2.5)$$

The distribution function of relaxation times $N(\lambda)$ is defined such that $N(\lambda) d\lambda$ represents the total viscosity of all Maxwell elements with relaxation times $\lambda$ and $\lambda+d\lambda$. In terms of relaxation times (1.2.3) can be written as

$$T_{ij} = 2 \int_{0}^{\infty} \frac{N(\lambda)}{\lambda} d\lambda \int_{-\infty}^{t} \exp \left[ -\frac{1}{\lambda} (t-t') \right] d\lambda \delta(T) dt'. \quad (1.2.6)$$

and the limiting viscosity at all rates of shear is

$$\mu = \int_{0}^{\infty} N(\lambda) d\lambda. \quad (1.2.7)$$

In introducing the relaxation function defined by

$$\nu(t-t') = \int_{0}^{\infty} \frac{N(\lambda)}{\lambda} \exp \left[ -\frac{1}{\lambda} (t-t') \right] d\lambda \quad (1.2.8)$$

The equation of state (1.2.8) takes the form

$$T_{ij} = 2 \int_{0}^{\infty} \nu(t-t') d\lambda \delta(T) dt'. \quad (1.2.9)$$
It is seen that the equation of state contains the time lag \( (t-t') \) and this is the most general equation of state for visco-elastic fluids. The Oldroyd equation of state (1.2.4) comes as a special case of this equation when is written in a particular form.

(d) Anisotropic fluids.

Anisotropic fluid is another type of incompressible visco-elastic material in which each particle of the fluid has a single preferred direction associated with it. This direction may change with position and time, the motion being governed by the fluid motion itself. Ericksen [18,31] put forward this theory under the assumption that the stress at any point is a function of the velocity gradient and of the preferred direction which is represented by a unit vector \( \eta_i \). He further assumed that the material derivation of the preferred direction depends upon the preferred direction itself together with the velocity gradient.) Ericksen [19,32,33,34] and Hand [35] studied various aspects of this theory.

Green [36] also gave a continuum theory of anisotropic fluids but in a slightly different form. By using the principle of invariance, the constitutive equations of such fluids, in Cartesian coordinate system, can be put down as

\[
\mathbf{T}_{ij} = (-\mathbf{\varepsilon}_{ij} + 2\mu \mathbf{d}_{ij}) + \eta_k \eta_j (\lambda_1 + \lambda_2 \mathbf{d}_{km} \eta_k \eta_m)
+ 2 \lambda_3 (d_{jk} \eta_k \eta_i + d_{ik} \eta_k \eta_j), \quad (1.2.10)
\]

where \( \eta_i = (\omega_{ij} \eta_j) + \lambda (d_{ij} \eta_j - d_{km} \eta_k \eta_m \eta_i) \). \( (1.2.11) \)
\( p \) is the scalar function of the space variables and time. 
\( \lambda, \mu, \mu_1 \) and \( \mu_2 \) are material constants independent of each other. \( \sigma_{ij} \) and \( \omega_{ij} \) are respectively symmetric and antisymmetric parts of the velocity gradient tensor. Exact solutions of simple shear flow and Poiseuille flow have been given for such fluids by Ericksen [31, 37]. Couette flow and Helical flow of these anisotropic fluids have been given by Verma [38]. Leslie [39] has given an exact solution for Hamel flow of these fluids when \( \mu_1 = 0 \). Leslie [40] studied the stability of the Couette flow of such fluids.

