Chapter 3

EFFECT OF SURFACE ROUGHNESS
ON THE SQUEEZE FILM LUBRICATION BETWEEN
CURVED ANNULAR PLATES

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3.1 Introduction

In the second chapter the combined effects of couple-stress and surface roughness on squeeze film between curved circular plates is analyzed. In this chapter the effect of surface roughness on the characteristics of squeeze film lubrication between curved annular plates is studied.

Earlier attempts for developing a theory of friction recognized that practically all surfaces manufactured are rough on the microscopic scale. The absolute height of the surface roughness asperities varies greatly depending on the method of surface preparation. It may range from $0.05 \mu m$ or less on polished surfaces to $10 \mu m$ on medium-machined surfaces [61]. Hence there are situations where the height of roughness asperities is of the same order as that of the mean separation in the lubricated contacts. Under these conditions, any realistic study of bearings must include the effect of surface roughness. Two types of averaged value methods have been proposed in the literature to mathematically model the large number of roughness asperities on the bearing surfaces. Patir and Cheng [62,63] proposed an average flow model of a randomly generated rough surface with known statistical properties. Christensen [13] developed a stochastic model for the study of fluid film lubrication of rough surfaces. Several investigators in the field (Raj and Sinha [64], Lin [65], Naduvanimani et al [27]) used this method to study the effect of surface roughness on the performance of various bearing systems.
Most of the engineering problems are pure squeeze film phenomenon. In view of this several investigators have studied the squeeze film phenomenon (Archibald [45], Hays [46], Jackson [66], Murti [50], Naduvinamani et al [67]). Gupta and Vora [68] have studied the squeeze film lubrication between curved annular plates with an assumption of perfectly smooth surfaces. The aim of this chapter is to study the effect of surface roughness on the squeeze film characteristics between curved annular plates. An exponential function suggested by Murti [50] is used to describe the curved film. This type of film configuration occurs in machine elements like clutch plates and collar bearings. The stochastic method for rough surfaces developed by Christensen [13] is used to derive stochastic Reynolds type equation and has been solved for the two types of one-dimensional roughness patterns. Closed form expressions of mean squeeze film pressure and mean load carrying capacities are obtained.

### 3.2 Mathematical Formulation and Solution

Figure 3.1 shows the configuration of the squeeze film geometry under consideration. The upper rough curved annular plate having circular pocket approaches the lower annular flat plate with a velocity \( (dh_0/dt) \).

To mathematically model the surface roughness, the film thickness \( H \) is considered to be made up of two parts

\[
H = h(r, t) + h_s(r, \theta, \xi),
\]

(3.2.1)
Fig. 3.1. Squeeze film configuration of curved circular plate with a concentric circular pocket.
where \( h(r, t) = h_0(t) e^{-\delta r^4}, \quad a \leq r \leq b \) denote the nominal smooth part of the film geometry. This type of film configurations is observed in machine elements like clutch plates, collar bearings etc. The other part of the film thickness \( h_s(r, \theta, \xi) \) is the part due to the surface roughness asperities measured from the nominal level and is regarded as a randomly varying quantity of zero mean. \( h_0(t) \) is the central film thickness, \( a \) and \( b \) are the inner and outer radii respectively, \( r \) is the radial co-ordinate and \( \beta \) is the curvature parameter, one can generate the concave film profiles for \( \beta > 0 \) and convex film profiles for \( \beta < 0 \).

With the usual assumptions of hydrodynamic lubrication applicable to thin films, the Navier- Stokes equations in polar co-ordinates reduce to

\[
\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu} \frac{\partial p}{\partial r},
\]

(3.2.2)

\[
0 = \frac{\partial p}{\partial z},
\]

(3.2.3)

\[
\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} = 0,
\]

(3.2.4)

where \( p \) is the fluid film pressure and \( \mu \) is the Newtonian viscosity.

