Chapter 2

EFFECT OF SURFACE ROUGHNESS AND COUPLE-STRESS ON SQUEEZE FILM CHARACTERISTICS BETWEEN CURVED CIRCULAR PLATES
2.1 Introduction

In this chapter the problem of the combined effects of couple-stress and surface roughness patterns on the characteristics of squeeze film lubrication between a curved circular plate and a flat plate is studied.

The squeeze film phenomena are widely observed in machine tools, gears, bearings, rolling elements and biolubrications. Owing to this number of articles on lubrication have appeared recently these include Archibald [45], Hays [46], Jackson [47], Moore [48], Gould [49], and Murti [50]. Later, Parkins and Wollom [51] made an experimental study of behaviour of an oscillating squeeze film. In most of the theoretical investigations of hydrodynamic lubrication, it has been assumed that the bearing surfaces are smooth. In the recent years a considerable amount of research in tribology has been devoted to the study of the surface roughness on hydrodynamic lubrication. This is mainly because of the fact that all solid surfaces are rough to some extent and generally the height of roughness asperities is of the same order as the mean separation between lubricated contacts. Hence, the effect of surface roughness plays a significant role in the analysis of the characteristics of the lubrication phenomena. Burton [10] studied the effect of surface roughness on the load supporting capacity of a lubricant film by postulating the sinusoidal variations in film thickness. Christensen [13] has developed a stochastic model for the study of hydrodynamic lubrication of rough surfaces. Prakash and Christensen [52] used the stochastic theory to study the surface roughness effects on squeeze film lubrication between two rectangular
plates. The hydrodynamic lubrication of rough journal bearing was studied by Christensen and Tonder [53]. Recently Lin et al [54] have studied the effect of surface roughness on the oscillating squeeze film behaviour of long partial bearings using Christensen’s stochastic theory for rough surfaces.

Most of the theoretical studies on the squeeze film lubrication between plane parallel plates or between the curved circular plates are based on Newtonian constitutive approximation for the lubricants. The experimental results suggest that, the addition of the small amount of long-chain polymer solution to Newtonian lubricant imparts the most desirable lubricant properties to the fluid. The use of additives stabilizes the flow properties and minimizes the sensitivity of the lubricant to change in shear rate. Various micro continuum theories have been proposed to describe the behaviour of the lubricants containing microstructure additives (Ariman and Sylvester [55]; Ariman et al, [56]). The micro continuum theory of Stokes [34] is the simplest generalization of the classical theory of fluids, which allows for the polar effects such as the presence of non-symmetric stress tensor, the couple stresses and the body couples. The study of squeeze film lubrication with fluids containing additives has been studied by many (Ramanaiah [57], Ramanaiah and Sarkar [58], Bujurke et al. [59]). Recently, Naduvinamani et al. [60] investigated the problem of squeeze film lubrication of a short porous journal bearing with couple-stress fluid as lubricant and reported some of the advantages of the couple-stress fluids over the Newtonian lubricants such as increased load carrying capacity and a delayed time of approach.
The purpose of this chapter is to study the combined effects of surface roughness and couple stresses on the performance characteristics of squeeze film in curved circular plates the expressions for mean squeeze film pressure, mean load carrying capacity and squeeze film time are given using stochastic method for rough surfaces.

2.2 Mathematical Formulation and the Solution of the Problem

Fig. 2.1 shows a schematic diagram of the squeeze film geometry under consideration. The upper rough curved circular plate approaching the lower flat circular plate with a squeezing velocity \( h_0 (= dh_0/dt) \). The lubricant in the film region is assumed to be Stokes couple-stress fluid. It is also assumed that, the body forces and body couples are absent. Under the usual assumptions of hydrodynamic lubrication applicable to thin films, the equations of motion for couple-stress fluid and continuity equation take the forms

\[
\frac{\partial p}{\partial r} = \mu \frac{\partial^2 u}{\partial r^2} - \eta \frac{\partial^4 u}{\partial z^4}, \tag{2.2.1}
\]

\[
\frac{\partial p}{\partial z} = 0, \tag{2.2.2}
\]

\[
\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \tag{2.2.3}
\]

where \( u \) and \( w \) are the velocity components in \( r \) and \( z \) directions respectively, \( p \) is the pressure in the film region, \( \mu \) is the Newtonian viscosity. \( \eta \) is the material constant characterizing the couple-stress and is of dimension of momentum the
Fig. 2.1. Geometry and configuration of curved circular plates.
ratio \((\eta/\mu)\) has the dimension of length squared and hence characterizes the material length of the fluid.