(a) Rivlin-Ericksen fluid

Equation of state for this kind of visco-elastic fluid was established by Rivlin-Ericksen [41] following the Reiner-Rivlin [25, 26] approach. All the tensors are defined with respect to Lagrangian coordinate system. The functional dependence of stress tensor on the gradient of velocity and acceleration will be invariant under the operation of translation and rotation. Such fluids do not have any direction of response to the applied stress. Constitutive equations in the simple case commonly encountered give,

\[
\mathbf{T} = \phi_0 \mathbf{I} + 2 \mu \mathbf{E} + \phi_2 \mathbf{D} + \phi_3 \mathbf{E}^2 + \phi_4 \mathbf{D}^2 + \phi_5 (\mathbf{D} \mathbf{D} + \mathbf{D}^2)
\]

\[
+ \phi_6 (\mathbf{E} \mathbf{D} + \mathbf{D} \mathbf{E}) + \phi_7 (\mathbf{E} \mathbf{D}^2 + \mathbf{D} \mathbf{E}^2) + \phi_8 (\mathbf{E}^2 \mathbf{D} + \mathbf{D}^2 \mathbf{E}) \tag{1.2.12}
\]

where \( \mathbf{D} \) is a tensor derived from the acceleration

\[
a = \left[ a_i \right] = \left[ \frac{\partial u_i}{\partial t} + u_m u_{i,m} \right] \tag{1.2.13}
\]
according to the following rule:

\[ D = \begin{bmatrix} D_{i,j} \end{bmatrix} = \begin{bmatrix} a_{i,j} + a_{j,i} + 2 \omega_{m_{i}} u_{m_{j}} \end{bmatrix} \]  
(1.2.14)

and \[ E = d_{i,j} = \frac{1}{2}(u_{i,j} + u_{j,i}) \]  
(1.2.15)

In our investigations, we have used the following simplified form of the Rivlin-Ericksen constitutive equations:

\[ \mathbf{T} = -\mu \mathbf{I} + 2 \mu d_{i,j} + \phi_{2} D + 4 \phi_{3} (d_{i,j})^{2} \]  
(1.2.16)

in which \( \mu \) is called Newtonian viscosity, \( \phi_{2} \) the coefficient of visco-elasticity and \( \phi_{3} \) the coefficient of cross-viscosity.

The constitutive equation (1.2.12) shows that a Rivlin-Ericksen fluid at rest is isotropic. It contains eight parameters which, in general, are functions of the invariants of the tensors of \( D \) and \( E \). The tensor \( D \) incorporates the elasticity of the fluid. If we retain the first two terms of (1.2.12), we get the constitutive equation for Newtonian fluids. Therefore, the linearity of Newtonian constitutive equation is broken by taking the powers and products of tensors \( D \) and \( E \). If we take \( \phi_{2} = 0 \), \( \phi_{3} = 0 \), \( i > 4 \) in equation (1.2.12) we get the Reiner-Rivlin constitutive equation.
1.3 Heat transfer in single and two phase flows

In modern engineering whether it is electrical, mechanical, chemical or atomic, the transfer of energy in the form of heat is an operation which is frequently encountered in almost all phases. Therefore, in recent years the study of heat transfer by laminar flow of viscous incompressible fluids has gained considerable importance. In order to study the phenomenon of heat transfer between a solid body and liquid or gaseous medium we must go into the understanding of the science of fluid motion. The flow of heat is superposed on the physical motion of the fluid and generally the flow field interacts with the temperature field. In order to derive the temperature distribution, the equation of motion need necessarily be combined with the equation of heat conduction. The temperature distribution around a hot body in fluid stream will often have the same character as that of a velocity distribution in boundary layer flow of fluids. The temperature of the stream will increase only over a thin layer in the immediate neighbourhood of the body and over a narrow wake behind it. We may say that the flow phenomenon and thermal phenomenon interact to a high degree.

The dynamics of multiphase systems include momentum, energy, mass and charge transfers between the phases, whether or not the process is influenced by the presence of a potential field.
A multiphase system consists of a fluid phase or fluid medium and a particulate phase of any number of chemical components. When the fluid is a gas, the particulate phase may consist of solid particles or liquid droplets or both. When the fluid medium is a liquid, the particulate phase may consist of solid particles, gas bubbles, or liquid immiscible to the fluid phase.

