CHAPTER VI

THERMO ELASTIC-PLASTIC AND CREEP TRANSITION IN AN ORTHOTROPIC ROTATING CYLINDER

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6.1 **Introduction**

The problem of thick-walled rotating cylinders without thermal effects has been discussed by Davis and Connelly [17] and Rimrott [99] in plastic theory and by Rimrott and Luke [100] and Dev [21] in creep theory. Gupta [35] analysed the above problem with no thermal effects by using Seth's transition theory discussed in Section (1.3).

In this chapter, the problem of elastic-plastic and creep transition of thick walled rotating cylinders under steady state temperature has been solved by using Seth's transition theory. The results for the combined effects of rotation and temperature have been calculated and discussed. In the absence of thermal effects, the results obtained are the same as given by Rimrott and Luke [99,100] and Gupta [35].

6.2 **Governing Equations**

We consider a thick-walled cylinder of internal and external radii 'a' and 'b' respectively, rotating about its axis with an angular velocity \( \omega \) and subjected to a steady state of temperature \( \Phi \) on the inner surface at \( \lambda = a \). The components of displacement in cylindrical coordinates are given by [35,36,140]:

\[
\begin{align*}
    u &= \lambda (1-\beta) , \\
    \psi &= 0 , \\
    \omega &= \varphi , \\
    \end{align*}
\]  \hspace{1cm} (6.2.1)
where $\beta$ is a function of $\lambda = (x^2 + y^2)^{\frac{1}{2}}$ only and $\alpha$ is a constant.

Substituting the values of $\psi$, $\psi$, $\omega$ in equation (1.2.6), we get,

$$
\varepsilon_{\lambda\lambda} = \frac{1}{2} \left[ 1 - (\lambda \beta^4 + \beta)^{\frac{1}{\alpha}} \right], \\
\varepsilon_{\phi\phi} = \frac{1}{2} \left[ 1 - \beta^2 \right], \\
\varepsilon_{zz} = \frac{1}{2} \left[ 1 - (1-d)^2 \right], \\
\varepsilon_{x\phi} = \varepsilon_{\phi z} = \varepsilon_{z\lambda} = 0.
$$

The generalized components of strain from equation (1.4.2) are

$$
\varepsilon_{\lambda\lambda} = \frac{1}{\eta m} \left[ 1 - (\lambda \beta^4 + \beta)^{\eta} \right]^m, \\
\varepsilon_{\phi\phi} = \frac{1}{\eta m} \left[ 1 - \beta^{\eta} \right]^m, \\
\varepsilon_{zz} = \frac{1}{\eta m} \left[ 1 - (1-d)^{\eta} \right]^m, \\
\varepsilon_{x\phi} = \varepsilon_{\phi z} = \varepsilon_{z\lambda} = 0.
$$

Using equation (6.2.3) in equation (1.5.2), the components of stress are

$$
\gamma_{\lambda\lambda} = \frac{c_{11}}{\eta m} \left[ 1 - (\lambda \beta^4 + \beta)^{\eta} \right]^m + \frac{c_{12}}{\eta m} \left[ 1 - \beta^{\eta} \right]^m + \frac{c_{13}}{\eta m} \left[ 1 - (1-d)^{\eta} \right]^m - \alpha_1 \sigma, \\
\gamma_{\phi\phi} = \frac{c_{21}}{\eta m} \left[ 1 - (\lambda \beta^4 + \beta)^{\eta} \right]^m + \frac{c_{22}}{\eta m} \left[ 1 - \beta^{\eta} \right]^m + \frac{c_{23}}{\eta m} \left[ 1 - (1-d)^{\eta} \right]^m - \alpha_2 \sigma, \\
\gamma_{zz} = \frac{c_{31}}{\eta m} \left[ 1 - (\lambda \beta^4 + \beta)^{\eta} \right]^m + \frac{c_{32}}{\eta m} \left[ 1 - \beta^{\eta} \right]^m + \frac{c_{33}}{\eta m} \left[ 1 - (1-d)^{\eta} \right]^m - \alpha_3 \sigma, \\
\gamma_{x\phi} = \gamma_{\phi z} = \gamma_{z\lambda} = 0.
$$
The equations of equilibrium are all satisfied except

\[ \frac{d}{d \lambda} \left( \tau_{A} \right) + \frac{\tau_{A} - \tau_{B}}{\lambda} + \sigma_{\omega} = 0. \quad (6.2.5) \]

where \( f \) is the density of the material.

The temperature field satisfying equation (1.5.a) and

\[ \Theta = \Theta_0 \quad \text{at} \quad \lambda = 0 \]

\[ \Theta = 0 \quad \text{at} \quad \lambda = b. \]

where \( \Theta_0 \) is constant, is given by \([34]\)

\[ \Theta = \Theta_0 \frac{\log \frac{b}{a}}{\log \frac{b}{a}}. \quad (6.2.6) \]

Using equations (6.2.4) and (6.2.6) in equation (6.2.5), we get a non-linear differential equation \( \beta \) as

\[
\rho (\beta + 1) \left[ 1 - \beta^p (\beta + 1)^n \right] \frac{\beta d \beta}{d \beta} + \rho (\beta + 1) \left[ 1 - \beta^p (\beta + 1)^n \right] \frac{\beta d \beta}{d \beta} + \frac{\bar{\alpha} \bar{m} \bar{n} \bar{b}}{\bar{m} \bar{n} \bar{b}} \left[ (\bar{c}_{1} - \bar{c}_{2}) \left[ 1 - (1 - d)^n \right] + (\bar{c}_{1} - \bar{c}_{2}) \left[ 1 - \alpha^m \right] + (\bar{c}_{3} - \bar{c}_{23}) \left[ 1 - \alpha^m \right] \right] \frac{\rho \bar{\omega}^2}{\bar{c}_{1} \bar{m} n \bar{b}^n} = 0.
\]

(6.2.7)

where \( \bar{p} \beta' = \rho \beta \) and \( \bar{\theta}_0 = \frac{\Theta_0}{\log \frac{a}{b}}. \)
For \( m = 1 \), which holds good for secondary stage of creep, equation (6.2.7) reduces to

\[
\left[ \rho (\rho + 1)^n + \frac{C_{12} \rho}{C_{22}} + \frac{\alpha_1 b_0}{C_{n} \rho^n} - \frac{1}{n C_{22} \rho^n} \left\{ (C_{11} - C_{22}) (1 - \rho (\rho + 1)^n) + (C_{12} - C_{22}) (1 - \rho^n) \right\} + (C_{12} - C_{22}) (1 - (\rho + 1)^n) + (C_2 - C_1) \rho + \mathcal{R} \frac{\partial^2 \mathcal{R}}{C_{n} \rho^n} \right] \frac{d \rho}{d \rho} + \rho \mathcal{R} (\rho + 1)^{n-1} = 0
\]  

(6.2.8)

The transition point of \( \beta \) in equation (6.2.7) are \( \rho \to 0 \pm \infty \).

