Chapter 1

INTRODUCTION
1.1 Hydrodynamic Lubrication

The theoretical study of hydrodynamic lubrication is the study of a particular form of the Navier-Stokes equations governing the pressure in the fluid film, which was first derived by Reynolds [1], in the wake of the experiment of Beauchamp Tower [2]. The experimental results of Tower were analyzed and interpreted by Reynolds. Tower's experiment showed the formulation of a thin film between the two lubricating surfaces for the first time.

The fluid mechanics theory is based on the continuum hypothesis concept that the fluid mass is distributed throughout the space such that the field theories become applicable mathematical tools in the description of fluid motions. The present chapter deals with the fundamental laws governing the conservation of mass, momentum and energy and serves as a basis for the solution of bearing problems in subsequent chapters. The governing equation for conservation of mass, momentum and energy for the flow region [3], without sources and sinks etc are

\[
\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{V} = 0, \quad \text{(1.1.1)}
\]

\[
\rho \frac{D\mathbf{V}_i}{Dt} = \mathbf{F}_i + T_{ij}, \quad \text{(1.1.2)}
\]

and \[
\rho \frac{De}{Dt} = \phi' - \text{div}\mathbf{q}, \quad \text{(1.1.3)}
\]

respectively, where \(D/Dt\) is the material time derivative, comma denote the covariant derivative, \(\rho\) is the density of the fluid, \(V_i\) the components of velocity
vector, \( F \), the components of body force per unit mass, \( T \), the components of stress vector, \( e \) the internal energy, \( q \) is the heat flux vector given by

\[
\vec{q} = -k' \frac{\partial T}{\partial n}
\]  

(1.1.4)

where \( k' \) is the thermal conductivity of the fluid, \( T \) the temperature and \( n \) the normal to the area element across which the heat flux is considered. \( \phi' \) denotes the heat flux in unit volume due to the heating during fluid motion, which depends upon the properties of the fluid material and internal heat sources if present in the flow field.

In the analysis of the problems considered in this thesis, it is assumed that, the lubricant is an incompressible fluid and for such fluids \( \rho \) is constant. Then the continuity equation (1.1.1) takes the form

\[
V_i = 0
\]  

(1.1.5)

Further, the study is confined to the isothermal case only. Also when the body forces are absent, the governing equations (1.1.2) and (1.1.5) in Cartesian coordinate system take the forms

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= \frac{1}{\rho} \frac{\partial p}{\partial x} + \gamma \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right], \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= \frac{1}{\rho} \frac{\partial p}{\partial y} + \gamma \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right], \\
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= \frac{1}{\rho} \frac{\partial p}{\partial z} + \gamma \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right],
\end{align*}
\]

(1.1.6) (1.1.7) (1.1.8)
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,
\] (1.1.9)

where \(u, v, w\) are the fluid velocity components in the \(x, y, z\) directions, respectively, \(p\) is the pressure and \(\nu = \mu/\rho\) is the kinematics viscosity of the fluid, \(\mu\) being the viscosity co-efficient of the fluid.

**Types of Lubrication**

A 'bearing', with reference to a machine or a structure, means contacting surfaces through which load is transmitted when surfaces are in relative motion, one desires to minimize friction and wear. Any substance when introduced between the moving surfaces that reduces the friction and wear is called lubricant. A lubricant can be a liquid (e.g. oils) or (e.g. graphite, teflon) or gas (e.g. pressurized air).

The objective of the lubrication is to reduce friction, wear and heating of machine parts, which are in relative motion. A lubricant, when introduced between the surfaces in relative motion, accomplishes these objectives. There are two types of bearings, namely, sliding element bearings (fluid film lubrication) and rolling element bearings (antifriction bearings) in which the objectives of lubrication are achieved. In fluid film lubrication, a very thin layer of fluid completely separates two solid surfaces, which are in relative motion. With the fluid film, this motion causes a shearing action that requires relatively small effort in the direction of motion. The surfaces are usually part of a bearing, which locates and supports a shaft and its loads. The pressure must be developed in the fluid film to support a
normal load. This pressure, developed in the fluid depends on the type of lubrication. The classification of lubrication based on the degree with which lubricant separated the surfaces is given in [4-6].

**Hydrodynamic lubrication**

The surfaces which are separated by the lubricant film is shown in (Fig 1.1(a)). The load tending to bring the surfaces together is supported by fluid pressure generated by relative motion of surfaces.