The film thickness is considered to be made up of two parts

\[ H = h(t) + h_s(r, \theta, \xi), \tag{2.2.4} \]

where \( h(t) = h_0 e^{-\beta t^2} \) denotes the nominal smooth part of the film thickness, \( h_s \) is due to the surface asperities measured from the nominal level and \( \xi \) is an index describing the definite roughness arrangements. \( h_0(t) \) is the central film thickness, \( r \) is the radial co-ordinate and \( \beta \) is the curvature parameter. Convex film can be generated for \( \beta < 0 \) and concave ones for \( \beta > 0 \).

The relevant boundary conditions for the velocity components are

\[ u(r, 0) = 0, \quad w(r, 0) = 0, \quad \tag{2.2.5} \]

\[ u(r, H) = 0, \quad w(r, H) = \frac{d h_0}{d t}, \quad \tag{2.2.6} \]

\[ \frac{\partial^3 u}{\partial z^3} \bigg|_{z=0} = \frac{\partial^3 u}{\partial z^3} \bigg|_{z=H} = 0. \tag{2.2.7} \]

Conditions (2.2.5) & (2.2.6) are no slip conditions, condition (2.2.7) is due to vanishing couple-stress fluid and \( dh_0/dt \) squeeze film velocity.

Solve equation (2.2.1) for \( u \) using boundary conditions (2.2.5-7). Later substitute this into continuity equation (2.2.3) and integrate with respect to \( z \) using boundary conditions (2.2.5) and (2.2.6), then we get modified form of Reynolds equation.
\[
\frac{1}{r} \frac{d}{dr} \left[ rf(H,l) \frac{dp}{dr} \right] = 12 \mu \frac{dh}{dt}, \quad (2.2.8)
\]

where
\[
f(H,l) = H^3 - \frac{12}{l^3} \left[ H - \frac{2}{l} \tanh \left( \frac{lh}{2} \right) \right].
\]

Taking the stochastic film average of equation (2.2.8) with respect to \( f(h_i) \), we get
\[
\frac{1}{r} \frac{d}{dr} \left[ rE \left( f(H,l) \frac{dp}{dr} \right) \right] = 12 \mu \frac{dh}{dt}, \quad (2.2.9)
\]
where \( E(\bullet) \) denote the expectancy operator defined by
\[
E(\bullet) = \int_{-\infty}^{\infty} (\bullet) f(h) dh.
\]

Let \( f(h_i) \) be the probability distribution function of the random variable \( h_i \). In most of the engineering applications, the rough surfaces are of Gaussian type and hence, the following polynomial function is chosen to approximate the Gaussian distribution;
\[
f(h_i) = \begin{cases} 
\frac{35}{32c^7}(c_i - h_i)^3, & -c \leq h_i \leq c \\
0, & \text{otherwise}
\end{cases} \quad (2.2.11)
\]

where \( c \) is the half total range of random film thickness at \( c = \pm 3 \sigma_i \) and \( \sigma_i \) is the standard deviation.

The relevant boundary conditions for the pressure are
\[
\frac{dE(p)}{dr} = 0 \quad \text{at} \quad r = 0. \quad (2.2.12)
\]
\[
E(p) = 0 \quad \text{at} \quad r = a. \quad (2.2.13)
\]
According to Christensen stochastic theory for rough surfaces, the two types of roughness structures are of interest.

**Circumferential Roughness**

In this model, the roughness is assumed to have the form of long narrow ridges and valleys running in \( \theta \)-direction. The film thickness can be described by a function of the form

\[
H = h(t) + h_s(r, \xi)
\]  
(2.2.14)

The modified Reynolds equation (2.2.9) takes the form

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{dE(p)}{dr} \right] = 2\mu \frac{dh_0}{dt}.
\]  
(2.2.15)

Introduce the following non-dimensional quantities and variables

\[
\bar{H} = \bar{h} + \bar{h}_s, \quad \bar{h} = \frac{h}{h_0} = e^{-\bar{h}^3}, \quad R = \frac{r}{\bar{a}}, \quad \bar{p} = \frac{E(p)h_0^3}{\bar{a}^2 \bar{h}_0}, \quad \tau = \frac{l}{h_0}, \quad \bar{p} = \beta \bar{a}^3, \quad C = \frac{c}{h_0}
\]

into equation (2.215) and integrate resulting equation using of boundary condition (2.2.12), we get

\[
\frac{d \bar{p}}{dR} = 6R E(\bar{H}, \tau), \quad (2.2.16)
\]

where

\[
F(\bar{H}, \tau) = \bar{H}^2 - \frac{12}{\tau^2} \left( \bar{H} - \frac{2}{\tau} \tanh(\tau \bar{H}/2) \right).
\]

