Chapter 7

NON-NEWTONIAN EFFECTS OF LUBRICANTS ON THE DYNAMIC CHARACTERISTICS OF HYPERBOLIC SLIDER BEARINGS: RABINOWITSCH FLUID MODEL
7.1 Introduction

In this chapter the theoretical study of non-Newtonian effects of an isothermal incompressible laminar flow of lubricant on the dynamic stiffness and damping characteristics of one-dimensional hyperbolic slider bearings are analyzed.

Modern lubricants often exhibit shear-thinning polymer as additives for some fluids and the viscosity can change by a factor of 10 to 100. Owing to the presence of macromolecules such an enormous change must not be ignored in lubrication problems, since in many cases these effects can be considered as dominant compared with thermal effects. Therefore, the influence of this non-Newtonian property on the performances of lubricating systems must be predicted. In recent studies many authors have investigated the rheological effects of fluids on the lubrication problems based upon different fluid models. Non-Newtonian characteristics in squeeze films have been investigated by Na [107] and Elkouh et al. [108] using the power-law model as the shear stress and shear strain-rate relation without including the inertial effects. Elkouh [109] has analyzed fluid inertia effects in non-Newtonian squeeze film, by using the power-law model. As some types of non-Newtonian fluids display viscoelastic characteristics Tichy and Winer [110] have analyzed the inertia effects in viscoelastic squeeze films between parallel circular plates with constant approaching velocity based on Lodge's rubber like liquid model. Hashimoto [111] has investigated the non-
Newtonian effects on the static characteristics of one-dimensional slider bearings in the inertial flow regime and their model is based on the Rabinowitsch model.

In the study of rolling elements by Bourgin and Gay [112], the load capacity is found to decrease with the nonlinear factor. In the study of squeeze film bearings by Hashimoto and Wada [113], the film pressure of circular bearings with pseudoplastic fluids under a sinusoidal squeeze motion is predicted including the effects of inertia forces. On the operation of journal bearings, the steady-state and thermodynamic performances are calculated by Rajalingham et al. [114], Bourgin and Gay [115] and Sheeja and Prabhu [116]. For the slider bearing characteristics, the steady performance is evaluated by Hsu and Saibel [117], Bourgin and Francois [118] and Francois [119] the optimal design of bearings is proposed by Auloge et al.[120] and the inertial effects on bearings are investigated by Hashimoto [111]. All the studies [116-120] are about slider bearings operating under steady squeeze action. Since slider bearings operate mainly upon the principle of wedge action, the analysis of dynamic squeezing action effect should be considered to assess the bearing stability behavior. Recently, many authors have investigated the rheological effects of fluids on the lubrication problems based upon different fluid models. The power-law fluid model is used to simulate the effects of non-Newtonian lubricants on the performance characteristics of journal bearings [38], and squeeze film behavior between two annular disks [121]. The couple-stress fluid model is adopted to study the rheological effects of lubricants on the characteristics of externally pressurized bearings [122], and
Journal bearings [123]. On the other hand, to describe the nonlinear relationship between shear stress and shear strain rate for the non-Newtonian lubricants, the Rabinowitsch fluid (cubic equation) model has also been introduced. In this model the following relationship holds for one-dimensional flows:

$$\tau_{xx} + k \tau_{xx}^3 = \mu_0 \frac{\partial u}{\partial x},$$  \hspace{1cm} (7.1.1)

where \( \tau_{xx} \) is the shear stress component, \( \mu_0 \) denotes the zero shear rate viscosity and is equivalent to the viscosity of Newtonian fluids and \( k \) means a nonlinear factor accounting for non-Newtonian effects. The cubic equation model describes dilatent fluids for \( k < 0 \), Newtonian fluids for \( k = 0 \) and pseudoplastic fluids for \( k > 0 \), respectively. In the experimental study Wada and Hayashi [38] have obtained a range of values of \( \mu_0 \) and \( k \) for different working conditions of mineral oils with polyisobutylene.