![Diagram of lubrication types](image)

- a) Hydrodynamic (Surfaces separated)
- b) Mixed film (Intermittent local contact)
- c) Boundary (Continuous and extensive local contact)

Figure 1.1 Three Basic Types of Lubrication

**Mixed film lubrication**

The surface peaks are intermittently in contact and there is a partial hydrodynamic support (Figure 1.1(b)), with proper design, surface wear can be minimized.
Boundary lubrications

Surface contact is continuous and extensive, but the lubricant is continuously “smeared” over the surface (Figure 1.1(c)).

Hydrostatic lubrication

A highly pressurized fluid film such as air or water is introduced into the load bearing area in order to separate the moving surfaces completely. Since the fluid is pressurized by external means, full separation can be achieved whether there is relative motion between the surface or not.

Elastic hydrodynamic lubrication

When the lubricating surfaces are non-conforming, (e.g. gear teeth, cam etc) a full film of lubricant is not formed, since the non-conforming surfaces tend to expel rather than entraps the fluid. At low speeds, this will be a boundary lubrication resulting in higher wear rates. The load creates a small contact path from elastic deflections of the surfaces. This small contact path can provide adequate flat surface to allow a full hydrodynamic film to form if the relative sliding velocity is sufficiently large. This condition is termed as elastic hydrodynamic lubrication; it depends on the elastic deflections of the surfaces and also on pressure dependent viscosity of the fluids.

1.2 Squeeze film Bearings

A bearing is a system of machine elements whose function is to support an applied load and to reduce friction when the surfaces are in relative motion. The
load may be radial, axial or combination of these. Bearings are classified according to the direction of applied load. The main objective of lubrication is to reduce friction, wear and heating of machine parts, which move relative to each other. Any interposed substance that reduces the friction and wear is called lubricant. The main objective of reducing friction, wear and heating is achieved using two types of bearings, namely, sliding element bearings (fluid film lubrication) and rolling element bearings. Bearings operating under fluid film lubrication are called fluid film bearings. The bearings which are operating on the principle of hydrodynamic lubrication are also called as ‘self-acting’ bearings. In these bearings, load is supported due to wedge effect of the fluid caused by the relative tangential motion between two surfaces. In an externally pressurized (also called hydrostatic) bearings, the load is supported due to pressure in the fluid, which is supplied from an external source. In addition to these two types, there is another class of fluid film bearing known as ‘squeeze-film’ bearing, which supports a load due to relative normal motion. In this type of bearings, the pressure can be generated in the fluid film without the wedge effect. In many cases, the relative normal or squeeze velocity may be predominant due to periodic external load, such as in the case of squeeze film damper. In this case, it is necessary to know the condition at which the lubricant film is likely to break down, since the load is time dependent. Some of the cases in which the technology of squeeze film lubrication is mainly observed are;
(i) The time of approach of disc clutch under lubricated conditions.

(ii) The mechanism of walking with rubber soles on wet or icy pavements.

(iii) The absorption of vibration in jet engines using annular squeeze films between the engine bearings and their support.

(iv) The rolling characteristics of an automobile tyre on a wet road.

In the design process of hydrodynamic bearings, some of the important characteristics of bearings such as load carrying capacity, flow requirement and power loss due to viscous friction are to be predicted accurately. These bearing parameters can be obtained only when the pressure in the fluid film is known. To determine the pressure in the film region, one has to solve a particular form of the Navier-Stokes equation along with the continuity equation after making the following basic assumptions [5]

1. Body forces are neglected. i.e. there are no extra fields of forces acting on the fluid.

2. The curvature of the surface is large compared with film thickness. Surface velocities need not be considered as varying in direction.

3. There is no slip at the solid boundaries. The velocity of the lubricant layer adjacent to the solid boundary of the bearings is the same as that of the boundary.

Further, the following sets of assumptions are made which enable theoretical analysis of models.

4. The lubricant flow in the film is laminar.
5. Fluid inertia is neglected.

6. The viscosity is constant throughout the film thickness.

The governing equation for the fluid film pressure can thus be obtained by using the above assumptions. This equation is generally called as Reynolds equation in the tribology literature. Normally, the parameters representing the various aspects of bearings, such as, bearing geometry, operating conditions and film shape, influence of the film forces under steady state conditions of laminar incompressible films etc are useful for detail study.

1.3 Surface Roughness

No solid surface is perfectly smooth on atomic scale. In other words, all solid surfaces are rough to some extent. The effect of surface roughness plays a significant role in the development of science and technology of tribology. As all solid surfaces are rough, the effect of surface topography must be taken into account in dealing with any contact situation of real surface.