The non-dimensional fluid film mean pressure \( \bar{p} \) is obtained by solving (2.2.16) using boundary condition (2.2.13) in the form
The load carrying capacity is obtained integrating mean squeeze film pressure over the plate, which is

\[ E(W) = 2\pi \int_0^R r E(p) dr. \]  

(2.2.18)

The dimensional mean load carrying capacity is

\[ W = \frac{E(W) h_0^3}{2\pi h_0 a^4} = \int_0^R 6 R \tilde{p} dR. \]  

(2.2.19)

The time taken to attain the film thickness \( h_{o2} \) from an initial film thickness \( h_{o1} \) under a constant mean load \( \bar{W} \) is obtained from (2.2.19)

\[ t = \left( \frac{1}{h_{o2}^2 - h_{o1}^2} \right) \frac{\bar{W}}{E(W)} 2\pi u a^4 \]  

(2.2.20)

which in non-dimensional form is

\[ T = \frac{t E(W) h_0^2}{2\pi u a^4} = \left( \frac{1}{h_{o2}^2} - \frac{1}{h_{o1}^2} \right) \bar{W}. \]  

(2.2.21)

**Radial Roughness**

In this model, the roughness is assumed to have the form of long narrow ridges and valleys running in \( r \)-direction. The film thickness can be described by a function of the form

\[ H = h(t) + \ell_1(\theta, \xi). \]  

(2.2.22)
The modified Reynolds equation (2.2.9) takes the form
\[
\frac{1}{r} \frac{d}{dr} \left[ rE(f(H, l)) \frac{dE(p)}{dr} \right] = 12 \mu \frac{dh_0}{dt}.
\] (2.2.23)

Introducing the non-dimensional parameters and variables and integrating the equation (2.2.23) using the boundary condition (2.2.12), we get
\[
\frac{d\bar{p}}{d\bar{R}} = \frac{6R}{E(F(H, \tau))}.
\] (2.2.24)

The non-dimensional fluid film mean pressure \( \bar{p} \) obtained by solving (2.2.24) using boundary condition (2.2.13) is
\[
\bar{p} = \int_0^1 \frac{6R}{E(F(H, \tau))} d\bar{R}.
\] (2.2.25)

The non-dimensional mean load carrying capacity is
\[
\bar{W} = \frac{E(W)h_0^3}{2\pi \mu \bar{h}_0^4} = \frac{1}{6} \int_0^1 6R\bar{p}d\bar{R}.
\] (2.2.26)

and the corresponding non-dimensional time-height relation is
\[
T = \frac{4E(W)h_0^3}{2\pi \mu a^4} = \left[ \frac{1}{\bar{h}_0^2} \right] \bar{W}.
\] (2.2.27)
2.3 Results and Discussion

In the present chapter, the combined effects of surface roughness and couple-stress on the characteristics of squeeze film lubrication between a curved circular plate and a flat plate is analyzed. The squeeze film characteristics are the functions of the non-dimensional roughness parameter $C(=c/h_0)$, couple-stress parameter $\tau (=l/h_0)$ and curvature parameter $\beta (=\beta b^2)$. The negative values of $\beta$ ($\beta < 0$) produce characteristic of the convex pad and positive values of $\beta$ ($\beta > 0$) generate that of concave pad. Since the ratio $\eta/\mu$ has the dimension of length squared and $h_0$ being the minimum film thickness, the non-dimensional couple-stress parameter $\tau$ gives a mechanism of interaction of the fluid with the bearing geometry. The numerical value of $\tau$ depends both on the chain length of the polar additives, $(\eta/\mu)^{\frac{1}{2}}$ and the minimum film thickness $h_0$. It is expected that the polar effects would be prominent either when the microstructure size of the polar additives is large or when the minimum film thickness is small.

Figures 2.2 and 2.3 show the variation of non-dimensional mean load carrying capacity $\bar{W}$ with curvature parameter $\beta$ for different values of $\tau$ for both concave and convex pads for a fixed roughness parameter $C=0.2$. The dotted curve in the graph corresponds to the Newtonian case. As the non-dimensional curvature parameter $\beta$ increases numerically the non-dimensional load $\bar{W}$ decreases for convex pads, where as it increases for concave pads. It is observed
that in both the cases the effect of couple-stress is to increases the load carrying capacity for both types of roughness pattern, as compared to the Newtonian case.

