Chapter 4

SURFACE ROUGHNESS EFFECT ON COUPLE-STRESS SQUEEZE-FILMS BETWEEN CURVED ANNULAR PLATES

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4.1 Introduction

In the previous chapter, we have studied the squeeze film lubrication between rough curved annular plates with lubricant to be a viscous fluid. In the present chapter, we analyze the combined effects of surface roughness and couple-stress on squeeze film between curved annular plates.

Squeeze film characteristics play an important role in many applications viz lubrication of machine elements and artificial joints etc. In view of their wide-ranging applications, numerous theoretical and experimental studies have been made [45-50] and [70]. Many investigators have used couple-stress fluid theory to analyze the performance of various bearing systems [71-74]. These studies have led to the predictions such as higher load carrying capacity, lower co-efficient of friction and delayed time of approach in comparison with the Newtonian case.

In most of the theoretical investigations of hydrodynamic lubrication, it has been assumed that the bearing surfaces are smooth. This is an unrealistic assumption for the bearings operating with small film thickness. In the recent years, a considerable amount of tribological research has been devoted to the study of surface roughness on hydrodynamic lubrication. This is mainly because of the fact that all solid surfaces are rough to some extent and generally the height of roughness asperities is of the same order as the mean separation between lubricated contacts. In the literature several lubrication models accounting for surface roughness effects have been proposed in order to seek a more realistic representation of bearing surfaces. Burton [10] studied the effect of surface
roughness on the load supporting characteristics of a lubricant film by postulating the sinusoidal variations in film thickness. Christensen [13] developed a stochastic model for the study of hydrodynamic lubrication of rough surfaces. Prakash and Christensen [52] used stochastic theory to study surface roughness effects on squeeze film lubrication between two rectangular plates. The hydrodynamic lubrication of rough journal bearings was studied by Christensen and Tonder [53]. Bujurke and Naduvinamani [59] have included the effect of surface roughness on the squeeze film lubrication between anisotropic porous rectangular plates and predicted the sensitivity of the maximum load to the roughness parameter. Many investigators used this theory to analyze the lubrication characteristics of various bearing systems such as slider bearings [75, 76], journal and squeeze film bearings [57, 60]. Since the effect of couple stresses is significant and the roughness cannot be avoided, it is worth to investigate the combined effects of these on the bearing performance.

The aim of this chapter is to extend the earlier analysis of squeeze films between curved annular plates studied by Gupta and Vora [68] to include the effect of surface roughness. The stochastic method for rough surface developed by Christensen is adopted. The generalized Reynolds type equation is derived and later the Christensen stochastic roughness model is introduced to account for the surface roughness effects. Expressions for mean squeeze film pressure, mean load carrying capacity and squeeze film time are obtained.
4.2 Mathematical Formulation of the Problem

A schematic diagram of the squeeze film geometry is shown in Figure 4.1 under consideration. The upper rough curved annular plate having circular pocket approaches the lower annular flat plate with velocity \( \frac{dh_0}{dt} \). The lubricant in the film region is assumed to be Stokes [34] couple-stress fluid. It is also assumed that, the body forces and body couples are absent. Under the usual assumptions of hydrodynamic lubrication applicable to thin films, the equations of motion for couple-stress fluid and continuity equation in the film region are equations (2.2.1-3).

To represent the surface roughness the expression for the film thickness is considered to be of two parts

\[
H = h(r,t) + h_s(r, \theta, \xi),
\]

(4.2.1)

where \( h(r,t) = h_0(t)e^{-\beta r^2}, \ a \leq r \leq b \) denote the nominal smooth part of the film geometry while \( h_s \) is the part due to the surface asperities measured from the nominal level and is regarded as a randomly varying quantity of zero mean, \( h_0(t) \) is the thickness at the center of the film, and \( a \ & b \) are respectively the inner and outer radii, \( r \) is the radial co-ordinate and \( \beta \) is the curvature parameter, convex film can be generated for \( \beta < 0 \) and concave ones for \( \beta > 0 \).

The relevant boundary conditions for the velocity components are

\[
\begin{align*}
    u &= 0, \quad w = 0, \\
    \frac{\partial^2 u}{\partial z^2} &= 0, \quad \text{at} \quad z = 0,
\end{align*}
\]

(4.2.2)
Fig. 4.1- Squeeze film configuration of curved circular plate with a concentric circular pocket.
and \[ u = 0, \quad w = \frac{dh_0}{dt}, \quad \frac{\partial^3 u}{\partial z^3} = 0. \text{ at } z = H. \] (4.2.3)

Since \( p \) is independent of \( z \), the solution of equation (2.2.1) subject to boundary conditions (4.2.2) and (4.2.3) is

\[ u = \frac{1}{2\mu} \frac{dp}{dr} \left[ z^2 - zH + \frac{2}{l^2} \left( 1 - \frac{\cosh((2z - H)l/2)}{\cosh(lH/2)} \right) \right], \] (4.2.4)

where

\[ l = \left( \frac{\mu}{\eta} \right)^{\frac{1}{2}} \]

is the couple-stress parameter.