In recent years a considerable work is being devoted to the study of the influence of surface roughness on the average pressure and load carrying capacity of a bearing in hydrodynamic lubrication. Mainly these efforts are concerned with the deviation of the mean pressure from that developed when the surfaces are smooth. The unrealistic assumption of the conventional lubrication theory (as all surfaces are perfectly smooth) has lead to the realization of roughness
phenomenon. Most of the bearing surfaces are rough to some extent when observed on the microscopic scale. In many cases, the roughness asperity heights are of the same order as the mean separation between the lubricated contacts. Now, it has been well established that, the surface roughness of the bearing surfaces significantly affects the bearing performance especially in the boundary or mixed lubrication [7,8]. Several approaches have been proposed to study the effects of surface roughness of the lubrication on the hydrodynamic lubrication of various bearings. The earliest analysis of solving the roughness problem for the particular bearing cases is by Davies [9] who employed a saw-tooth curve to mathematically model the roughness asperities on the bearing surface. The Fourier series type approximation is used by Burton [10] to model the large number of roughness asperities on the bearing surfaces. Later Mitchell [11] described the surface roughness by a high frequency sine curve. This was perhaps found to be the first published investigation in this regard. The random character of the surface roughness was recognized by several investigators and have used a Stochastic approach to mathematically model the surface roughness of the bearing surfaces [12-15]. Tonder [16] theoretically analyzed the transition between surface distributed waviness and random roughness. The beta probability density function for the random variable to characterize the surface roughness was used by Tzeng and Saibel [8]. This distribution approximates the Gaussian distribution with a good degree of accuracy for certain particular cases and it is symmetrical in nature having zero mean.
Further studies by Elrod [17] classified the surface roughness into two classes depending on the characteristic wavelength of the striations as Reynolds roughness and Stokes roughness. Tonder [18] studied, the lubrication of a slider bearing taking into account of crossed striations in transverse and longitudinal directions and found that a transverse component on the stationary surface results in increase of load, friction and side flow, where as longitudinal component increases both friction and flow in the sliding directions and reduces side flow. Christensen [13] developed stochastic models for hydrodynamic lubrication of rough surfaces. This approach has formed the basis for the analysis of rough bearings studied by several authors [19-28].

The part of the present thesis deals with the effect of surface roughness on the performance characteristic of squeeze film and porous squeeze film bearings, using the Stochastic method developed by Christensen [13] for the hydrodynamic lubrication of rough surfaces. The study of surface roughness has a greater importance in the study of porous bearings since the surface roughness is inherent in the process used for their manufacture.

1.4 Non-Newtonian Fluids

In the year 1687, Isaac Newton wrote a simple equation defining the viscosity of a fluid as the co-efficients of proportionality between the shear stress
and the velocity gradient. Newton's equation adequately describes the flow of gases and liquids of low molecular weights such as air, water, hydrocarbons etc. The description of the flow of such Newtonian fluids was well established in the middle of 19th century. This description is based on use of the laws of conservation of mass and momentum in conjunction with Newton's constitutive law for fluids. However Newton's equation cannot describe the flow of liquids containing polymers, especially, when we deal with very large molecules, whose typical molecular weights may range from $10^5$ to $10^8$ [29]. The flow behaviour of these material deviates significantly from a "Newtonian" behaviour and hence they are called non-Newtonian fluids. In general, the non-Newtonian fluids are classified in to three broad types [30].

\textit{i. Time-independent Fluids}

These are the fluids for which the rate of shear at any point is some function of the shearing stress at that point. These fluids can be subdivided in to the following three groups [31]:

(a) Pseudoplastic (shear thinning)

The viscosity decreases with increase in shear rate. This is fully reversible without time lag.

(b) Dilatant (shear thickening)

The viscosity increases with increasing shear rate.
(c) Viscoplastic

A finite yield stress is required to initiate the flow.

The flow curves and viscosity / shear rate curves are shown in the Fig. 1.4 for these types of fluids.

ii. Time dependent fluids

These are fluids for which relation between shear stress and shear rate depends on the time since when the fluid has been sheared or on its previous history.

iii. Visco-elastic fluids

These are the fluids having the characteristics of both solids and exhibit partial elastic recovery after deformation. Many rheological formulae have been suggested in the literature for the description of one or more features of non-Newtonian behaviour. Some of them have limited use and care must be taken in the application of formulae, especially when the motion is anything other than simple steady shearing motion. The names of Ostwalde-de Waele, Ellis Sisko, Casson, Reiner, Oldroyd, Harries, Maxwell are known for their formulae in the literature on this subject.
Couple-stress Fluids