Motivated by the above investigation, the present study is to theoretically investigate the non-Newtonian effects of an isothermal incompressible laminar flow lubricant on the dynamic stiffness and damping characteristics of one-dimensional hyperbolic slider bearings. In the derivation of the non-Newtonian Reynolds type equation considering transient motion of the slider, the bearing squeeze action is taken in to account, and the Rabinowitsch model is used as a constitutive equation for non-Newtonian fluids. Applying a small perturbation technique to the Reynolds equations, both the steady state and dynamic performances are predicted. To reveal the non-Newtonian effects of lubricants,
bearing characteristics, including the steady film pressure, load carrying capacity and the dynamic stiffness and damping co-efficients are presented and compared with Newtonian lubricant case.

7.2 Mathematical Formulation

The geometrical configuration of a one-dimensional hyperbolic slider bearing is shown in Figure 7.1. The non-Newtonian lubricant is taken to be a Rabinowitsch fluid. It is assumed that the flow is isothermal, incompressible and laminar, and the lubricant inertia effect is small. According to the thin-film theory of hydrodynamic lubrication, equations of continuity and motion in Cartesian coordinates reduces to

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 , \quad (7.2.1)
\]

\[
\frac{\partial p}{\partial x} = \frac{\partial \tau_{xx}}{\partial z} , \quad (7.2.2)
\]

\[
\frac{\partial p}{\partial z} = 0 . \quad (7.2.3)
\]

where \( u, w \) are velocity components of lubricant in the \( x \) and \( z \) directions respectively and \( p \) is the pressure. The constitutive relationship between the shear stress component and the shear strain rate for a Rabinowitsch fluid obeys the cubic equation (7.1.1). The boundary conditions for the velocity components are

\[
z = 0: \quad u = U, \quad w = 0, \quad (7.2.4a)
\]

\[
z = h: \quad u = 0, \quad w = V, \quad (7.2.4b)
\]
Fig. 7.1. Geometrical configuration of a hyperbolic slider bearing.
where $U$ is the sliding velocity and $V$ be the squeeze velocity ($V = \partial h/\partial t$).

Denoting the pressure gradient $f = \frac{\partial p}{\partial z}$, integrating equation (7.2.2) with respect to $z$ yields

$$\tau_{xz} = f z + d_1.$$  (7.2.5)

Substituting equation (7.2.5) into the constitutive equation (7.1.1) leads to

$$\frac{\partial u}{\partial z} = \frac{1}{\mu_0} \left[ \frac{1}{2} f z + d_i \right] + k \left( f z + d_i \right)^2.$$ (7.2.6)

Integrating equations (7.2.6) with respect to $z$, the velocity component $u$ is obtained in the form

$$u = \frac{1}{\mu_0} \left[ \frac{1}{2} f z^2 + d_i z + k \left( \frac{1}{4} f^3 + f^2 d_i z^3 + \frac{3}{2} f d_i z^2 + d_i^2 z \right) \right] + d_2.$$ (7.2.7)

Applying condition (7.2.4) the integration constants $d_1$ and $d_2$ can be determined.

The velocity component is expressed as

$$u = \frac{1}{\mu_0} \left[ \frac{1}{2} f F_1 + k f^3 F_2 \right] + U \left[ 1 - \frac{z + k f^2 F_3}{h (1 + 0.25 k f^2 h^2)} \right],$$ (7.2.8)

where

$$F_1 = z(z - k),$$

$$F_2 = \frac{1}{4} z^4 - \frac{1}{2} h z^3 + \frac{3}{8} h^2 z^2 - \frac{1}{8} h^3 z,$$ (7.2.9)

$$F_3 = z^3 - \frac{3}{2} h z^2 + \frac{3}{4} h^2 z.$$

Substituting equation (7.2.8) into equation (7.2.1) and integrating with respect to $z$
and using boundary conditions on $w$, the Reynolds-modified equation

$$\frac{\partial}{\partial x} \left[ h^3 \frac{\partial h}{\partial x} + \frac{3}{20} kh^5 \left( \frac{\partial h}{\partial x} \right)^3 \right] = 6\mu_0 U \frac{\partial h}{\partial x} + 12\mu_0 \frac{\partial h}{\partial t}$$

(7.2.10)
is obtained.