The relevant boundary conditions are

\[
u = 0, \quad w = 0 \quad \text{at} \quad z = 0,
\]

(3.2.5)

\[
u = 0, \quad w = \frac{dh_0}{dt} \quad \text{at} \quad z = H.
\]

(3.2.6)

The Reynolds equation governing the squeeze film pressure is obtained by solving
equation (3.2.2) for $u$ subject to the boundary conditions (3.2.5) and (3.2.6), using it in (3.2.4) and integrating it over the fluid film thickness

$$\frac{1}{r} \frac{d}{dr} \left[ r H^3 \frac{dp}{dr} \right] = 12 \mu \frac{dh_0}{dt}.$$  \hspace{1cm} (3.2.7)

Let $f(h_s)$ be the probability density function of the stochastic film thickness $h_s$.

Taking the stochastic average of equation (3.2.7) with respect to $f(h_s)$, the stochastic Reynolds equation is obtained in the form

$$\frac{1}{r} \frac{d}{dr} \left[ r E \left( H^3 \frac{dp}{dr} \right) \right] = 12 \mu \frac{dh_0}{dt},$$  \hspace{1cm} (3.2.8)

where $E(\cdot)$ is the expectancy operator as defined in (2.2.10).

In view of the Christensen [13] stochastic theory for the hydrodynamic lubrication of rough surfaces, two types of one dimensional roughness patterns are of interest, namely (a) the circumferential roughness pattern having the form of long narrow ridges and valleys running in the $\theta$-direction and (b) the radial roughness pattern having the form of long narrow ridges and valleys running in the radial direction.

For one-dimensional circumferential roughness pattern, the film thickness $H$ assumes the form

$$H = h(r,t) + h_\theta(r,\xi).$$  \hspace{1cm} (3.2.9)

In this case the stochastic Reynolds equation (3.2.8) takes the form

$$\frac{1}{r} \frac{d}{dr} \left[ \frac{r}{E(1/H^3)} \frac{dE(p)}{dr} \right] = 12 \mu \frac{dh_0}{dt},$$  \hspace{1cm} (3.2.10)
The use of binomial theorem and the integration of equation (3.2.11) give expressions for $E(H^{-3})$ and $(E(H^{-3}))^{-1}$ valid up to second moment as

$$E(H^{-3}) = \frac{3h^2 + 2c^2}{3h^3},$$  \hspace{1cm} (3.2.12)

$$\left( E(H^{-3}) \right)^{-1} = h^3 - \frac{2}{3} hc^2.$$  \hspace{1cm} (3.2.13)

For one-dimensional radial roughness pattern, the film thickness $H$ assumes the form

$$H = h(r,t) + h_s(\theta, \xi).$$  \hspace{1cm} (3.2.14)

The stochastic Reynolds equation (3.2.8) takes the form

$$\frac{1}{r} \frac{d}{dr} \left[ r E(H^3) \frac{dE(p)}{dr} \right] = 12 \mu \frac{dh_i}{dt},$$  \hspace{1cm} (3.2.15)

where

$$E(H^3) = \frac{35}{32} c^3 \int (h + h_s)^3 (c^2 - h_s^2)^3 dh_s.$$  \hspace{1cm} (3.2.16)

Equation (3.2.10) and (3.2.15) together can be written as

$$\frac{1}{r} \frac{d}{dr} \left[ r G(h, c) \frac{dE(p)}{dr} \right] = 12 \mu \frac{dh_s}{dt},$$  \hspace{1cm} (3.2.17)
where

\[ G(h, c) = \begin{cases} h^3 - \frac{2}{3}hc^2 & \text{for the circumferential roughness pattern} \\ h^3 + \frac{1}{3}hc^2 & \text{for the radial roughness pattern.} \end{cases} \]

The boundary conditions for the mean fluid film pressure are

\[ E(p) = 0 \text{ at } r = a, \quad (3.2.18) \]
\[ E(p) = 0 \text{ at } r = b. \quad (3.2.19) \]