The boundary condition requires that

(i) \( \Gamma_{z}^{a} = 0 \) at \( \xi = a \) and \( \xi = b \).  

(ii) The resultant force normal to the plane \( \xi = \text{constant} \) must vanish i.e.

\[
\int_{a}^{b} \xi \Gamma_{z}^{z}, d \xi = 0
\]

(6.2.10)

6.3 Asymptotic Solution Through \( \rho \to \pm \infty \).

In this section it is shown that \([35-44, 57, 135]\) the asymptotic solution through the principal stress leads from elast to plastic state at the transition point \( \rho \to \pm \infty \). We define the transition function \( \mathcal{R} \) as,
Substituting the value of $\eta_{\text{eq}}$ from equation (6.2.4) in equation (6.3.1), we get

$$R = 1 - \frac{\eta_{\text{eq}}'}{c_{11} + c_{12} + c_{13}} - \frac{(c_{11} - c_{12})(\rho_{+} + \rho_{-})}{c_{11}(c_{11} + c_{12} + c_{13})} - \frac{\eta_{\text{eq}}'}{c_{11}(c_{11} + c_{12} + c_{13})}$$  \hspace{1cm} (6.3.1)

Taking logarithmic differentiation of equation (6.3.2) with respect to $x$, we have

$$\frac{d \log R}{dx} = \frac{d}{dx} \left[ \frac{\eta_{\text{eq}}'}{c_{11} + c_{12} + c_{13}} \right]$$  \hspace{1cm} (6.3.3)

Substituting the value of $\frac{dp}{d\beta}$ from equation (6.2.8) in equation (6.3.3), we get

$$\frac{d \log R}{d\beta} = \frac{\eta_{\text{eq}}'}{c_{11} + c_{12} + c_{13}} \left[ \frac{\eta_{\text{eq}}'}{c_{11} \beta n} + \frac{1}{n c_{11} \beta n} \left\{ (c_{11} - c_{12}) (1 - \beta \eta_{\text{eq}} + \rho_{+} + \rho_{-}) + (c_{12} - c_{13}) \right\} + \frac{\alpha_{1} \beta_{0}}{c_{11} \beta n} \right]$$  \hspace{1cm} (6.3.4)
Asymptotic value of equation (6.3.4) as $\rho \to \pm \infty$ is,

$$\frac{d}{d\rho} \log R = -\frac{(c_{11} - c_{22})}{c_{11}} \frac{1}{\rho} \quad (6.3.5)$$

Integration of equation (6.3.5), yields

$$R = \frac{c_{11} - c_{22}}{c_{11}} - \frac{c_{11} - c_{22}}{c_{11}} \quad (6.3.6)$$

where $v_0$ is a constant of integration.

From equation (6.3.1) and (6.3.6), we obtain

$$\gamma = \frac{(c_{11} + c_{12} + c_{13})}{\gamma} \left[ 1 - \frac{c_{11} - c_{22}}{c_{11}} \right] - \frac{c_{11} - c_{22}}{c_{11}} \alpha_1 \Phi - \frac{\rho^3 \gamma^2}{2} \quad (6.3.7)$$

The value of material constant in the transition range is given by [136].

$$\gamma = \frac{1}{n} \left[ \frac{c_{11} c_{22} c_{33} - c_{11} c_{33}^2 - c_{33} c_{12}^2 + 2 c_{12}^2 c_{23} - c_{12}^2 c_{23}}{c_{12} c_{33} - c_{23}^2} \right] \quad (6.3.8)$$

Using equation (6.3.8) in equation (6.3.7), we have

$$\gamma = \frac{\left[ c_{11} c_{22} - c_{23}^2 \right] \left[ c_{11} + c_{13} + c_{12} \right] \gamma}{c_{11} c_{22} c_{33} - c_{23}^2 c_{11} - c_{33} c_{12}^2 + 2 c_{12}^2 c_{23} - c_{12}^2 c_{23}} - \frac{\left( c_{11} - c_{22} \right) \alpha_1 \Phi}{c_{11}} - \frac{\rho^3 \gamma^2}{2} \quad (6.3.9)$$
Substituting equation (6.3.9) in equation (6.2.6), we get transitional stress \( \gamma_{\phi} \) as

\[
\gamma_{\phi} = \frac{[c_{22}c_{33} - c_{23}^2][c_{11} + c_{22} + c_{13}]\gamma}{c_{11}c_{22}c_{33} - c_{11}c_{23}^2 - c_{33}c_{12}^2 + 2c_{12}c_{23} - c_{13}c_{22}}
\]

\[
- \left(\frac{c_{11}c_{21}}{c_{11}}\right)\alpha_1 \beta b \left[ \log \frac{\xi}{\beta_b} + 1 \right] = \frac{\omega_b^2 \lambda^2}{\gamma}.
\]  

Equation (6.2.4) gives

\[
\gamma'_{zz} = \frac{c_{31}}{c_{11} + c_{21}} (\gamma'_{AA} + \gamma'_{B}) + \kappa_1 [\alpha_1 + \alpha_2] \frac{c_{31}}{c_{11} + c_{21}} - \alpha_3 + 2\kappa_1.
\]

where

\[
\kappa_1 = \frac{1}{2} \left[ \left( c_{32} - \frac{c_{31}(c_{12} + c_{22})}{(c_{11} + c_{21})} \right) \gamma B + \left[ c_{33} - \frac{c_{31}(c_{13} + c_{23})}{(c_{11} + c_{21})} \right] c_{zz} \right].
\]

and

\[
c_{zz} = \frac{1}{\gamma} \left[ 1 - (1-d)^n \right].
\]