There are many important multiphase systems in nature as well as in engineering and technological processes. Some of them are listed below:

(i) Atmosphere of the earth.
(ii) Paper-pulp suspensions in paper industry.
(iii) Red cell suspensions in blood.
(iv) Gas solid particle systems:
    pneumatic conveyors, dust collectors, fluidised beds, heterogeneous reactors, metallised propellant rockets, aerodynamic ablation, xerography, cosmic dusts, nuclear fallout problems.
(v) Gas-liquid droplet system:
    atomizers, scrubbers, dryers, absorbers, combustors, agglomeration, air pollution, gascooling evaporation, cyro-pumping.
(vi) Liquid gas bubble systems:
    absorbers, evaporators, scrubbers, air lift pump, cavitation, flotation, aeration.
(vii) Liquid-liquid droplet systems:
Extraction, homogenizing, emulsifying.

(viii) Liquid solid particles systems:
fluidized beds, flotation, sedimentation.

Interest in problems of mechanics of systems
with more than one phase has developed rapidly in recent
years. The mathematical description of such diverse systems
must, of course, vary widely. In problems of fluidization,
for example, the bulk concentration of particles is large
whereas in problems of dust flow this will be small. This
factor may bring considerable simplification to the theory
of dusty gas flows since the effect of one particle on
another is not so pronounced and a good approximation might
be expected by assuming the motion of one solid particle
not to be influenced by the surrounding particles which will
be, in general many particle-diameters away. The work
of the present thesis is concerned with flows under this
simplifying assumption.

Studies of dynamics of multiphase systems have
followed two methods of approach:

(a) Treating the dynamics of single particles and
then trying to extend to a multiple particle
system in an analogous manner as in molecular
(kinetic) theory.

(b) Modifying the continuum mechanics of single
phase fluids in such a way as to account for the
presence of particles.
In our work we have tried to express the properties of multiphase systems in terms of those of their components following the approach (b).

Heat transfer by gas-dust suspensions in duct flow has been the subject of many studies because of the anticipated large heat transfer coefficient due to high volumetric specific heat the demand for high heat-transfer coefficient in gas-cooled reactors.

To obtain the heat-transfer performance and the temperature of dust particles and of liquid it is necessary to set down two energy equations as given by \( \text{boo} [42], \) one for the dust particles and one for the liquid dust mixture. Neglecting eddy diffusivity of heat they are given as

\[
\frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} + \omega_p \frac{\partial T_p}{\partial z} = G (T - T_p) + G_h (T_{\text{w}} - T_p) \quad (1.3.1)
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \omega \frac{\partial T}{\partial z} + \frac{m \nu c_p}{\rho C} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{\beta_1}{\rho} (T_p - T) \quad (1.3.2)
\]

\[
G = \frac{\gamma_p \eta}{m N c_p v_p} \quad G_h = \frac{\gamma_h A_p E_p}{m N c_p v_p} \quad \beta_1 = \frac{m \nu c_p G}{\rho C}
\]

\( T_p \) is the temperature of the dust particle, \( T \) is that of
the liquid, \( T_w \) that of the solid wall, \( h_p \) is the heat-transfer coefficient for flow over the dust particles. \( A_p \) is the surface area of the dust particles, \( V_p \) is the volume of dust particle, \( \sigma \) is the Stefan-Boltzmann constant of radiation, \( \varepsilon \) is the emissivity of the surface of the dust particles, \( \rho N \) is the density of the dust particles, \( \rho \) is the liquid density, \( c \) is the specific heat of liquid, \( \nu \) is the kinematic viscosity of liquid, \( u, v \) and \( w \) and \( u_p, v_p, w_p \) are the velocity components of liquid and dust particles respectively.