The closed form solution of equation (3.2.17) subject to boundary conditions (3.2.18) and (3.2.19) is obtained in the form

\[ E(p) = \frac{6\mu h_0}{h} \left[ \frac{1}{G(h, c)} \int_a^b \frac{r}{G(h, c)} \, dr - \frac{1}{R} \int_a^b \frac{1}{G(h, c)} \, dr \right]. \quad (3.2.20) \]

Introducing the following non-dimensional parameters

\[ \overline{H} = \overline{h} + \overline{h}, \quad \overline{h} = \frac{h}{h_0} = e^{\overline{r}} \overline{x}, \quad \overline{h} = \frac{h}{h_0}, \quad R = \frac{r}{b}, \quad \overline{p} = \frac{E(p)h_0^3}{b^2 \mu h_0}, \quad C = \frac{c}{h_0}, \quad \overline{\beta} = \beta b^2, \]

into equation (3.2.20), the non-dimensional mean squeeze film pressure \( \overline{p} \) is obtained in the form

\[ \overline{p} = 6 \left[ \frac{1}{G(\overline{h}, C)} \int_a^b \frac{1}{G(h, C)} \, dR - \frac{1}{G(\overline{h}, C)} \int_a^b \frac{1}{G(\overline{h}, C)} \, dR \right]. \quad (3.2.21) \]

where
\[ G(h, C) = \begin{cases} \frac{h^3 - \frac{2}{3}hC^2}{3} & \text{for the circumferential roughness pattern} \\ \frac{h^3 + \frac{1}{3}hC^2}{3} & \text{for the radial roughness pattern.} \end{cases} \]

The mean load carrying capacity is obtained by integrating the mean squeeze film pressure \( p \) over the annular plate

\[ E(W) = 2\pi \int_a^b E(p) \, dr. \]  

(3.2.22)

which in non-dimensional form is

\[ \overline{W} = \frac{E(W)h_0^3}{2\pi \mu h_0 h^4} = 3 \int_{a/b}^{b} \frac{R^3}{G(h, C)} dR - 3 \int_{a/b}^{b} \frac{1}{RG(h, C)} dR. \]  

(3.2.23)

The time taken to attain the film thickness \( h_{02} \) from an initial film thickness \( h_{01} \) under a constant mean load \( \overline{W} \) is obtained as

\[ t = \left( \frac{1}{h_{02}^2 - h_{01}^2} \right) \frac{\overline{W}}{E(W)} \pi \mu b^4. \]  

(3.2.24)

which in non-dimensional from is given by

\[ T = \frac{t E(W) h_0^2}{\pi \mu b^4} = \left[ \frac{1}{h_{02}^2} - \frac{1}{h_{01}^2} \right] \overline{W}. \]  

(3.2.25)

where \( \overline{h}_{01} = \frac{h_{01}}{h_0}, \overline{h}_{02} = \frac{h_{02}}{h_0} \).

In the limiting case of \( C = 0 \), the squeeze film characteristics obtained in equations (3.2.21), (3.2.23) and (3.2.25) reduce the results obtained by Gupta and Vora [68] for the smooth case.
3.3 Results and Discussion

In the present chapter three non-dimensional parameters $C$, $\bar{P}$ and $a/b$ are of importance. The parameter $C$ arises due to the presence of surface roughness. The parameter $\bar{P}$ is the non-dimensional curvature parameter. The negative values of $\bar{P}$ produce convex pad geometry whereas positive values of $\bar{P}$ generate the concave pad geometries and $a$ and $b$ are respectively the inner and outer radii. The effect of pocket on load carrying capacity is characterized by the parameter $a$, the load carrying capacity of the bearing decreases for increase of pocket radius $a$.