Using boundary condition (6.2.9) and (6.2.10) in equation (6.3.9) and (6.3.11), we get
\[ v_0 = \left(1 - \frac{\alpha \beta_0 (c_{11} - c_{21})}{c_{11}} \frac{c_{11} c_{22} c_{33} - c_{11} c_{23}^2 - c_{33} c_{12}^2 + 2 c_{12} c_{23} - c_{13} c_{22}}{c_{22} c_{33} - c_{23}^2} \frac{\alpha}{c_{11} + c_{12} + c_{13}} \cdot \gamma \right) \]\[ (6.3.12) \]

\[ \frac{w^2}{\tilde{E}} = \left[ \frac{\alpha \beta_0 (c_{11} - c_{21}) (y_0)}{c_{11}} \left(\frac{c_{11} c_{22} c_{33} - c_{11} c_{23}^2 - c_{33} c_{12}^2 + 2 c_{12} c_{23} - c_{13} c_{22}}{c_{22} c_{33} - c_{23}^2} \frac{\alpha}{c_{11} + c_{12} + c_{13}} \cdot \gamma \right) - \frac{(c_{11} - c_{21})}{c_{11} + c_{21}} \right] \]

\[ \left[ b^2 - a^2 (\frac{y_0}{c_{11}}) - \frac{(c_{11} - c_{21})}{c_{11} + c_{21}} \right] \]

\[ (6.3.13) \]

and

\[ k_1 = - \frac{c_{21}}{c_{11} + c_{21}} \frac{\frac{1}{4} \frac{3}{4} (a^2 + b^2)}{4} - \tilde{\beta}_0 \left[ (\alpha_1 + \alpha_2) \frac{c_{21}}{c_{11} + c_{21}} - \alpha_3 \right] \left[ \frac{a^2 - b^2}{4} - \frac{a_3^2}{2} \log \frac{y_0}{y_1} \right] \]

\[ (6.3.14) \]

Substituting the value of \( \nu_0 \) and \( k_1 \) from equation (6.3.12) and (6.3.14) in equation (6.3.9) - (6.3.11), we get transitional stresses as
\[
\begin{align*}
G'_{\Delta K} &= \frac{(C_{22} C_{33} - C_{23}^2)}{(C_{11} C_{22} C_{33} - C_{33} C_{12}^2 - C_{11} C_{23}^2 + C_{11} C_{22} + C_{11} C_{23}) \gamma \left[ 1 - \left( \frac{a}{\lambda} \right) \frac{C_{11} - C_{21}}{C_{11}} \right]} \left\{ 1 - \frac{C_{11} - C_{21}}{C_{11}} \gamma \left[ 1 - \left( \frac{a}{\lambda} \right) \frac{C_{11} - C_{21}}{C_{11}} \right] \right\} \\
G'_{\delta \Phi} &= \frac{(C_{22} C_{33} - C_{23}^2)}{(C_{11} C_{22} C_{33} - C_{33} C_{12}^2 - C_{11} C_{23}^2 + C_{11} C_{22} + C_{11} C_{23}) \gamma \left[ 1 - \left( \frac{a}{\lambda} \right) \frac{C_{11} - C_{21}}{C_{11}} \right]} \left\{ 1 - \frac{C_{11} - C_{21}}{C_{11}} \gamma \left[ 1 - \left( \frac{a}{\lambda} \right) \frac{C_{11} - C_{21}}{C_{11}} \right] \right\} \\
\Gamma_{Z Z} &= \frac{C_{31}}{C_{11} + C_{21}} \left( G'_{\Delta K} + G'_{\delta \Phi} \right) - \frac{C_{31}}{C_{11} + C_{21}} \frac{2 \lambda^2 a^2}{\kappa} \left( \frac{a^2 + b^2}{\kappa} \right) + \frac{2 \lambda^2}{\kappa} \left[ \left( \alpha_1 + \alpha_2 \right) \frac{C_{31}}{C_{11} + C_{21}} - \alpha_3 \right] - \frac{2 \lambda^2}{\kappa} \left[ \left( \alpha_1 + \alpha_2 \right) \frac{C_{31}}{C_{11} + C_{21}} - \alpha_3 \right] \left[ \frac{a^2}{\kappa} - b^2 \right] - \frac{2 \lambda^2}{\kappa} \left[ \left( \alpha_1 + \alpha_2 \right) \frac{C_{31}}{C_{11} + C_{21}} - \alpha_3 \right] \left[ \frac{a^2}{\kappa} - b^2 \right] \left[ \frac{a^2}{\kappa} \log a_1 b \right].
\end{align*}
\]
It is found that the value of \( \gamma_{A-H} - \gamma_{H} \) is maximum at \( x=a \), which means that the yielding of the cylinder will take place at the internal surface. In this case equation (6.3.15) gives,

\[
\left| \gamma_{A-H} - \gamma_{H} \right| = \frac{\left( c_{31}c_{33} - c_{33}^2 \right)}{c_{11} \left( c_{11}c_{33} - c_{33}^2 - c_{33} c_{21}^2 - c_{21}^2 c_{11}^2 - 2 c_{33} c_{11} + 2 c_{33} c_{21} - c_{21} c_{11} \right)} \times \left\{ 1 - \alpha' \beta_0 \left( \frac{c_{11} - c_{21}}{c_{11}} \right) \right\} \gamma \frac{c_{11} c_{22} c_{33} - c_{11} c_{23}^2 - c_{33} c_{22}^2 + 2 c_{11} c_{23} - c_{21} c_{22}}{c_{22} c_{33} - c_{23}^2 \cdot (c_{11} + c_{21} + c_{31}) \cdot \gamma} - \alpha' \beta_0 \left( \frac{c_{11} - c_{21}}{c_{11}} \right) \right) \equiv \gamma_1
\]

(6.3.16)