Before investigating the interaction phenomenon, it is necessary that in addition to the consideration of equation of motion, the rate of work equation for fluid elements in motion must be established. In case the fluid is incompressible, the rate of work equation or the energy balance is determined by the internal energy, the heat conduction phenomenon, the convection of heat with the stream of the fluid and the heat produced by friction. If the fluid is compressible then the heat due to the work of expansion or contraction because of change in volume is an additional contribution. Heat due to radiation may be neglected, because, at moderate temperature its contribution is small. In the light of the above we recapitulate some of the existing temperature distributions in steady and unsteady situations.
(a) Steady viscous flows:

The exact solution of the temperature distribution for the flow of viscous incompressible fluid in convergent and divergent channels has been worked out by Millsaps and Pohlhansen [43]. They used the transformation

$$T = \frac{\theta(\theta)}{\lambda} + T_w$$

(1.3.3)

Where $T_w$ is the temperature of the walls which is same for both the walls. It has been found that the for divergent channels, the temperature distribution oscillates with maxima and minimum above and below the wall temperature.

Squire [44] has discussed approximately the temperature distribution in the round laminar jet by neglecting the dissipations and using transformation.

$$T = \frac{\theta(\theta)}{\lambda}$$

Seigel and Savinon [45] have presented a theoretical analysis of the temperature distribution in a rectangular duct in fully developed laminar flow, with uniform heat generation within all or part of the broad walls. The Broad walls have a nonzero thermal conductivity over their width and the unheated short walls are assumed to be constant and the velocity distribution of the fluid is considered to be known. Heat conduction in the broad walls
is expressed in terms of an integral equation that is coupled with convective energy equation for the fluid. They obtained analytical solutions for three cases (a) the entire broad wall is internally heated, (b) the heating does not extend to the corner, and (c) the heating extends beyond the corner into the side wall. It is found that if high peak temperatures are to be avoided on the walls, internal heat generation in or beyond the corners should be avoided and the wall material should have a high thermal conductivity.

The momentum and energy equations of a non-dissipative but viscous fluid having constant properties has been solved by Sastry[46] by means of complex variable method using conformal mapping when the fluid flows in a channel with cross section bounded by confocal ellipses. A constant axial pressure gradient and a constant heat source function are assumed. Expressions for average field velocity and mean temperature are obtained analytically. They have given the results for an elliptical cross section also.

Hwang, Knieper and Fan[47] have studied the effects of viscous dissipation on the temperature profile of a laminar, fully developed flow between two flat plates using numerical methods. It has been assumed that the heat flux at the wall is constant along the length of the
flat plates without any restrictions on the values of the
Prandtl number. Mercer, Pearce and Hitchcock [48] have
analysed the conditions where the momentum and thermal
boundary layers are simultaneously developing in the
entrance region between parallel flat plates. The
analysis is a numerical finite difference solution of the
momentum and energy equations in stream function
coordinates.

The temperature distribution in plane
laminar one dimensional flow with the usual assumption
of convection parallel to the flow and conduction transverse
to the flow has been obtained by Hahnemam [49]. Using
the separation of variable technique, he has expressed
the fluid temperature as a product of an exponential
term which decays in the direction of the flow, and a
power series which expresses the temperature variation
transverse to the flow.

Rao, Ramacharyulu and Krishnamurthy [50] have
solved a simplified form of the energy equation for
laminar force convection in elliptical ducts. They have
found the expressions for the average and local Nusselt-
numbers and wall temperatures for (a) constant wall
temperature and (b) constant heat flux at the wall.
Deavours [51] also obtained an exact solution for the
fluid temperature due to laminar heat transfer in parallel
plate flow for an arbitrary velocity profiles.
The temperature distribution of a viscous liquid flowing through a circular pipe has been calculated by Graetz [52] and Nusselt [53] by solving the equation

\[ \frac{D \Theta}{D t} = k \nabla^2 \Theta \quad \text{where} \quad \Theta = \frac{T - T_i}{T_o - T_i} \]

The origin is taken at the centre of the pipe whose temperature changes from \( T_o \) to \( T_i \). The problem in which the wall of the circular pipe is kept at a uniform temperature gradient and the fluid allowed to flow through it is solved by Nusselt, Eagle and Ferguson [54] in the form

\[ T = A \zeta + g(\zeta) \quad (1.3.4) \]

where \( A \) is a constant temperature gradient along the \( \zeta \)-axis.