Integration of equation (2.2.3) across the fluid film and the use of boundary conditions (4.2.2), (4.2.3) and the expression (4.2.4) for \( u \), gives the modified Reynolds equation in the form

\[ \frac{1}{r} \frac{d}{dr} \left[ rf(H,l) \frac{dp}{dr} \right] = 12\mu \frac{dh_0}{dt}, \] (4.2.5)

where

\[ f(H,l) = H^3 - \frac{12}{l^2} \left[ H - \frac{2}{l} \tanh \left( \frac{lH}{2} \right) \right]. \]

Following the Christensen stochastic approach of rough surface, taking the expected value of both side of Reynolds equation (4.2.5), and one can get the stochastic Reynolds type equation;

\[ \frac{1}{r} \frac{d}{dr} \left[ rE \left( f(H,l) \frac{dp}{dr} \right) \right] = 12\mu \frac{dh_0}{dt}, \] (4.2.6)

where
\[ E(\bullet) = \int_{-\infty}^{\infty} f(h_s) dh_s. \]

and \( f(h_s) \) is the probability distribution function of the random variable \( h_s \) as defined in the equation (2.2.11).

The relevant boundary conditions for the pressure are

\[
E(p) = 0 \quad \text{at} \quad r = a,
\]

(4.2.7)

\[
E(p) = 0 \quad \text{at} \quad r = b.
\]

(4.2.8)

In the context of stochastic theory, the following two types of roughness structures are of special interest.

**Circumferential Roughness**

In this model, the roughness is assumed to have the form of long narrow ridges and valleys running in \( \theta \)-direction. The film thickness can be described by a function of the form

\[ H = h(r,t) + h_s(r,\xi). \]

(4.2.9)

Then the modified Reynolds equation (4.2.6) takes the form

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{dE(p)}{dr} \right] = 12\mu \frac{dh_s}{dt}.
\]

(4.2.10)

**Radial Roughness**

In this model, the roughness is assumed to have the form of long narrow ridges and valleys running in \( r \)-direction. The film thickness can be described by a function of the form

\[ H = h(r,t) + h_s(\theta,\xi). \]

(4.2.11)
and in this case the modified Reynolds equation (4.2.6) is

\[ \frac{1}{r} \frac{d}{dr} \left[ rE(f(H,t)) \frac{dE(p)}{dr} \right] = 12\mu \frac{dh_0}{dt}. \]  

(4.2.12)

**Solution of the Problem**

Equations (4.2.10) and (4.2.12) together can be written in the form

\[ \frac{1}{r} \frac{d}{dr} \left[ rG(H,l) \frac{dE(p)}{dr} \right] = 12\mu \frac{dh_0}{dt}, \]  

(4.2.13)

where

\[ G(H,l) = \begin{cases} (E[I/f(H,l)])^{-1} & \text{for circumferential roughness,} \\ E(f(H,l)) & \text{for radial roughness.} \end{cases} \]

Integrating equation (4.2.13) twice with respect to \( r \) and making use of the boundary conditions (4.2.7) and (4.2.8), the mean squeeze film pressure in the film region is obtained as

\[ E(p) = \mu \dot{h}_0 \left[ \frac{1}{b} \int_{a}^{b} \frac{6r}{G(H,l)} \frac{dr}{rG(H,l)} - \int_{a}^{b} \frac{1}{rG(H,l)} \frac{6r}{dr} \right]. \]  

(4.2.14)

Introducing the following non-dimensional parameters and variables

\[ H = \bar{h} + \bar{h}_s, \quad \bar{h} = \frac{h}{h_0} = e^{-\beta \rho^2}, \quad \bar{h}_s = \frac{h_s}{h_0}, \quad R = \frac{r}{b}, \quad \bar{p} = \frac{E(p)h_0}{b^2 \mu h_0}, \quad C = \frac{c}{h_0}, \quad \tau = \frac{l}{h_0}, \quad \bar{p} = \beta b^3 \]

in the above, we get the non-dimensional mean film pressure in the form

\[ \bar{p} = \int_{a}^{b} \frac{6R}{G(H,\tau)} dR - \int_{a}^{b} \frac{1}{RG(H,\tau)} \frac{dR}{G(H,\tau)} \int_{a}^{b} \frac{6R}{G(H,\tau)} dR, \]  