Substituting the value of \( \gamma \) in terms of \( \gamma_1 \) in equation (6.3.13) and (6.3.15), we get orthotropic transitional stresses and a relation between \( \omega \) and temperature as,

\[
\gamma_{A-H} = \gamma_1 \frac{c_{11} - c_{21}}{c_{11}} \left[ 1 - \left( \frac{c_{11} - c_{21}}{c_{11}} \right) \right] + \alpha' \beta_0 \left[ \frac{\left( c_{11} - c_{21} \right)^2}{c_{11}} \right] \left( \frac{\log q_1}{\log q_{1b}} \right) + \frac{\rho q_2 b^2 (a^2 - b^2)}{2}
\]

\[
\gamma_{P-H} = \gamma_1 \frac{c_{11}}{c_{11} - c_{21}} \left[ 1 - \left( \frac{c_{11} - c_{21}}{c_{11}} \right) \left( \frac{c_{11} - c_{21}}{c_{11}} \right) \right] + \alpha' \beta_0 \left[ \frac{\left( c_{11} - c_{21} \right)^2}{c_{11}} \right] \left( \frac{\log q_1}{\log q_{1b}} \right) + \frac{\rho q_2 (a^2 - b^2)}{2}
\]

\[
\gamma_{Z-Z} = \gamma_1 \frac{c_{31}}{c_{11} + c_{21}} \left( \gamma_{A-H} + \gamma_{P-H} \right) + (\alpha_1 + \alpha_2) \frac{c_{31}}{c_{11} + c_{21}} - \alpha_2 \left[ \frac{\rho q_2 b^2}{b^2 - a^2} \left( \frac{a^2 + b^2}{a^2} \right) \log q_{1b} \right]
\]

\[
= \frac{c_{31}}{c_{11} + c_{21}} \cdot \frac{\rho q_2 b^2}{b^2 - a^2} \left( \frac{a^2 + b^2}{a^2} \right)
\]

(6.3.17)
and

\[
\frac{p_{V_{3}}}{2} = \left[ \frac{\alpha_1 \theta_0 (c_{11} - c_{22}) (b_0)}{c_{11}} - \left( \frac{c_{11} - c_{22}}{c_{11}} \right) \left( \frac{c_{11} - c_{22}}{c_{11}} \right) \right] \frac{\gamma_3 (c_{11} - c_{22}) + \alpha_1 \theta_0 (c_{11} - c_{22}) + \beta_0 b_0^2}{2} \frac{c_{11} - c_{22}}{c_{11}} \left( \frac{b_0}{a} \right)^2 c_{11} - c_{22} \left( \frac{b_0}{a} \right)^2 \frac{c_{11} - c_{22}}{c_{11}} \right]
\]

where \( \alpha_1, \theta_0 = \beta_0 \).

For fully plastic case, we have the relation, \( c_{12} = c_{13} = c_{21} = c_{23} = c_{32} \), and \( c_{31} = c_{62} = c_{53} \) Seth [136], equation (6.3.17) becomes,

\[
\begin{align*}
\gamma_4 &= \frac{\gamma_1 c_{11}}{c_{11} - c_{22}} \left[ 1 - \left( \frac{c_{11} - c_{22}}{c_{11}} \right) \right] + \frac{\alpha_1 \theta_0}{\log a_1 b} \left[ \left( 1 - \left( \frac{c_{11} - c_{22}}{c_{11}} \right) \right) \right] + \gamma_3 (a_1^2 - b_1^2) \\
\gamma_5 &= \frac{\gamma_1 c_{11}}{c_{11} - c_{22}} \left( 1 - \left( \frac{c_{11} - c_{22}}{c_{11}} \right) \right) + \frac{\alpha_1 \theta_0}{\log a_1 b} \left[ \left( 1 - \left( \frac{c_{11} - c_{22}}{c_{11}} \right) \right) \right] + \gamma_3 (a_1^2 - b_1^2) \\
\gamma_{zz} &= \frac{c_{33}}{c_{11} + c_{22}} \left( \frac{\gamma_4}{c_{11}} + \gamma_5 \right) - \frac{\gamma_3}{c_{11} + c_{22}} \left( \frac{c_{33}}{c_{11} + c_{22}} - \alpha_3^2 \right) \left[ \frac{c_{11} - c_{22}}{c_{11} + c_{22}} \right] - \frac{\gamma_3}{c_{11} + c_{22}} + \frac{\gamma_3}{2} (a_1^2 + b_1^2) \\
\end{align*}
\]

(6.3.18)

and

\[
\frac{p_{V_{3}}}{2} = \left[ \frac{\alpha_1 \theta_0 (c_{11} - c_{22}) (b_0)}{c_{11}} - \left( \frac{c_{11} - c_{22}}{c_{11}} \right) \left( \frac{c_{11} - c_{22}}{c_{11}} \right) \right] \frac{\gamma_3 (c_{11} - c_{22}) + \alpha_1 \theta_0 (c_{11} - c_{22}) + \beta_0 b_0^2}{2} \frac{c_{11} - c_{22}}{c_{11}} \left( \frac{b_0}{a} \right)^2 c_{11} - c_{22} \left( \frac{b_0}{a} \right)^2 \frac{c_{11} - c_{22}}{c_{11}} \right]
\]
Particular Case

For isotropic materials, the constants reduce to two only

Sokolinikoff [145] i.e. \( c_{11} = c_{22} = c_{33} \), \( c_{12} = c_{21} = c_{32} = c_{31} = -\frac{c_{11} - 2c_{12}}{2} \), \( \alpha = c_{12}, \mu = -\frac{c_{11} - c_{12}}{c_{11}} \), \( \alpha_1 = \alpha_2 = \alpha_3 \)

Equation (6.3.17) becomes,

\[
\gamma_{AA} = \frac{\gamma_i}{c} \left[ 1 - (\alpha/\alpha_i)^c \right] + \frac{\alpha_1\theta_0}{\log \alpha/\alpha_i} \left[ 1 - (\alpha/\alpha_i)^c \right] + c \log \alpha/\alpha_i \] + \frac{\kappa_w^2 (a^2 - b^2)}{2},
\]

\[
\gamma_{\phi\phi} = \frac{\gamma_i}{c} \left[ 1 - (1-c)(\alpha/\alpha_i)^c \right] + \frac{\alpha_1\theta_0}{\log \alpha/\alpha_i} \left[ 1 - (1-c)(\alpha/\alpha_i)^c \right] + c \log \alpha/\alpha_i - c \] + \frac{\kappa_w^2 (a^2 - b^2)}{2},
\]

\[
\gamma_{zz} = \frac{\gamma_i}{c} \left[ 1 - (1-c)(\alpha/\alpha_i)^c \right] + \frac{\alpha_1\theta_0}{\log \alpha/\alpha_i} \left[ 1 - (1-c)(\alpha/\alpha_i)^c \right] + c \log \alpha/\alpha_i - c \] - \frac{\kappa_w^2 (a^2 + b^2)}{2},
\]

\[
\gamma_i = (2-c\gamma_i) \gamma \left[ 1 - \frac{(1-c)\alpha_1\theta_0 c^2}{(2-c)\gamma} - \frac{\kappa_w^2 a^2 c}{2(2-c)\gamma} \right] - \frac{1-c}{2-c} \cdot \frac{\kappa_w^2 (a^2 + b^2)}{2},
\]

and

\[
\frac{\kappa_w^2}{2} = \left[ \frac{\alpha_1\theta_0 c (b_0)}{c} - \frac{\log b_0}{c} \right] \left[ \frac{\gamma_i}{c} + \frac{\alpha_1\theta_0 c}{2} + \frac{\kappa_w^2 a^2}{2} + \frac{\alpha_1\theta_0}{c} \right]
\]