Schenk and Dumore [55] have obtained the temperature distribution for flow through a circular tube when the effect of finite transmissivity of the wall is included. The heat transfer in a concentric circular annulus when the heat flux at either or both of the walls varies around the periphery but is independent of axial distance has been analyzed by
Sutherland and Kays [56] have discussed the laminar as well as the turbulent regions. The wall temperature variations are found to be substantial in both turbulent and laminar flows. Mori and Nakayama [57] have analysed the velocity and temperature fields of a fully developed laminar flow in a straight pipe rotating about a parallel axis under a constant wall temperature gradient. The flow and temperature fields have been divided into a core region, where the effect of a secondary flow is predominant. Bhattacharyya and Roy [58] have investigated the temperature distribution in the thermal entrance region for fully developed laminar flow in a pipe. Jones [59] has analysed the temperature field in a source free fluid, flowing along a circular tube with negligible viscous dissipation for (a) a step change in wall temperature and (b) a ring heat source on an otherwise insulated wall. Kalb and Seader [60] have analysed numerically the steady and fully developed heat transfer in a laminar flow through a curved circular tube under the assumption that the transport coefficients and constants, the energy dissipation is negligible and the wall temperature is uniform but varies linearly in the axial direction. Ou and Cheng [61] have derived analytical expressions for fluid and wall temperature distributions in a circular pipe with parabolic velocity profile considering viscous dissipation.
Soo [42] has obtained solutions of the transient forced convection energy equations of dust particles and of liquid in circular pipe with given constant wall temperature when the inlet temperatures of dust particles and of liquid are constant across the flow. He has reduced this problem to a Sturm-Liouville system with two linear homogeneous boundary conditions. He has shown that the asymptotic Nusselt numbers for the case of uniform wall temperature are the same for both two-phase and single-phase systems.

We investigate unsteady flow cases with or without heat transfer for different geometries with a view to analyse how we can precipitate or slow down the heat transfer in liquids.
(b) Unsteady viscous flows:

Unsteady flow plays an important role in present day technology. It has application in turbomachinery, aerospace technology and chemical engineering problems etc. It can be broadly divided into two parts: (i) Flow due to unsteady motion of the plate or plate temperature depends on time, (ii) Oscillatory free-stream or oscillatory surface temperature. The assumptions under which these problems are studied in the literature have been discussed in text books by Rosenhead [62], Poi [63], and Schlichting [64] etc. The recent work in the field is presented below so that the development in this field of research is clearly understood.

The first solution of a Navier-Stokes equations governing the flow past an impulsively started infinite flat plate in a stationary viscous incompressible fluid was given by Stokes [3] which is known as Rayleigh's problem also. The corresponding MHD Stokes problem for horizontal plates was presented by Rossow [65], Chang and Yen [66], Ludford [67], Drake [68]. On taking into account the free convection currents and under usual Boussinesq approximation, the flow past an infinite vertical plate as shown by Soundalgekar [69] govern the following equations in non-dimensional form.
\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G \theta , \quad (1.3.6)
\]

\[
P \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} , \quad (1.3.7)
\]

\[
t < 0 , \quad u(y,t) = 0 , \quad \theta(y,t) = 0 , \\
t > 0 , \quad u(0,t) = 0 , \quad \theta(0,t) = 1 , \\
\quad u(\infty,t) = 0 , \quad \theta(\infty,t) = 0 , \quad (1.3.8)
\]

where \( G = \frac{\nu g \beta (T_w' - T_{w0})}{u_0^3} \), \( P = \frac{\mu \lambda h}{k} \), \( t = \frac{t' u_0^2}{\nu} \),

\[
\mu = \frac{\mu'}{u_0} \quad \gamma = \frac{y' u_0}{\nu} \quad \theta = \frac{T - T_{w0}'}{T_w' - T_{w0}'} \quad \eta = \frac{y}{2 \nu k} ,
\]