The variation of relative load difference $R_w = \left( \frac{W_{\text{rough}} - W_{\text{smooth}}}{W_{\text{smooth}}} \right) \times 100$ for different values of $\tilde{\beta}$ and couple-stress parameter $\tau$ is presented in Table 2.1, for both types of roughness patterns. It is observed that $R_w$ increases for increasing values of roughness parameter $C$ in case of circumferential roughness pattern. However, $R_w$ decreases with increasing values of $C$ in the case of radial roughness pattern. Further it is observe that $R_w$ decreases for increasing values of $\tau$ for both types of roughness patterns. A significant decrease in $R_w$ is observed for increasing values of concave and convex pad $\tilde{\beta}$. The trend is observed for all the values of $\tau$.

Figures 2.4 and 2.5 show the variation of non-dimensional load carrying capacity $\overline{W}$ with $\tilde{\beta}$ for different values of roughness parameter $C$, for both concave and convex pads. The dotted curve in the graph corresponds to the smooth case. In concave pad as $\tilde{\beta}$ increases, the load carrying capacity $\overline{W}$ increases but in the case of convex pad $\overline{W}$ decreases with increasing curvature $\tilde{\beta}$.

The effect of roughness is to increase the load carrying capacity in the direction of circumferential roughness and to decrease it in the direction of radial roughness pattern compared to smooth case.
The squeeze film time for any curved plate can be easily calculated from equations (2.2.21) and (2.2.27) by reading appropriate value of $\bar{W}$ from figures 2.2, 2.3, 2.4 and 2.5.

2.4 Conclusions

In the present analysis the combined effect of surface roughness and couple stresses on the characteristic of squeeze film lubrication between curved circular plates is presented on the basis of the Stokes micro continuum theory for the couple-stress fluids and Christensen stochastic theory of hydrodynamic lubrication. It is found that the effect of couple stresses is to increase the load carrying capacity and the squeeze film time as compared to the corresponding Newtonian case. These results are more pronounced for concave pads. From the above observation we find that roughness effects are marginal for radial roughness pattern whereas they are more pronounced for those with circumferential roughness pattern. In the limiting case $\tau \to \infty$ and for $C = 0$ the analysis corresponds to classical squeeze film of curved circular plates studied by Murti [50]. These are some of the useful predictions for the design parameters and consequently these results can be exploited in predicting squeeze film behavior in machine elements like gears and cylindrical rollers.
Fig. 2.2. Non-dimensional mean load carrying capacity $\overline{W}$ as a function of the normalized curvature parameter $\overline{\beta}$ for different values of couplestress parameter with $C=0.2$ for circumferential roughness pattern.
Fig. 2.3. Non-dimensional mean load carrying capacity $\bar{W}$ as a function of the normalized curvature parameter $\bar{\beta}$ for different values of couple-stress parameter with $C=0.2$ for radial roughness pattern.
Fig. 2.4. Non-dimensional mean load carrying capacity $\overline{W}$ as a function of the normalized curvature parameter $\bar{\beta}$ for different values of roughness parameter $C$ with $\tau = 5$ for circumferential roughness pattern.
Fig. 2.5. Non-dimensional mean load carrying capacity $\overline{W}$ as a function of the normalized curvature parameter $\overline{\beta}$ for different values of roughness parameter $C$ with $\tau = 5$ for radial roughness pattern.
Table 2.1 Relative load difference $R_w$: (i) For concave Pad ($\bar{p} > 0$)

<table>
<thead>
<tr>
<th>$\bar{p}$</th>
<th>$C$</th>
<th>$\tau = 10$</th>
<th>$\tau = 20$</th>
<th>$\tau = 30$</th>
<th>$\tau = 40$</th>
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<td>B</td>
<td>A</td>
<td>B</td>
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<td>0.01</td>
<td>0.1</td>
<td>0.824 -0.422</td>
<td>0.755 -0.396</td>
<td>0.741 -0.391</td>
<td>0.736 -0.389</td>
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<td>2.867 -1.393</td>
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<td>6.772 -3.062</td>
<td>6.624 -3.018</td>
<td>6.569 -3.003</td>
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<td>0.1</td>
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<td>13.230 -5.507</td>
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(ii) For convex pad ($\bar{p} < 0$)

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A: CIRCUMFERENTIAL ROUGHNESS
B: RADIAL ROUGHNESS