The stresses for fully plastic state are obtained by taking \( c \to \infty \) [35-52, 57, 135] in equation (6.3.19) as,

\[
\gamma_{AA} = \frac{\gamma_i \log \beta/\alpha_i + \kappa_w^2 (a^2 - b^2)}{2},
\]

\[
\gamma_{\phi\phi} = \gamma_i \left[ \log \beta/\alpha_i + 1 \right] + \frac{\kappa_w^2 (a^2 - b^2)}{2},
\]

\[
\gamma_{zz} = \frac{1}{\alpha_1} \left( \gamma_{AA} + \gamma_{\phi\phi} \right) - \frac{\alpha_1\theta_0}{2 \log \alpha/\alpha_i} \left[ \log \beta/\alpha_i - \frac{2}{b^2 a^2} \left( \frac{a_0^2}{4} - \frac{b_0^2}{4} - \frac{a_0^2 b_0}{2} \log a b \right) \right] \gamma_i
\]

- \frac{\kappa_w^2 (a^2 + b^2)}{2}.\]
where \( p = \frac{\alpha_{1}}{\theta_{0}} \) and \( a_{1} = 2 \alpha E \).

When there is no thermal effect i.e. \( \theta_{0} = 0 \), the equation (6.3.20) become as,

\[
\gamma_{1} = 2\gamma - \alpha_{1} \bar{\theta}_{0} \\
\frac{\mu^2}{2} = \frac{(\alpha_{1} \alpha_{2} \bar{\theta}_{0})}{(b^2 - \alpha^2)} \log \frac{b}{a}
\]

This expressions are the same as obtained by Gupta [35].

As a numerical illustration, curves have been drawn for Barytes material. The constants for the material have the following values [64]

\[
C_{11} = 907, \quad C_{12} = 273, \quad C_{13} = 275, \quad C_{22} = 800, \quad C_{23} = 468, \\
C_{33} = 1074, \quad C_{44} = 121, \quad C_{55} = 243, \quad C_{66} = 283.
\]

In figure (6.1), curves have been drawn for \( \frac{W_{y}}{h_{1}} \) to give yielding of the cylinder for different wall thickness ratio's.
It is seen that cylinders having smaller radii ratio require lesser angular speed for yielding as compared to cylinders having higher radii ratio's. Cylinder made of orthotropic material becomes fully plastic at a higher angular speed than the cylinder made of isotropic material. With the inclusion of thermal effects, isotropic cylinder require lesser angular speed to become fully plastic than the cylinder made of orthotropic material, but at high temperature [see figure (6.2)] much lesser angular speed is required to become fully plastic for cylinder made of isotropic material as compared to orthotropic material.

In figure (6.3), (6.4), (6.5) and (6.6), curves for transitional and fully plastic state have been drawn for radial, circumferential and axial stresses with respect to radii ratio \( \frac{a}{\alpha} \) and for various combinations of \( \frac{B}{\kappa} \) and \( \frac{W}{\kappa} \) taken from figure (6.2). It can be seen that maximum circumferential stress occurs at internal surface for fully plastic state (orthotropic material) whereas it is maximum for orthotropic transitional circumferential stress at the outer surface at high temperature.

6.4 **Asymptotic Solution Through** \( \rho \rightarrow -1 \).

It has been shown \([35-52, 57, 135]\) that transition function through the stress-difference at the transition point \( \rho \rightarrow -1 \), gives
the creep stresses, we define the transition function \( R_1 \) as,

\[
R_1 = C_{aA} - C_{a \Phi} = \frac{1}{n^m} \left[ (C_{11} - C_{21}) \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m + (C_{12} - C_{22}) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m \right] + (C_{33} - C_{32}) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m + (a_2 - a_1) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m.
\]  

(6.4.1)

Taking logarithmic differentiation of equation (6.4.1), with respect to \( \beta \), we have

\[
\frac{d \log R_1}{d \beta} = \frac{m \beta^{-1}}{n^m R_1} \left[ (C_{11} - C_{21}) \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^{m-1} \left( \frac{\beta d \beta}{d \beta} + (P + 1) \right) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m + (C_{12} - C_{22}) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m + (a_2 - a_1) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m \right] - \frac{(a_2 - a_1) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m}{m \beta^{-1} \frac{n^m \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m}{n^m R_1}}.
\]

(6.4.2)

Substituting the value of \( \frac{d \beta}{d \beta} \) from equation (6.2.7) in equation (6.4.2), we get

\[
\frac{d \log R_1}{d \beta} = \frac{m \beta^{-1}}{C_{11} \left[ (C_{11} - C_{21}) \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^{m-1} + (C_{12} - C_{22}) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m + (C_{33} - C_{32}) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m + (a_2 - a_1) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m \right] - \frac{m \beta^{-1}}{C_{11} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m + (C_{12} - C_{22}) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m + (C_{33} - C_{32}) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m + (a_2 - a_1) \frac{m}{n} \left( \frac{1}{1 - \beta^\eta_1 (P + 1)^n} \right)^m}.
\]

(6.4.3)
Asymptotic value at $\rho \to -1$ in equation (6.4.3), we have

$$\frac{d \log R_i}{d \rho} = \frac{m \beta^\alpha [1 - \beta^\alpha]^{m-1} [(C_{10} - C_{21}) C_{12} - (C_{42} - C_{43}) C_{11}]}{C_{11} \left[ (C_{11} - C_{21}) + (C_{42} - C_{43}) (1 - \beta^\alpha)^m + (C_{43} - C_{23}) (1 - (1 - \delta)^n)^m \right] + (\delta - \xi) n^{\delta \delta}} +$$

$$+ \frac{m \beta^\alpha}{C_{11} \beta} \left[ (C_{11} - C_{21}) + (C_{42} - C_{43}) (1 - \beta^\alpha)^m + (C_{43} - C_{23}) (1 - (1 - \delta)^n)^m + (\delta - \xi) n^{\delta \delta} \right].$$