\( T_w' \) being the temperatures of the plate and \( T_{w0}' \) the temperature of the fluid far away from the plate. The closed solution were derived by Laplace transformation technique for \( G > 0 \) i.e. cooling of the plate by free-convection currents. It has been observed that at large values of \( G > 0 \), separation may occur near the plate in case of air and skin friction is also more. The rate of heat transfer is directly proportional to \( \sqrt{P} \) and inversely proportional to \( \sqrt{\eta} \). The corresponding problem at
normal temperature and pressure for no free-convection has been solved by Illingworth [70], Elliott [71]. Taking the presence of free convection currents as an initial conditions the stokes problem for an infinite vertical plate has solved by Soundalgekar and Pop [72,73] and the problem of motion of fluid past a vertical plate accelerated uniformly in its own plane has been analysed by Soundalgeskar and Gupta [74] for isothermal and constant heat flux case. However, for rarefield medium this problem for horizontal plate (under first order velocity slip and temperature-jump boundary condition) has been analysed by Eckert and Droke [75] and for vertical plate it has been studied by Soundalgekar, Pohsnerkar and Patil [76]

In all these papers, the effects of viscous dissipation are neglected and hence the governing equations are couples and linear. When viscous dissipation effects are included, the governing equations are couples and non-linear and hence exact solutions for the Stokes problem are not possible. The governing equations in non-dimensional forms now become:

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G \theta, \quad (1.3.9)
\]

\[
\rho \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + \rho E \left( \frac{\partial u}{\partial y} \right)^2. \quad (1.3.10)
\]
where
\[
\begin{align*}
t &= t'(c^2 \nu)^{1/3}, \quad y = y'(c \nu^2)^{1/3}, \\
\theta &= \frac{T' - T_{\infty}}{T_w - T_{\infty}}, \quad \rho = \frac{\mu c_p \nu}{k}, \quad G = \frac{\beta (T_w - T_{\infty})}{c}, \\
E &= \frac{(\nu \sigma)^{1/3}}{c_p (T_w - T_{\infty})}. \tag{1.3.11}
\end{align*}
\]

The boundary conditions are
\[
\begin{align*}
u(y,t) &= 0, \quad \theta(y,t) = 0, \quad \text{for } t < 0, \\
\nu(0,t) &= t, \quad \theta(0,t) = 1, \quad \text{for } t > 0, \\
\nu(\infty,t) &= 0, \quad \theta(\infty,t) = 0, \quad \text{for } t > 0. \tag{1.3.12}
\end{align*}
\]

The effects of free-steam oscillations on the flow past horizontal bodies for small amplitude oscillations were studied by Moore [11], Lighthill [18]. The case of finite amplitude assumptions was studied by Lin [79]. The oscillatory flow near a stagnation point was studied independently by Rott [80], Glaart [81]. The numerical solution of oscillatory flow past a semi-infinite plate has been presented by Nickerson [82]. In his paper Ishigaki [83] has discussed the oscillatory flow near a two-dimensional stagnation point and the results for skin-friction and surface temperature in case of an insulated flat plate has been given by Ishigaki [84]. The heat transfer aspect of flat plate and constant temperature and for small and finite-amplitude case has been discussed by...
Ishigaki [85], and the two-dimensional stagnation point flow for both small and finite-amplitude case has been by Ishigaki [86]. It has been observed that the time-mean heat transfer decreases at low frequency but slightly increases at high frequency. In Ishigaki [87], the unsteady Couette flow with one of its plate oscillating about a constant mean velocity has been discussed. Mori and Tokuda [88] have studied the effects of free-stream oscillations a heat transfer in flow past a cylinder, the theoretical results have been confirmed by experiments also. Yoshizawa [89] has investigated the unsteady viscous flows past a semi-infinite flat plate on the basis of the Stokes approximation. The local shear stress is calculated at the leading edge region and the far down stream region. Unsteady viscous flow due to the impulsive motion of a flat plate, with special reference to the initial period is analysed by Yoshizawa [90]. He has shown that some of the undetermined constants in the present solution can be determined by matching procedure, if the unsteady Stokes solution for this problem is obtained asymptotically near leading edge.