Number of theories have been proposed to explain the peculiar behaviour of fluids, which contain microstructures such as additives, suspensions or granular matter. Eringen's [33] micropolar fluid theory defines the rotational field in terms of kinematically independent rotation vector called micro rotation vector for setting up of stress- strain rate constitutive equations. However, the theory of couple-stress fluid given by Stokes [34] defines the rotational field in terms of the velocity field, thereby reducing considerably the number of material constants in the constitutive equations characterizing fluid material. This theory introduces a second order gradient of velocity vector, instead of kinematically independent rotation vector in the constitutive relationship between stress rates.
Stokes theory of couple-stress fluids is the simplest generalization of the classical theory of fluids which allows for the polar effects such as the presence of a non-symmetric stress tensor, couple stresses and body couples. The constitutive equations for couple-stress fluids proposed by Stokes are

\[ T_{(i,j)} = -p\delta_{ij} + \lambda_c d_{k,k}\delta_{ij} + 2\mu d_y, \]  
\[ T_{(i)} = -2\eta W_{y,\ast k} - \frac{\rho}{2} \varepsilon_{yi} G_s, \]  
and \[ M_y = 4\eta w_{ji} + 4\eta' w_{i,j}, \]

where

\[ d_y = \frac{1}{2}[V_{i,j} + V_{j,i}], \]  
\[ W_y = -\frac{1}{2}[V_{i,j} + V_{j,i}], \]  
\[ w_i = \frac{1}{2}\varepsilon_{ik} V_{k,j}. \]

\( T_{(i,j)} \) and \( T_{(i)} \) are the symmetric and anti-symmetric parts of stress \( T_y \), \( M_y \) the couple-stress tensor, \( W_y \) the vorticity tensor, \( d_y \) the deformation rate tensor, \( \varepsilon_{yi} \) the alternating unit tensor, \( V_i \) the components of velocity vector, \( G_s \) the body couple \( \delta_y \) the kronecker delta, \( w_i \) the velocity vector, \( \rho \) the density, \( p \) the pressure, \( \lambda_c \) and \( \mu \) are the material constants of the dimension of viscosity, \( \eta \) and \( \eta' \) are the material constants having the dimensional of length squared and characterizes the microstructure size.
The governing equations in Cartesian co-ordinates for three-dimensional steady incompressible flow of Stokes couple-stress fluids in the absence of body couples and body forces are

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$  \hspace{1cm} (1.4.7)

Momentum equations:

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \gamma \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] - k_c \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]^2 u,$$  \hspace{1cm} (1.4.8)

$$\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \gamma \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] - k_c \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right]^2 v,$$  \hspace{1cm} (1.4.9)

$$\frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \gamma \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] - k_c \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right]^2 w,$$  \hspace{1cm} (1.4.10)

where $u$, $v$ and $w$ are the velocity components in the $x$, $y$ and $z$ directions, respectively, $\gamma = (\mu/\rho)$ is the kinematics viscosity and $k_c = (\eta/\mu)$.

**Rabinowitsch fluid model**

Modern lubricants often exhibit shear thinning due to the presence of high molecular weight polymers as additives. For some fluids the viscosity can change by a factor of 10 to 100, owing to the presence of macromolecules such an enormous change must not be ignored in lubrication problems, since in many cases this effect can be considered as dominant compared with thermal effects. Therefore, the influence of this non-Newtonian property on the performances of
lubricating systems must be predicted. To account for the fact, the viscosity of a fluid changes with the shear rate, the Reiner "generalized Newtonian fluid". As quoted by Metzner [35], Bird [36] or Person [37], this model is very useful for applications.

It is well known that the viscosity of pure mineral oils decreases as their temperature increases. To reduce this detrimental effect, lubricating oils are added with "viscosity index improvers", with higher molecular weight polymers. Correlatively, these lubricants exhibit a nonlinear relationship between the shear stress and the rate of shear, as usual for polymer solutions. Wada and Hayashi [38] have shown that the Rabinowitch [39] empirical model would fit reasonably well for the viscosity behavior of mineral oils added with polyisobutylene, with shear rates ranging from 0 to $10^6$ s$^{-1}$, which is representative of the actual working conditions of lubricants. In addition, the Rabinowitsch fluid is a particular case of the Ellis model, which has been studied extensively, see for instance Reiner [40] or Bird [41]. On the other hand, to describe the nonlinear relationship between shear stress and shear strain rate for the non-Newtonian lubricants, the Rabinowitsch fluid (cubic equation) model has also been introduced. In this model the following relationship holds for one-dimensional flows:

$$\tau_{xx} + k \tau_x^3 = \mu_0 \frac{\partial u}{\partial z},$$

where $\mu_0$ denotes the zero shear rate viscosity and is equivalent to the viscosity of Newtonian fluids and $k$ a nonlinear factor accounting for non-Newtonian effects.
The cubic equation model describes dilatent fluids for $k < 0$, Newtonian fluids for $k = 0$ and pseudoplastic fluids for $k > 0$, respectively. In the experimental study Wada and Hayashi [38] have obtained a range of values of $\mu_0$ and $k$ for different working conditions of mineral oils with polyisobutylene.