(4.2.15)
and the load carrying capacity of squeeze film is found by integrating pressure over the plate

\[ E(W) = 2\pi \int_0^\infty r E(p)dr \]  

which in non-dimensional form is given by

\[ \bar{W} = \frac{E(W)h_0^3}{2\pi\mu h_0 b^4} = \int_0^1 \frac{3R^3}{4} dR - 3 \int_0^1 \frac{R}{RG(H,\tau)} dR \]  

(4.2.17)

The time taken to attain the film thickness \( h_{02} \) from an initial film thickness \( h_{01} \) under a constant mean load \( \bar{W} \), is obtained from (4.2.17) as

\[ t = \left( \frac{1}{h_{02}^2 - h_{01}^2} \right) \frac{\bar{W}}{E(W)\pi \mu b^4} \]  

(4.2.18)

which in non-dimensional form is

\[ T = \frac{t E(W) h_0^2}{\pi \mu b^4} = \left[ \frac{1}{\bar{h}_{02}^2} - \frac{1}{\bar{h}_{01}^2} \right] \bar{W}, \]  

(4.2.19)

where

\[ \bar{h}_{02} = \frac{h_{02}}{h_0}, \bar{h}_{01} = \frac{h_{01}}{h_0}, \]

\[ G(H,\tau) = \left\{ \begin{array}{ll} [E(1 F(H,\tau))]^{-1} & \text{for circumferential roughness,} \\ E(F(H,\tau)) & \text{for radial roughness.} \end{array} \right. \]
4.3 Results and Discussion

Using Christensen [13] stochastic theory for rough surface and Stokes [34] couple-stress fluid model for lubricant we analyze the combined effects of surface roughness and couple-stress on the characteristics of squeeze film lubrication between curved circular plate and a flat plate. The couple-stress parameter \( l = (\mu/\eta)^{1/2} \) is the dimension of length-squared and may be regarded as the chain length of polar additives in the lubricant. Hence, it provides the microstructure 'size' effect on the mechanism of interaction of the lubricant with the bearing geometry.

The squeeze film characteristics are the functions of the various non-dimensional quantities such as roughness parameter \( C(=c/h_0) \), couple-stress parameter \( \tau(=l/h_0) \) and curvature parameter \( \bar{\beta}(=\beta \theta) \). The negative values of \( \bar{\beta} \) \((\bar{\beta} < 0)\) produce convex pad geometry and positive values of \( \bar{\beta} \) \((\bar{\beta} > 0)\) generate that of concave pad.

Figures 4.2 and 4.3 show the variation of non-dimensional pressure \( \bar{p} \) as a function of the non-dimensional radial co-ordinate \( R \) for different values of roughness parameter \( C \), for both concave and convex pads. The dotted curves in the graphs correspond to the smooth case \((C=0)\). It is observed that the point of maximum pressure is asymmetrically located and is shifted toward the outer edge for concave pad \((\bar{\beta} > 0)\) and this shift is toward the inner edge for convex pad \((\bar{\beta} < 0)\). This shift of the pressure peak is due to the wedge effect and is toward the...
minimum film thickness. The important observation is that the effect of roughness is to increase the pressure for circumferential roughness pattern and to decrease it for radial roughness pattern as compared to the corresponding smooth cases. The variation of non-dimensional mean squeeze film pressure $\bar{p}$ as function of $R$ for different values of $\tau$ for both concave ($\beta=0.25$) and convex ($\beta=-0.25$) pad geometries is shown in Figures 4.4 and 4.5 for radial and circumferential roughness patterns respectively. The dotted curves in the graphs correspond to Newtonian case ($\tau \to \infty$). It is found that the effect of couple-stress is to increase the squeeze film pressure as compared to the corresponding Newtonian case. Further, the increase in $\bar{p}$ is more pronounced in the case of concave pad geometries as compared to the convex pad geometries for both circumferential and radial roughness patterns.

The variation of non-dimensional mean load carrying capacity $\bar{W}$ as a function of the normalized curvature parameter $\bar{\beta}$ for different values of roughness parameter $C$ for two values of $a/b$ for both concave and convex pads is depicted in Figures 4.6 and 4.7 for radial and circumferential roughness patterns respectively. The dotted curves in the graphs correspond to the smooth case. It is observed that $\bar{W}$ increases (decreases) for concave (convex) pad geometries as $\bar{\beta}$ increases. Further it is observed that $\bar{W}$ decreases for increasing values of $a/b$ in both the cases. The effect of roughness is to increase the load carrying capacity in
the case of circumferential roughness pattern and to decrease it in the case of radial roughness pattern as compared to smooth case.