Integration of equation (6.4.4) yields

$$R_i = A_0 \beta^{-\alpha (C_{11} - C_{21})} \exp \left( f_1 + f_2 + f_3 \right).$$ (6.4.5)

where $A_0$ is a constant of integration and

$$f_1 = \frac{m \beta^\alpha}{C_{11} \beta} \left[ (C_{11} - C_{21}) C_{12} - (C_{42} - C_{43}) C_{11} \right] \int [1 - \beta^\alpha]^{m-1} d \beta$$

$$f_2 = \frac{m \beta^\alpha}{C_{11} \beta} \left[ \alpha_1 - \alpha_2 - \alpha_3 (C_{11} - C_{21}) \right] \int \frac{d \beta}{\beta} \left[ (C_{11} - C_{21}) + (C_{42} - C_{43}) (1 - \beta^\alpha)^m + (C_{43} - C_{23}) (1 - (1 - \delta)^n)^m + (\delta - \xi) n^{\delta \delta} \right]$$

$$f_3 = \frac{m \beta^\alpha}{C_{11} \beta} \int \frac{A^2 \beta^\alpha}{\beta} \
$$

$$\left[ (C_{11} - C_{21}) + (C_{42} - C_{43}) (1 - \beta^\alpha)^m + (C_{43} - C_{23}) (1 - (1 - \delta)^n)^m + (\delta - \xi) n^{\delta \delta} \right].$$
Using equation (6.4.5) in equation (6.2.5) and integrating, we get

\[ \gamma_{\lambda} = -\frac{\rho^2 \lambda^2}{3} - A_0 \int \frac{(\xi - \xi_0)}{C_0} \cdot e^{\lambda (f_1 + f_2 + f_3)} \, d\lambda + A_1 \]  
(6.4.6)

where \( A_1 \) is a constant of integration.

The asymptotic value of \( \beta \) as \( P \to -1 \) is \( \frac{D}{\lambda} \), \( D \)
being a constant, equation (6.4.6) becomes, as

\[ \gamma_{\lambda} = -\frac{\rho^2 \lambda^2}{3} - A_0 \int \frac{(\xi - \xi_0)}{C_0} \cdot e^{\lambda (f_1 + f_2 + f_3)} \, d\lambda + A_1 \]  
(6.4.7)

where

\[ f_1 = -\frac{m^n \rho^n \{(C_1 - C_2) (C_2 - (C_2 - C_2) C_3)\}}{C_0} \int \frac{\lambda^{n-1} (1-D^n \lambda^n) \cdot \lambda^{n-1}}{(C_1 - C_2) + (C_2 - C_3) (1-D^n \lambda^n) + (C_3 - C_3) (1-D^n) \lambda^n + (C_3 - C_3) (1-D^n) \lambda^n + (C_3 - C_3) \lambda^n} \, d\lambda \]

\[ f_2 = -\frac{m^n \rho^n \{(C_1 - C_2) \}}{C_0} \int \frac{\lambda^{n-1} (1-D^n \lambda^n) \cdot \lambda^{n-1}}{(C_1 - C_2) + (C_2 - C_3) (1-D^n \lambda^n) + (C_3 - C_3) (1-D^n) \lambda^n + (C_3 - C_3) (1-D^n) \lambda^n + (C_3 - C_3) \lambda^n} \, d\lambda \]

\[ f_3 = -\frac{m^n \rho^n \{(C_1 - C_2) \}}{C_0} \int \frac{\lambda^{n-1} (1-D^n \lambda^n) \cdot \lambda^{n-1}}{(C_1 - C_2) + (C_2 - C_3) (1-D^n \lambda^n) + (C_3 - C_3) (1-D^n) \lambda^n + (C_3 - C_3) (1-D^n) \lambda^n + (C_3 - C_3) \lambda^n} \, d\lambda \]
Using boundary conditions (6.2.9) in equation (6.4.7), we get

\[ A_0 = \frac{-\int_{a}^{b} b^2 \left( \frac{c_1 - c_2}{c_{11}} \right)^{-1} \exp (f_1 + f_2 + f_3) \, dx}{2 \int_{a}^{b} \left( \frac{c_1 - c_2}{c_{11}} \right)^{-1} \exp (f_1 + f_2 + f_3) \, dx} \]

and

\[ A_1 = \frac{\int_{a}^{b} b^2 \left( \frac{c_1 - c_2}{c_{11}} \right)^{-1} \exp (f_1 + f_2 + f_3) \, dx}{2 \int_{a}^{b} \left( \frac{c_1 - c_2}{c_{11}} \right)^{-1} \exp (f_1 + f_2 + f_3) \, dx} \]

Substituting the values of \( A_0 \) and \( A_1 \) in equation (6.4.7) and (6.4.5), we have

\[ \gamma_{AA} = -\frac{\int_{a}^{b} b^2 (x^2 - a^2) \, dx}{2} \]

\[ + \frac{\int_{a}^{b} b^2 \left( \frac{c_1 - c_2}{c_{11}} \right)^{-1} \exp (f_1 + f_2 + f_3) \, dx}{2 \int_{a}^{b} \left( \frac{c_1 - c_2}{c_{11}} \right)^{-1} \exp (f_1 + f_2 + f_3) \, dx} \]

(6.4.8)

\[ \gamma_{\phi \phi} = \gamma_{AA} \]

\[ + \frac{\int_{a}^{b} b^2 \left( \frac{c_1 - c_2}{c_{11}} \right)^{-1} \exp (f_1 + f_2 + f_3) \, dx}{2 \int_{a}^{b} \left( \frac{c_1 - c_2}{c_{11}} \right)^{-1} \exp (f_1 + f_2 + f_3) \, dx} \]

(6.4.9)

From equation (6.2.4) \( \gamma_{zz} \) can be written as,
\[ \tau_{zz} = \frac{c_{32}}{c_{12} + c_{22}} \left( \tau_{ML} + \tau_{M0} \right) + \alpha \left[ \alpha_x + \alpha_z \right] \frac{c_{32}}{c_{12} + c_{22}} \] (6.4.10)

where

\[ k_1 = \left\{ \left[ c_{33} - c_{23} \frac{(c_{13} + c_{23})}{c_{12} + c_{22}} \right] \frac{1}{c_{12} + c_{22}} + \left[ c_{33} - c_{23} \frac{(c_{13} + c_{23})}{c_{12} + c_{22}} \right] \right\}^{1/2} \]

and

\[ c_{zz} = \frac{1}{h_n} \left[ 1 - (1 - \delta)^n \right] \]