The film thickness function is expressed in the form.

$$h(x, t) = \frac{h_m(t)}{h_p(x)}$$

(7.2.11)

where $h_p$ denotes the profile function of a slider depending upon the location and $h_m$ the minimum film thickness varying with the time.

For the hyperbolic slider considered in Figure 7.1, the dimensionless film thickness is given by

$$h^*(x^*, r') = \frac{h^*_m(r')}{h^*_p(x^*)}$$

$$= \frac{h^*_m(r')}{1 + \delta' x^*},$$

(7.2.12)

The dimensionless quantities are film thickness function $h^* (= h/h_m)$, minimum film thickness $h^*_m (= h_m/h_m)$, wedge parameter $\delta'(= d/h_m)$, time $r' (= \omega t)$ and coordinate $x^* = x/L$.

Dimensionless form of equation (7.2.10) is

$$\frac{\partial}{\partial x^*} \left[ h^* \frac{\partial h^*}{\partial x^*} + \frac{3\alpha'}{20} h^* \left( \frac{\partial h^*}{\partial x^*} \right)^3 \right] = -6\delta' + 12\sigma' \frac{dh^*_m}{dt},$$

(7.2.13)

where $\alpha' (= k\mu_0 U^2 / h_m^2)$ is the dimensionless nonlinear factor accounting for non-Newtonian effects, and $\sigma' (= \omega L / U)$ denotes the dimensionless squeeze number.
responsible for squeezing-action effects. As the value of $\alpha'$ is set equal to zero, the classical Reynolds equation for a hyperbolic slider bearing with Newtonian fluids is recovered. Assume the runner undergoes a small-amplitude oscillation about its steady-state position, perturb the dimensionless film thickness and pressure in the form

$$h^* = 1 + \alpha e^{i\tau}, \quad (7.2.14)$$

$$p^* = p_0^* + \delta p_0^* e^{i\tau}, \quad (7.2.15)$$

where $p^* (= p h^2_{ma}/\mu_0 UL)$ is the non-dimension film pressure, $p_0^* (= p_0 h^2_{ma}/\mu_0 UL)$ is the steady film pressure, $p_1^* (= p_1 h^2_{ma}/\mu_0 UL)$ perturbed film pressure and $\varepsilon$ is the small amplitude of oscillation. Substituting equations (7.2.14) and (7.2.15) into the equation (7.2.13) and neglecting higher order terms in $\varepsilon$, we get two dimensionless Reynolds-type equations for the steady-state performance and the dynamic stiffness and damping characteristics, respectively.

$$\frac{d}{dx^*} \left[ \frac{1}{h_p^3} \frac{\partial^3 p_1^*}{\partial x^*} + \frac{3\alpha'}{20} \frac{1}{h_p^3} \left( \frac{\partial^3 p_0^*}{\partial x^*} \right)^3 \right] = -\frac{6\delta'}{(1 + \delta x^*)^2} \quad \text{(Steady state)} \quad (7.2.16)$$

$$\frac{\partial}{\partial x^*} \left[ \frac{1}{h_p^3} \left[ 1 + \frac{9\alpha'}{20} \left( \frac{dp_0^*}{dx^*} \right)^2 \right] \frac{\partial p_1^*}{\partial x^*} \right] = -\frac{6\delta'}{(1 + \delta x^*)^3} + \frac{12\sigma^* i}{(1 + \delta x^*)}$$

$$-\frac{3}{20} \left[ \frac{1}{h_p^3} \left( \frac{dp_0^*}{dx^*} \right)^3 \frac{1}{h_p^*} \right] \quad \text{(perturbed state)} \quad (7.2.17)$$
7.3 Steady-State Characteristics

The steady-state film pressure can be obtained by solving the steady Reynolds-type equation (7.2.16) with the boundary conditions neglecting the effect of cavitation;

\[ p_0^* = 0 \quad \text{at } x^* = 0, \quad (7.3.1) \]

\[ p_0^* = 0 \quad \text{at } x^* = 1. \quad (7.3.2) \]

In order to obtain an analytical approximate expression, a small perturbation is used in the present study. Based upon the perturbation technique, the steady pressure \( p_0^* \) is expanded for \(-1 < \alpha' < 1\);

\[ p_0^* = p_{00}^* + \alpha p_{01}^*. \quad (7.3.3) \]