Figures 4.8 and 4.9 show the variation of non-dimensional mean load carrying capacity $\overline{W}$ as a function of normalized curvature parameter $\beta$ (for radial and circumferential roughness patterns respectively) for various values of couple-stress parameter $\tau$ with different values of $a/b$ for both concave and convex pads. The effect of pocket size on load carrying capacity is characterized by the parameter $a$, the load carrying capacity of the bearing decreases for increase of pocket radius $a$. The mean load carrying capacity increases (decreases) for concave (convex) pads for increasing values of the curvature parameter, $\beta$. It is also observed that the effect of couple stresses is to increase the $\overline{W}$ for both types of roughness patterns as compared to the corresponding Newtonian case.

The variation of non-dimensional squeeze film time $T$ with the non-dimensional final central film thickness $\overline{h}_{02}$ for different values of $\tau$ is depicted in Figures 4.10 and 4.11 for radial and circumferential roughness patterns respectively. It is observed that $T$ is large for couple-stress lubricant as compared to the corresponding Newtonian case for both concave and convex pad geometries. Figure 4.12 shows the variation of $T$ with $\overline{h}_{02}$ for various values of $C$ for both types of roughness patterns with $\beta = 0.2$ (concave pad geometry). It is observed that squeeze film time is lengthened for the circumferential roughness pattern as compared to the smooth case ($C=0$). Whereas the effect of radial roughness
pattern is to reduce $T$ as compared to the corresponding smooth case. However, the numerical computations show that the effect of $C$ on the variations of $T$ with $\bar{h}_{02}$ is marginal for convex pad geometries.

4.4 Conclusions

Using Stokes microcontinuum theory for couple-stress fluids and the Christensen stochastic model for the rough surfaces the combined effects of surface roughness and couple stresses on the squeeze film characteristics of curved annular plates are presented. The generalized stochastic non-Newtonian Reynolds-type equation has been derived. It is found that the effect of couple-stress is to increase the squeeze film pressure, the mean load carrying capacity and squeeze film time as compared to the corresponding Newtonian case. These results are more pronounced for concave pads compared with convex pads. For the radial roughness pattern the influence of roughness is marginal whereas it is more pronounced for the circumferential roughness pattern. In the limiting case $\tau \to \infty$ and $C = 0$ the analysis reduces to the analysis of classical squeeze film lubrication of curved annular plates studied by Gupta and Vora [68]. The useful prediction is that the squeeze film bearings with curved annular plates having surface roughness and couple-stress fluid as lubricant sustain larger load for a longer time as compared to the corresponding Newtonian case by which it improves the bearing performance.
Fig. 4.2. Variation of non-dimensional mean squeeze film pressure $\bar{p}$ as a function of $R$ for different values of roughness parameter $C$ with $x = 5$ for radial roughness pattern.
Fig. 4.3. Variation of non-dimensional mean squeeze film pressure $\bar{p}$ as a function of $R$ for different values of roughness parameter $C$ with $\tau = 5$ for circumferential roughness pattern.
Fig. 4.4. Variation of non-dimensional mean squeeze film pressure $\bar{p}$ as function of $R$ for different values of couple-stress parameter $\tau$ with $C=0.2$ for radial roughness pattern.
Fig. 4.5. Variation of non-dimensional mean squeeze film pressure $\bar{p}$ as function of $R$ for different values of couple-stress parameter $\tau$ with $C=0.2$ for circumferential roughness pattern.
Fig. 4.6. Non-dimensional mean load carrying capacity $\overline{W}$ with $\overline{\beta}$ for various values of roughness parameter $C$ with different values of $a/b$ with $x = 5$ for radial roughness pattern.
Fig. 4.7. Non-dimensional mean load carrying capacity \( \bar{w} \) with \( \bar{\beta} \) for different roughness parameter \( C \) with different values of \( a/b \) with \( \kappa = 5 \) for circumferential roughness pattern.
Fig. 4.8. Non-dimensional mean load carrying capacity $\bar{W}$ as a function of normalized curvature parameter $\bar{\beta}$ for various values of couple-stress parameter with different values of $a/b$ and $C=0.2$ for radial roughness pattern.
Fig. 4.9. Non-dimensional mean load carrying capacity $\bar{W}$ as a function of normalized curvature parameter $\overline{\beta}$ for various values of couple-stress parameter with different values of $a/b$ and $C=0.2$ for circumferential roughness pattern.
Fig. 4.10. Variation of non-dimensional squeeze time $T$ with $h_0^2$ for different values of couple-stress parameter $\tau$ for fixed values of $a/b=0.4$ and $C=0.2$ for radial roughness pattern.
Fig. 4.11. Variation of non-dimensional squeeze time $T$ with $\overline{h_{02}}$ for different values of couple-stress parameter $\tau$ for fixed values of $a/b=0.4$ and $C=0.2$ for circumferential roughness pattern.
Fig. 4.12. Variation of non-dimensional squeeze time $T$ with $\bar{h}_{02}$ for different roughness parameters $C$ with $a/b = 0.4$, and $\tau = 5$. 