The constant \( k_1 \) is determined from the boundary condition (6.2.10) and from equation (6.4.10), as

\[ k_1 = -\frac{c_{32}}{c_{12} + c_{22}} \left( a^2 + b^2 \right) + \frac{\beta}{(b - a^2)} \left[ \alpha_x + \alpha_z \right] \frac{c_{32}}{c_{12} + c_{22}} \left[ \frac{a^2}{4} - \frac{b^2}{4} - a^2 \log q_b \right] \]

Equations (6.4.8) - (6.4.10) give the creep transitional stresses for a hollow rotating cylinder. These equations correspond to only one stage of creep, if all the three stages are to be taken into accounts, we shall add the incremental values [35-42, 57, 135] of \( (\tau_{ML}, \tau_{M0}) \). Thus from equation (6.4.5), we have

\[ \tau_{ML} - \tau_{M0} = \alpha \frac{3(c_{13} - c_{23})}{c_{11} + c_{22}} \left( f_1 + f_2 + f_3 \right) \]

(6.4.11)
Where $m, n$ having three different sets of values each corresponding to one stage of creep.

It is found that the value of $|\gamma_{s1} - \gamma_{s2}|$ is maximum at $s = a$, therefore yielding of the cylinder starts at the internal surface. In this case equation (6.4.9) reduce to

$$|\gamma_{s1} - \gamma_{s2}| = \frac{\rho w^2 (b^2 - a^2) \frac{d}{\alpha} (\frac{c_1 - S_2}{c_1}) | \exp \left( f_1 + f_2 + f_3 \right) | |_{s=a} \leq \gamma_2$$

$$2 \int_{a}^{b} \frac{d}{\alpha} (\frac{c_1 - S_2}{c_1})^{-1} \exp \left( f_1 + f_2 + f_3 \right) \, dk$$

(6.4.12)

where $\gamma_2$ is the yield stress.

For steady state of creep $m = 1$, equation (6.4.8) - (6.4.10) reduces to

$$\gamma_{s1} = - \frac{\rho w^2 (b^2 - a^2)}{a} + \frac{\rho w^2 (b^2 - a^2) \int_{a}^{b} \frac{d}{\alpha} (\frac{c_1 - S_1}{c_1})^{-1} \exp \left( f_1 + f_2 + f_3 \right) \, dk}{2 \int_{a}^{b} \frac{d}{\alpha} (\frac{c_1 - S_1}{c_1})^{-1} \exp \left( f_1 + f_2 + f_3 \right) \, dk}$$

$$\gamma_{s2} = \gamma_{s1} + \frac{\rho w^2 (b^2 - a^2) \frac{d}{\alpha} (\frac{c_1 - S_1}{c_1}) \exp \left( f_1 + f_2 + f_3 \right) |_{s=a} \leq \gamma_2$$

$$2 \int_{a}^{b} \frac{d}{\alpha} (\frac{c_1 - S_1}{c_1})^{-1} \exp \left( f_1 + f_2 + f_3 \right) \, dk$$

(6.4.13)
\[ \tilde{\chi}_{zz} = \frac{C_{32}}{C_{12} + C_{22}} (\tilde{\chi}_{M} + \tilde{\chi}_{\phi}) + \left[ \left( \chi_{1} + \chi_{2} \right) \frac{C_{32}}{C_{12} + C_{22}} - \alpha_{3} \right] - \frac{C_{32} \cdot \gamma \omega^{2} (a^{2} + b^{2})}{C_{12} + C_{22}} - \frac{2 \beta_{0}}{b \cdot a^{2}} \left[ \left( \chi_{1} + \chi_{2} \right) \frac{C_{32}}{C_{12} + C_{22}} - \alpha_{3} \right] \left[ \frac{a^{2} - b^{2}}{a} - \frac{a^{2} \log a + b}{2} \right] \]

where

\[ f_{1} = - \frac{\mu \beta^{n}}{C_{11}} \left[ (C_{11} - C_{21}) C_{12} - (C_{12} - C_{22}) C_{11} \right] \int \frac{\rho \cdot d\rho}{(C_{11} - C_{21}) + (C_{21} - C_{22})(1 - D \psi_{n}) + (C_{12} - C_{22})(1 - D \psi_{m}) + (C_{11} - C_{21}) \eta \theta_{0}} \]

\[ f_{2} = - \frac{\mu \beta^{n}}{C_{11}} \left[ (C_{11} - C_{21}) C_{12} - (C_{12} - C_{22}) C_{11} \right] \int \frac{\rho \cdot d\rho}{(C_{11} - C_{21}) + (C_{21} - C_{22})(1 - D \psi_{n}) + (C_{12} - C_{22})(1 - D \psi_{m}) + (C_{11} - C_{21}) \eta \theta_{0}} \]

\[ f_{3} = - \frac{\mu \beta^{n}}{C_{11}} \left[ (C_{11} - C_{21}) C_{12} - (C_{12} - C_{22}) C_{11} \right] \int \frac{\rho \cdot d\rho}{(C_{11} - C_{21}) + (C_{21} - C_{22})(1 - D \psi_{n}) + (C_{12} - C_{22})(1 - D \psi_{m}) + (C_{11} - C_{21}) \eta \theta_{0}} \]

**Particular Case**

When there is no thermal effect i.e., \( \theta_{0} = 0 \), the equation (6.4.13) become as,

\[ \tilde{\chi}_{M} = - \frac{f_{1} S^{2}(\lambda^{2} - a^{2})}{a} + \frac{f_{2} S^{2}(b^{2} - a^{2})}{a} \int \frac{\rho \cdot d\rho}{(C_{11} - C_{21}) \cdot F_{1}(\lambda) \cdot \exp(f_{3}) \cdot d\lambda} \]

\[ \tilde{\chi}_{\phi} = \tilde{\chi}_{M} + \frac{f_{1} S^{2}(b^{2} - a^{2}) \cdot (C_{11} - C_{21})}{a} \cdot F_{1}(\lambda) \cdot \exp(f_{3}) \]

\[ \int \frac{\rho \cdot d\rho}{(C_{11} - C_{21}) \cdot F_{1}(\lambda) \cdot \exp(f_{3})} \]