The effects of free-stream oscillations on flow past an infinite porous plate with constant suction were studied by Stuart [91]. He found that these exist a reverse type of flow. He also studies the heat transfer problem. The MHD aspects of Stuart's problem was studied
by Suryaparkashrao [92,93]. The case of variable suction and free stream oscillation on an infinite porous plate was studied by Messina [94]. Reddy [95] studied the effects of rarefaction of the medium on Stuart's problem and the corresponding MHD problem was studied by Soundalgekar [96]. Kelley [97] studied the flow past an infinite porous plate with time-dependent suction and showed that the mean flow is affected by the frequency of the oscillations. The effect of magnetic field, rarefaction of the medium and visco-elastic property on oscillatory flows have been studied by Soundalgekar and his co-workers [98 to 104]. Nanda and Sharma [105] considered unsteady free convective flow past a semi-infinite plate in which the effect of oscillatory wall temperature was calculated. In another paper Nanda and Sharma [106] showed that the similarity solution exists for the problem under reference [107]. Soundalgekar [108] presented an analysis of the two-dimensional flow of an incompressible fluid past an infinite porous plate when (i) the suction velocity normal to the plate is constant, (ii) the free stream velocity oscillates in time about a constant mean (iii) the plate temperature is constant (iv) the difference between the temperature of the plate and free stream is moderately larger, resulting in free convection currents. Approximate solution for coupled non-linear equations are obtained for velocity and temperature fields. Expressions for the mean
velocity, the mean temperature and the mean skin friction are derived. There is a reverse flow of the mean velocity profile of fluids with small , in the boundary layer close to a plate which is being heated by the free convection current. The mean skin friction increases with more cooling of the plate and decreases with more heating of the plate. Soundalgekar [109] first studied the effect of free-convection currents and free-stream oscillations on the flow past an infinite vertical porous plate with constant suction. The problem is governed by the following coupled non-linear equations in non-dimensional form:

\[ \frac{1}{4} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{4} \frac{\partial \theta}{\partial t} + C \theta + \frac{\partial^2 u}{\partial y^2}, \quad (1.3.13) \]

\[ \frac{\rho}{4} \frac{\partial \theta}{\partial t} - \frac{\rho}{4} \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + pE \left( \frac{\partial u}{\partial y} \right)^2 \quad (1.3.14) \]

and the boundary conditions are:

\[ u = 0, \quad \theta = 1 \quad \text{at} \quad y = 0 \quad (1.3.15) \]

\[ u = U(t) = 1 + \epsilon e^{i\omega t}, \quad \theta = 0 \quad \text{as} \quad y \to \infty. \]
The equations are solved in closed form for mean velocity, mean temperature, mean skin-friction, mean rate of heat transfer, the transient velocity and transient temperature, the amplitude and phase of the skin-friction and the rate of heat transfer. The effect of different parameters like $P$, $E$, the frequency and heating or cooling of the plate have been discussed. It is observed that in the presence of the plate being heated by free convection currents, there exist a reverse type of flow in case of air. Because of greater viscous dissipative heat for $G > 0$ or the greater cooling of the plate, the amplitude of the skin friction increases, where as owing to the greater viscous dissipative heat $G < 0$ or to greater heating of the plate, the amplitude of skin-friction decreases. But the amplitude of the rate of heat transfer increases owing to greater heating or cooling of the plate. However, it is more when the plate is cooled than when the plate is heated. All those oscillatory free convection and free-stream flows are important from engineering point of view.
1.4 Scope of the thesis

Time dependent flows of Newtonian or non-Newtonian fluids have been reported in the literature in the last. As many of these have been found to be applicable in Bio-Physics and other important engineering problems, like acoustics, biomedical engineering, lubrication, design of thrust bearing and diffusers etc., research workers have shown interest again in recent years. Bauer [110] has discussed the non-linear steady flow case under constant reabsorption. He has also presented the pulsatile flow for constant and exponentially decaying reabsorption across the wall. Wang [111] discussed the pulsatile flow between a porous plate channel and the corresponding problem of heat transfer was studied by Haiti.