Substituting (7.3.3) into equation (7.2.16) results in two equations governing \( p_{00}^* \) and \( p_{01}^* \);

\[
\frac{d}{dx^*} \left[ \frac{1}{h_p^*} \frac{dp_{00}^*}{dx^*} \right] = \frac{-6\delta'}{(1 + \delta x^*)^2}, \quad (7.3.4)
\]

\[
\frac{d}{dx^*} \left[ \frac{3}{20} h_p^{*4} \left( \frac{dp_{00}^*}{dx^*} \right)^3 + \frac{1}{h_p^*} \frac{dp_{01}^*}{dx^*} \right] = 0. \quad (7.3.5)
\]

Solving these equations, one can obtain the steady pressure components

\[ p_{00}^* = \frac{2}{\delta'} \left[ \left( 1 + \delta x^* \right)^3 - 1 \right] - \frac{2}{\delta'} \left[ \left( 1 + \delta x^* \right)^4 - 1 \right] \frac{\left( 1 + \delta x^* \right)^3 - 1}{\left( 1 + \delta x^* \right)^4 - 1}. \quad (7.3.6)\]

\[ p_{01}^* = -\frac{3}{20} \int_0^1 \frac{1}{(1 + \delta x^*)^2} \int_0^{x^*} dx^* + \frac{3}{20} \left( 1 + \delta x^* \right)^4 - 1 \right] \frac{1}{\left( 1 + \delta x^* \right)^4 - 1} \int_0^1 \left( 1 + \delta x^* \right)^5 f_{00} dx^*. \quad (7.3.7)\]
The steady load-carrying capacity of the bearing is calculated by integrating the pressure over the film region:

\[ W_0 = B \int_{x=0}^{L} p_0 \, dx . \]  
(7.3.8)

Expressing in a dimensionless form, one has the steady-state load-carrying capacity:

\[ W_0^* = W_0 h_0^2 / \mu_0 U L^2 = B \int_{x=0}^{L} p_0^* \, dx^* . \]  
(7.3.9)

The friction force exerted on the sliding surface is

\[ F_f = B \int_{x=0}^{L} [r_{xx} \, x_{x=0}^0] \, dx . \]  
(7.3.10)

The flow rate is obtained from

\[ Q_0 = B \int_{x=0}^{h} u \, dz . \]  
(7.3.11)

Expressing in terms of dimensionless quantities, the friction force and flow rate are given by

\[ F_f^* = F_f h_0 / \mu_0 U L B = \int_{x=0}^{L} \left\{ \frac{1}{h_0^2} + \alpha \frac{1}{0.5 h_0^2 f_0^*} + \frac{1}{2} h_0^2 f_0^* \right\} \, dx , \]  
(7.3.12)

and

\[ Q_0^* = Q_0 / h_0 U B = \frac{1}{2} h_0^* - \frac{1}{12} h_0^* f_0^* - \frac{\alpha'}{80} h_0^2 f_0^* , \]  
(7.3.13)

respectively.

### 7.4 Dynamic Stiffness and Damping Characteristics

With the known steady pressure, the perturbed film pressure for the dynamic stiffness and damping characteristics can be analyzed using the perturbed
Reynolds-type equation (7.2.17) satisfying boundary conditions:

\[ p_1' = 0 \quad \text{at } x' = 0. \]  
\[ p_1' = 0 \quad \text{at } x' = 1. \]  