(6.4.14)
\[ \eta_{zz} = \frac{C_{32}}{C_{12} + C_{22}} \left( \eta_{xx} + \eta_{yy} \right) - \frac{C_{32}}{C_{12} + C_{22}} \cdot \frac{p_1 b_2^2}{2} \cdot (a_2^2 + b_2^2) \]

where

\[ f_2(\alpha) = \left[ \frac{(C_{11} - C_{12}) + (C_{12} - C_{22}) \{1 - 2\eta \alpha \}^2}{(C_{11} - C_{12}) + (C_{12} - C_{22}) \{1 - 2\eta \alpha \}^2} \right] \]

and

\[ f_3 = -\int \frac{\lambda d\lambda}{\left( \frac{(C_{11} - C_{12}) + (C_{12} - C_{22}) \{1 - 2\eta \alpha \}^2}{(C_{11} - C_{12}) + (C_{12} - C_{22}) \{1 - 2\eta \alpha \}^2} \right)} \]

These equations are the same as obtained by Gupta and Bhardwaj [46].

6.5 Isotropic Steady State of Creep

For isotropic material, the constants reduces to two only \[ [145] \] i.e.,
\[ C_{11} = C_{22} = C_{33} , \quad C_{12} = C_{21} = C_{23} = C_{32} = C_{13} = C_{31} = C_{23} = C_{32} = C_{11} - 2C_{12}, \]
\[ \mu = \frac{C_{11} - C_{12}}{2} , \quad \nu = C_{12} , \quad C = \frac{C_{11} - C_{12}}{C_{11}} , \]
\[ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha (3\nu + 2\mu) . \]

Equations (6.4.13) becomes,

\[ \eta_{xx} = -\frac{p_1 b_2^2}{2} \cdot \left( \alpha^2 - a^2 \right) \]
\[ + \frac{\rho \omega^3 (b^2 - a^2) \int_{\alpha}^{\lambda} \exp \left( -C_1 \lambda + C_2 \lambda^m \right) \cdot d\lambda}{2 \int_{\alpha}^{\lambda} \exp \left( -C_1 \lambda + C_2 \lambda^m \right) \cdot d\lambda} . \]
\[
\begin{align*}
\gamma_{\theta \phi} &= \gamma_{\phi \theta} + \frac{\phi b^2 (b^2-a^2)}{a \cdot \exp \left( -c_2 \kappa^\eta \right)} \cdot \frac{c(n-1)-2n}{2} \cdot \frac{c(n-1) - 2n - 1}{a} \cdot \exp \left( -c_1 \kappa^\eta + c_2 \kappa^\eta \right) \cdot d\kappa. \\
\gamma_{zz} &= \frac{1-c}{2-c} \left( \gamma_{\phi \phi} + \gamma_{\theta \theta} \right) + \frac{2c \sum_{i=0}^{\infty} \left[ \frac{a_i^2}{u} - \frac{b_i^2}{u} - \frac{a_i^2 b_i}{u} \right]}{2 \cdot (2-c)} - \frac{c_2 \theta_0}{(2-c)} \\
&\quad - \frac{1-c}{2-c} \cdot \frac{\phi b^2 (a^2 + b^2)}{2}.
\end{align*}
\]

where
\[
\begin{align*}
c_1 &= \frac{\eta \phi b^2 c}{2 a \omega \rho \kappa^\eta (n+2)}, \quad c_2 = \frac{c_2 \theta_0}{2 \mu \rho \kappa^\eta}.
\end{align*}
\]

For incompressible material i.e. \( c \to 0 \), equation (6.5.1) reduce to
\[
\begin{align*}
\gamma_{ll} &= -\frac{\phi b^2 (a^2-a^2)}{2} + \frac{\phi b^2 (b^2-a^2)}{a} \cdot \exp \left( c_2 \kappa^\eta \right) \cdot d\kappa, \\
\gamma_{\phi \phi} &= \gamma_{\theta \theta} + \frac{\phi b^2 (b^2-a^2)}{a} \cdot \exp \left( c_2 \kappa^\eta \right) \cdot d\kappa, \\
\gamma_{zz} &= \frac{1}{2} \left( \gamma_{\phi \phi} + \gamma_{\theta \theta} \right) + \frac{2 \times E \theta_0}{(b^2-a^2)} \left[ \frac{a_i^2}{u} - \frac{b_i^2}{u} - \frac{a_i^2 b_i}{u} \right] - \frac{\phi b^2 (a^2+b^2)}{u}.
\end{align*}
\]

where
\[
C_2 = \frac{3 \times E \theta_0}{\eta \gamma \rho \kappa^\eta}
\]

When there is no thermal effect, equation (6.5.2) become,
These expressions are the same as obtained by Gupta [35].

The stresses for an elastic rotating hollow cylinder are obtained by putting \( \eta = 1 \) in equation (6.5.3) as,

\[
\begin{align*}
\sigma_{\theta\theta} &= -\frac{\rho \omega^2}{\bar{\alpha}^2} (\bar{\alpha}^2 - a^2) + \frac{\rho \omega^2}{\bar{\alpha}^2} \frac{(b^2 - a^2) \bar{\alpha}^{2n}}{(b^{2n} - a^{2n})}, \\
\sigma_{\phi\phi} &= \sigma_{\theta\theta} - \frac{\rho \omega^2}{\bar{\alpha}^2} \frac{(b^2 - a^2) \bar{\alpha}^{2n}}{(b^{2n} - a^{2n})}, \\
\sigma_{zz} &= \frac{1}{\bar{\alpha}^2} (\sigma_{\theta\theta} + \sigma_{\phi\phi}) - \frac{\rho \omega^2}{\bar{\alpha}^2} (a^2 + b^2).
\end{align*}
\]

These expressions are the same as obtained by Rimrott and Luke [100] at time \( t = 0 \).
Fig.(6.1) Transitional yielding of the isotropic and Orthotropic cylinders
Fig(5.2) Relation between $\frac{W^*}{Y_1}$ and $\frac{P_o}{Y_1}$ for various thickness ratio's.
Fig(63) Distribution of orthotropic transitional plastic stresses due to rotation and temperature through the wall of the cylinder (Barytes material).
Fig(6.4) Distribution of plastic stress due to rotation and temperature through the wall of the cylinder (Isotropic transition state).
Fig. (65) Distribution of plastic stress due to rotation and temperature through the wall of the cylinder (Orthotropic fully plastic state).