**Conducting Fluids**

Many processes and phenomena in science, engineering and technology involve motion of electrically conducting fluids in the presence of a magnetic field. MHD flows and waves occur commonly in astrophysical, geophysical and industrial environments and also have engineering applications like MHD power generator, MHD pumps, MHD lubrication and propulsion.

MHD is the study of the motion of electrically conducting liquids in the presence of a magnetic field. It deals with the introduction of electromagnetic and hydrodynamic forces to yield the effects of the magnetic field on the flow.

The basic equations of MHD are the Maxwell equations, Ohm's law, the equation of continuity, the equation of motion with the $(J \times B)$ body force. The four basic Maxwell equations are the following [42]

\[
\nabla \times H = J \\
\nabla \cdot H = 0 \\
\nabla \times B = -\frac{\partial B}{\partial t} \\
\n\nabla \cdot J = 0 
\]

Ohm's law: $J = E_c (E + \nabla \times B)$
Equation of motion is given by
\[
\rho \left[ \frac{\partial V}{\partial t} + (V \cdot \nabla) V \right] = -\nabla p + \nabla \cdot \tau + J \times B - \rho \nabla G_p
\]
\[
= -\nabla(p + \frac{B^2}{2\mu p} + (B \cdot \nabla) \frac{B}{\mu p} + \nabla \cdot J - \rho \nabla G_p),
\]
where \( \tau \) is the mechanical stress tensor, \( G_p \) is the gravitational potential.

1.5 Darcy Law

The flow of Newtonian fluid in the isotropic porous medium is governed by the Darcy's law. This law was established empirically by Darcy [43] in the year 1856. Subsequently it has been verified experimentally by numerous investigators. This law states that the mean fitter velocity, called Darcy velocity, \( q^* \) is proportional to the pressure gradient i.e.,
\[
q^* = -\frac{k}{\mu} \nabla p^*,
\]
where the proportionality constant \( k \) is related to the permeability of the porous material (Muskat [44]) and it has the dimension of length squared, and hence,
characterizes the pore size; \( \mu \) is the viscosity of the fluid and \( p' \) is the pressure in the porous medium. Equation (1.5.1) is well known in the literature as the Darcy’s law for isotropic porous media.

The equation of continuity for incompressible flow of Newtonian fluid in the porous medium is given by

\[
div q^* = 0.
\]  

(1.5.2)

**Modified Darcy Law**

For the flow of couplestress fluids through isotropic media, equation (1.5.1) is modified as

\[
q^* = \frac{k}{\mu(1 - \beta_i)} \nabla p^*,
\]  

(1.5.3)

where \( \beta_i = (\eta/\mu)/k \) represents the ratio of microstructure size to the pore size.

As the parameter \( \beta_i \) tends to zero, the modified Darcy equation (1.5.3) reduces the corresponding Newtonian case (1.5.1). If \( (\eta/\mu)^{1/2} \approx \sqrt{k} \), i.e., \( \beta_i \approx 1 \), then the microstructure additives present in the lubricant block the pores in the porous layer and thus reduces the Darcy flow through the porous matrix. When the microstructure size is very small compared to the pore size, i.e. \( \beta_i << 1 \), the additives percolate into the porous matrix.

The study of physics of flow through porous media has attracted the attention of many researchers mainly due to its applications in many areas of science and engineering, namely, soil mechanics, ground water hydrology,
petroleum engineering, water purification, industrial filtration, ceramic engineering and the study of power metallurgy. In addition to these, it has an application in the study of porous bearings. The porous material of the bearing function as a reservoir from which the bearing gap will be filled with lubricant during operation and in which the lubricant is stored during stand still. This method should ensure an efficient functioning of the bearing throughout its life without any further maintenance. In addition to this, the lubricant stored in the porous material serves as an additional supply of lubricant when the oil is lost in the film region during its operation. This ensures the running of porous bearings hydro dynamically for a longer time than the conventional bearings without periodic maintenance.