After integration the perturbed pressure is given by

\[ p_1' = i\sigma' \int_0^{x'} f_1' dx' + \int_0^{x'} f_2' dx' + \int_0^{x'} f_3' dx' + C_1 \int_0^{x'} f_4' dx', \]  

where

\[ C_1 = C_{iv} + iC_{iu}, \]  
integration constants

\[ C_{iv} = -\left[ \frac{\int_0^{x'} f_2' dx' + \int_0^{x'} f_3' dx'}{\int_0^{x'} f_4' dx'} \right], \]  

\[ C_{iu} = -\sigma' \frac{\int_0^{x'} f_1' dx'}{\int_0^{x'} f_4' dx'} , \]

\[ f_1 = \frac{240h_p^4 \log(1 + \delta' x')}{\delta' (20h_p^2 + 9\alpha' f_0)} , \]  

\[ f_2 = \frac{120h_p^4}{20h_p^2 + 9\alpha' f_0^2} , \]  

\[ f_3 = \frac{-15f_0 (4h_p^2 + \alpha' f_0^2)}{20h_p^2 + 9\alpha' f_0^2} , \]  

\[ f_4 = \frac{20h_p^5}{20h_p^2 + 9\alpha' f_0^2} . \]
where \( f'_0 = \frac{d\rho'_0}{dx}. \) By integrating the perturbed film pressure the perturbed film force is

\[
 F'_x = B \int_{x_0}^{x_f} p_0 dx. \tag{7.4.10}
\]

Expressing this in a dimensionless form, one obtains

\[
 F'_x = F_c h^3_{w_0} / \mu_k UL^2 B \left[ \left( \int \int f_3'(dx'^2) + \int \int f_3'(dx'^2)^2 + C_{ir} \int f_4'(dx'^2) \right) + i \left( \sigma' \int f_1'(dx'^2)^2 + C_{ir} \int f_4'(dx'^2)^2 \right) \right]. \tag{7.4.11}
\]

The perturbed film force due to the perturbed film pressure is now expressed in terms of linearized stiffness and damping coefficients in accordance with the linearized perturbation theory

\[
 F_c \omega e^{i\tau} = -k_c h_{w_0} \omega e^{i\tau} - D_c \frac{d(h_{w_0} \omega e^{i\tau})}{dt}. \tag{7.4.12}
\]

which in dimensionless form is

\[
 F'_x = -K_x' - i\sigma'D_x'. \tag{7.4.13}
\]

Equating the real part and the imaginary parts, the dynamic stiffness and damping coefficients are obtained, respectively.

\[
 K_x' = k_c h^3_{w_0} / \mu_k UL^2 B = -R(F'_x). \tag{7.4.14}
\]

\[
 D_x' = Dc h^3_{w_0} / \mu_k L^2 B = \frac{1}{\sigma'} \frac{1}{d \left( F'_x \right)}. \tag{7.4.15}
\]
7.5 Results and Discussion

The non-Newtonian effects of an isothermal incompressible laminar-flow of a lubricant on the dynamic characteristics of one-dimensional hyperbolic slider bearings is analyzed. Based upon the Rabinowitsch fluid model, the non-Newtonian properties are characterized by the dimensionless nonlinear factor $\alpha'$. As $\alpha'=0$, the Newtonian case is recovered. For $\alpha'<0$, the fluid is dilatant. For $\alpha'>0$, it characterizes pseudoplastic behavior. In the present analysis bearings characteristics are evaluated for values of the following parameters:

Wedge parameter $\delta'=0.25-2.75$, Nonlinear factor $\alpha' = -0.1-0.1$, and Squeeze number $\sigma' = 0.1-1$.

Steady-State Characteristics

Figure 7.2 shows the variation of non-dimensional steady film pressure $p_0^*$ with dimensionless co-ordinate $x^*$ for different values of non-linear factor $\alpha'$ at wedge parameter $\delta'=1$. It is observed that, the effect of a pseudoplastic lubricant ($\alpha'>0$) is to decrease $p_0^*$, whereas the dilatent behavior ($\alpha'<0$) increases the steady film pressure.

The variation of non-dimensional steady performances with non-linear factor $\alpha'$ at wedge parameter $\delta'=1$ is depicted in Figure 7.3. It is observed that increasing values of $\alpha'$, degreases the value of flow rate $Q_0^*$. A marginal decrease in $W_0^*$ is observed for the increasing values of $\alpha'$ whereas a marginal increase in non-dimension friction force $F_f^*$ with increasing in $\alpha'$. 
Dynamic Characteristics

Fig. 7.4 shows the variation of dimensionless dynamic stiffness coefficient $K_\ast$ and the damping stiffness coefficient $D_\ast$ with nonlinear factor $\alpha'$ for $\delta' = 1$ and $\sigma' = 1$. The results represent the Newtonian case for $\alpha' = 0$. The values of $K_\ast$ and $D_\ast$ are found to decrease with increasing value of the nonlinear factor. Since the steady load carrying capacity increases with decreasing $\alpha'$, higher values of damping co-efficients are obtained for negative values of the nonlinear factor $\alpha'$.