Squeeze Film Pressure

The effect of radial roughness on the variation of non-dimensional squeeze film pressure $\bar{p}$ with the non-dimensional radial co-ordinate is depicted in Figure 3.2 for various values of $P$. The dotted curves in the graph correspond to the smooth surfaces ($C=0$). It is observed that the point of maximum pressure is asymmetrically located and is shifted toward the outer edge for concave pad ($\bar{P} > 0$) and is shifted toward the inner edge for the convex pad ($\bar{P} < 0$) for all values of $C$. Further it is also found that, the effect of radial roughness is to shift the point of maximum pressure toward the inlet edge. This effect is more pronounced for larger values of $\bar{P}$. The effect of radial roughness pattern is to decrease $\bar{p}$ as compared to the corresponding smooth case.
Figure 3.3 shows the effect of circumferential roughness pattern on the variation of $p$ with $R$ for both concave and convex pad geometries. It is observed that, the effect of circumferential roughness pattern is to shift the point of maximum pressure toward the outlet edge. Further it is observed that, the effect of circumferential roughness pattern is to increase $p$ as compared to the corresponding to the smooth case for both concave and convex pad geometries. These effects are more accentuated for higher values of $\tilde{\beta}$.

**Load Carrying Capacity**

The variation of non-dimensional load carrying capacity $\bar{W}$ with the curvature parameter $\tilde{\beta}$ for different values of $C$ is depicted in Figure 3.4 for the radial roughness pattern. It is observed that the variation of $\bar{W}$ in the present case is similar to the case of rough curved circular plate studied by Naduvinamani and Gurubasavaraj [69]. The effect of radial roughness is to decrease $\bar{W}$ as compared to the corresponding smooth case. As the curvature parameter $\tilde{\beta}$ increases numerically $\bar{W}$ decreases for convex pads whereas it is found for concave pads. A decrease in $\bar{W}$ is observed for increasing values of $a/b$ for both concave and convex pad geometries.

Figure 3.5 depicts the variation of $\bar{W}$ with $\tilde{\beta}$ for different values $C$ for the circumferential roughness pattern. It is observed that, the effect of circumferential roughness pattern is to increase $\bar{W}$ as compared to the corresponding smooth case ($C=0$) for both concave and convex pad geometries.
For a given load the non-dimensional squeeze film time $T$ to attain a film thickness $\tilde{h}_{02}$ from an initial film thickness of $\tilde{h}_{01}$ can be calculated from equation (3.2.25).

### 3.4 Conclusions

On the basis of the Christensen [13] stochastic theory for the hydrodynamic lubrication of rough surfaces, the effect of surface roughness on the performance of squeeze film lubrication between curved annular plates is presented. In the limiting case of $C = 0$, the results reduce to those of smooth case studied by Gupta and Vora[68]. The effect of radial (circumferential) roughness pattern is to shift the point of maximum pressure toward the inlet (outlet) edge and also observed that the mean load carrying capacity increases (decreases) for the circumferential (radial) roughness patterns as compared to the corresponding smooth case, for both concave and convex pad geometries. These effects are more pronounced for larger values of $\tilde{\beta}$. The type of geometry considered in this chapter has an application in machine elements like clutch plates and collar bearings and it has application in situations where the edges of bearings are worn out due to high relative velocities and hence has practical importance.
Fig. 3.2. Non-dimensional pressure $\bar{p}$ as a function of $R$ for different values of roughness parameter $C$ for various values of curvature parameter $\bar{\beta}$ for radial roughness pattern.
Fig. 3.3. Non-dimensional mean squeeze film pressure $\bar{p}$ as a function of $R$ for different values of roughness parameter $C$ for various values of curvature parameter $\bar{\beta}$ for circumferential roughness pattern.
Fig. 3.4. Non-dimensional mean load carrying capacity $\overline{W}$ with $\bar{\beta}$ for different roughness parameter $C$ and various values of $a/b$ for radial roughness.
Fig. 3.5. Non-dimensional mean load capacity $\bar{W}$ with $\bar{\beta}$ for different values of roughness parameter $C$ and for various values of $a/b$ for circumferential roughness.