The unsteady flow of dusty-gas in a pipe flow has been presented by Kishore and Pande [112]. The flow past an oscillating plate in rotating frame of dusty gas has been studied by Datta and Jana [113]. The unsteady flow of a dusty viscous liquid in a rotating channel has been presented by Dube and Sharma [114]. Singh and Dube [115] Dube and Sharma [116] have solved the unsteady flow of dusty gas in pipe and channel respectively.
In the present thesis, solutions of several specific flow problems with or without heat transfer are obtained when either the temperature or the pressure is a function of time. Care has been taken to choose those problems which may be amenable to experiment. In the two phase flow problems, which form the second chapter of the thesis, attempts have been made to express the properties of such systems in channel and pipe flows by modifying the continuum mechanics of single-phase fluids in such a way as to account for the presence of particles. In the two-phase flow heat transfer in a channel when the inlet temperature varies linearly with time, for example, it is found that the effect of the presence of the particles is to increase the heat transfer. In the two-phase flow heat transfer in a circular pipe when the inlet temperature varies periodically with time, a new solution have been found to a heat transfer problem discussed by Soo [42]. The effect of various parameters on the amplitudes of the dust particles, liquid dust mixture and clear liquid is calculated. It is found that the effect of the dust particles is to flatten the temperature profile and consequently increase the heat transfer. The phase lags are same for single phase and two-phase systems. Also, it is seen that with the increase of the Reynolds number, the phase lag decreases and it increases with the increase of the inlet frequency.
In case of single-phase flow heat transfer, firstly the problem of unsteady combined free and forced convection flow of a viscous incompressible fluid between two horizontal parallel walls with a linear axial temperature variation is considered. Initially the walls and the fluid are at the same temperature and there is no flow. The effect of the dimensionless physical parameters characterizing the flow on the velocity, the skin-friction and temperature distribution has been discussed. It is found that the velocity profiles and temperature profiles are symmetric due to the presence of buoyancy force. The skin friction at the two plates has been calculated for different times. Secondly, an exact solution of the transient forced convection energy equation of a viscous incompressible fluid with fully developed flow in a parallel plate channel is obtained under prescribed boundary conditions when the inlet temperature varies sinusoidally with time. Expression for the dimensionless temperature distribution between the plates is obtained. The case of homogeneous temperature distribution on the wall is discussed in detail. Further, in the case of unsteady convection effects on flow between two horizontal parallel walls with linear axial temperature variation when the pressure drops with time, the effect of the Grashof number of velocity, and temperature along the channel have been obtained. It is found that with the increase of the Grashof number the velocity and temperature both increase in the lower half.
and decreases in the upper half of the channel and the situation is reversed for negative values of the Grashof number. The rate of change is precipitated with the passage of time.

Unsteady flow of a Rivlin-Erickson liquid in a rotating channel is investigated when (a) pressure gradient is constant, (b) pressure gradient varies linearly with time, (c) pressure gradient varies periodically with time. The effect of the rotation parameter and the elastic number on the velocity of the fluid is studied. It is found that the Coriolis forces conspire to produce a flow situation against the pressure gradient and that the elastic parameter decreases the speed of flow in all these cases.

Mostly, the technique of Laplace transformation has been employed to solve differential equations in the thesis. In Chapter II, problem (b), a new solution to a heat transfer problem discussed by Professor Soo [42] on page 155 of his book has been obtained. One of the consequences of problem (c), in Chapter IV, (i.e. when the frequency of the pressure gradient matches with twice the reciprocal of Ekman number, the liquid assumes extremely high velocity) can be exploited to transport fluids at very high speeds. This can have potential use in chemical and petroleum industry.
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The contents of part (b) of Chapter II were presented at the 22nd Congress of ISTAM held at S.V.R., College of Engineering and Technology, Surat in December, 1977.