The variation of dimensionless dynamic stiffness coefficient $K_\ast$ with wedge parameter $\delta'$ for different values of $\alpha'$ is shown in Figure 7.5. A marginal effect of $\alpha'$ on $K_\ast$ is observed for smaller values of $\delta'$. However a noticeable increase (decrease) in $K_\ast$ is observed for dialatant (pseudoplastic) fluids.

Figure 7.6 shows the variation of dimensionless damping coefficient $D_\ast$ with wedge parameter $\delta'$ under squeeze number $\sigma' = 1$. For the bearing with smaller $\delta'$, the effect of $\alpha'$ on the damping coefficient is found to be small. But, for moderate values of wedge parameter ($\delta' > 0.75$), increasing value of $\delta'$ increases noticeably the value of $D_\ast$. It is also observed that negatively decreasing value of $\alpha'$ increases $D_\ast$ and positively increasing values of $\alpha'$ decreases the value of damping co-efficient $D_\ast$.

To get further insight into the effect of squeeze action on the bearing, the variation of dynamic characteristics with squeeze parameter $\sigma'$ for $\delta' = 1$ is
predicted in Figure 7.7. It is observed that $D^*_e$ decreases with squeeze parameter $\alpha'$. Further $D^*_e$ increases for dialatent fluids ($\alpha'<0$) where as it decreases for pseudoplastic ($\alpha'>0$) fluids compared to the corresponding Newtonian case.

7.6 Conclusions

The non-Newtonian effects of an isothermal incompressible lubricant on the dynamic stiffness and damping characteristics of hyperbolic slider bearings, neglecting the fluid inertia and cavitation effects are presented. On the basis of the Rabinowitsch (cubic equation) model incorporated with the bearing squeeze action, the non-Newtonian Reynolds-type equation considering transient motion of the slider is derived. Applying perturbation technique to the Reynolds-type equation both the steady-state performances and the dynamic characteristics are obtained. As the value of non-linear factor ($\alpha'$) is equal to zero, bearing characteristics reduce to Newtonian lubricant case.

The influence of various parameters (factors) upon the steady and dynamic characteristics is significantly apparent and these are the nonlinear factor of a non-Newtonian lubricant, the wedge parameter of a hyperbolic slider profile, and the squeeze number of bearing squeezing action. Lower steady flow rate, higher steady load carrying capacity, dynamic stiffness and damping coefficients are predicted for the bearing lubricated with a dilatent fluid ($\alpha'<0$). However, the
reverse trend is observed for the Pseudoplastic lubricant ($\alpha' > 0$). For fixed wedge parameter, the effects of non-Newtonian characteristics on the damping behaviors are more pronounced under high squeeze numbers.
Fig 7.2. Variation of dimensionless steady film pressure $p_0^*$ with dimensionless co-ordinate $x^*$ for different values of nonlinear factor $\alpha$ with $\delta^* = 1$. 
Fig. 7.3. Variation of dimensionless steady characteristics with nonlinear factor $\alpha$ for $\delta = 1$. 

- $Q^*$
- $F^*$
- $W^*$
Fig. 7.4. Variation of dimensionless dynamic coefficients $K_e^*$ and $D_e^*$ with nonlinear factor $\alpha$ for $\delta = 1$ and $\sigma = 1$. 
Fig. 7.5. Variation of dimensionless dynamic stiffness coefficient $K_e^*$ with wedge parameter $\delta$ with $\sigma = 1$. 
Fig. 7.6. Variation of dimensionless dynamic damping coefficient $D_e^*$ with wedge parameter $\delta$ for $\dot{\sigma} = 1$. 
Fig. 7.7. Variation of dimensionless dynamic damping coefficient $D^*$ with squeeze parameter $\sigma'$ for $\delta = 1$. 