CHAPTER II
WAITING TIME DISTRIBUTION OF BIRTH FOR A PREFERRED SEX

2.1 Introduction:

The concept that male issues later bear (shoulder) the responsibilities of the family is slowly withering in most of the educated families. The boys getting the highest educational facilities simply because they become a source of income of the family cannot be relied upon. In spite of the strict legislative measures taken with regard to the banning of the pre-sex determination tests, unknowing things are happening so distressingly that a countless number of abortions take place for different reasons. But the abortions of the couples married can mostly happen only when sex determination tests are conducted unlawfully. It is quite barbaric in act to know that just born female babies are dumped in the dustbins. The cruelty of man is sadly indescribable. Why such an apathetic treatment of innocent babies! The selective measures have undone the things. Human values are thrown to the winds.

The fertility problem of a particular generation may be passed on to the next generation too. It may so happen that the biological needs of the people may affect the birth rates of a particular sex. The science alone can answer the challenges posed by the unscientific
method of selective procedure of the sex. The decline in the chromosomes deciding the sex of a baby may bring unforeseen demographic picture in a distracted manner. Anything against the nature will result in peculiar changes. Who knows the wrong birth control methods may later lead to only a male population for some more years. Without the female population the generations cannot continue. The sex bias is found in the form of dehumanizing tendencies. Atrocities on women folk are on the rise and abduction, brutal rapes and ruthless killing are increasing. It is not the problem of sex preference alone. The ill-treatment meted to the fairer sex at home and in the society holds a gory scenario. Mercy killings caste-differences, non-availability of the brides in certain communities and such other evil practices have their own role in downsizing the population of the women folk.

Every couple eagerly waits for the child after conception. And usually they are more eager to know the sex of the child. Nowadays sex detection is banned they don't have any choice to wait until the birth occurs. It affects a lot on fertility and the number of children that a family has. Some may go for births until they get the required sex. It may cause the health of the mother, financial status, etc. Some prefer boy some prefer girl and some say “doesn’t matter” whether
boy or girl. Now our study is concerned with i.e. the time required to get the preferred child (male/female). It is generally believed that parental sex preferences have little impact on fertility and other aspects of household behaviour. Even though sex preferences may be weak, its existence itself is sufficient to warrant the attention of the policy makers.

There are differences in the economic costs or benefits associated with boys and girls. The fact that parents care about the sex of their children is established by showing the dependence of the tendency to have more children and the sex composition of earlier children. Sex preferences are also demonstrated in less developed countries through relationships between length of birth intervals and the child sex. The "Parental preference of sex affects divorce, child custody, marriage, shotgun marriage when the sex of the child is known before birth, child support payments, and the decision of parents not to have any more children."

Almost all families prefer boys to girls, just because "Boys will carry the family name forward", "Boys will take care of their parents", "Boys will bring money into the family", "Boys will continue the family line". Boys are considered as investment and girls are considered as expenditure, but there are few who prefer girls also.
Sex is the most important demographic variable. Parents seem to care about the sex of their children. Ignorance, inability to understand the equality between the sexes, misunderstanding of the importance of the sexes are some of the important causes for the sex bias. The tendency for parents to prefer boy to girl child or vice versa is because of certain importance such as the difference in the economic costs or emotional reasons such as benefits associated with boys and girls. If boys and young men can be expected to contribute more than girls, this may be a reason for preferring boys to girls. Sex preferences affect fertility in a statistically significant way (Ben-Porath and Welch, 1976). Decline in fertility is due to the control of sex (in the presence of sex preference and gender detection tests). We find higher fertility among those who did not get the preferred sex composition. When gender detection tests become more affordable, women are more likely to have sex-selective abortions (childbirths) than having unselective abortions or undergoing childbirths without gender detection tests (Kim 2005; Leung 1994; Davies and Zhang 1997). Gupta and Bhat (1997) explored the relations between fertility decline and the net manifestation of sex bias in India. Bhat and Zavier (1999) showed that son preference is more in the northern plains and central uplands of India. The
analysis of Ladusingh et al. (2006) revealed that there is moderate son preference invariant of residence and socio-economic background, but not at the cost of balance sex composition of a boy and a girl and son preference is stronger among illiterate and non working women and women above 30 years of age. Hang and Kohler (2003) used data from German General Social Survey to find the determinants of the preferred sex distribution of desired first and additional children as well as the influence of the sex of previous children on parents' intended and actual parity progressions.

Singh (1964) developed the waiting time distribution for the first birth. Biswas and Srestha (1985) obtained the modified waiting time distribution for the first and for the subsequent births. In the present chapter, the probability distributions are derived under the assumptions i) that mean waiting time to get preferred child (sex) is greater than the mean waiting time to get a first birth and ii) birth for a preferred sex has not observed before the length pre-reproductive period. In the present paper whatever be the preference, we obtain the waiting time distribution of the preferred sex, considering the waiting time T as a continuous variable and also a discrete variable. For the validity of the models developed, we have
collected the data, considering the families with two kinds of children and tested with chi-square goodness of fit test.

2.2 Waiting time Distribution as a continuous variable:

Let $T$ be a positive random variable representing an individual’s waiting time until the occurrence of birth of a preferred sex. The distribution of $T$ will be denoted by

$$F(t) = P(T \leq t),$$

$F(t)$ has an associated density $f(t)$ which is written conditionally as

$$f(t/a\gamma = \lambda) = \lambda e^{-\lambda(t-\gamma)}, \quad t > \gamma, \lambda > 0 \tag{2.2.1}$$

Here as per the assumption (ii), we know that the birth for a preferred sex has not been observed before $\gamma$, i.e. $T \geq \gamma$. Where $\gamma$ is the length of the pre-reproductive period. Then $T$ is the amount of additional waiting time needed for birth of a preferred sex to observe. Therefore we have $P(T = t/T \geq \gamma) = f(t/\lambda)$. We assume that $\Lambda$ is a fecundity level which varies from individual to individual and it follows Pearson type III distribution. That is

$$f(\lambda) = \frac{\beta^\alpha e^{-\beta\lambda} \lambda^{\alpha-1}}{\Gamma\alpha}, \theta < \lambda < \infty$$

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The unconditional distribution for the waiting time $T$ is thus obtained as

$$f(t) = \int f(t/\lambda = \lambda)f(\lambda)d\lambda$$

$$= \frac{\alpha\beta^\alpha}{(t - \gamma + \beta)^{\alpha+1}}, \quad \alpha > 0, \beta > 0, t > \gamma \quad (2.2.2)$$

The parameters of (2.2.2) are derived by the method of maximum likelihood estimation (mle). The log-likelihood function of (2.2.2) is

$$\log L = \sum_{i=0}^{n} \log n \alpha + n \alpha \log \beta - (\alpha + 1) \sum_{1}^{r} \log(t_k - \gamma + \beta) \quad (2.2.3)$$

Differentiating with respect to $\alpha$, $\beta$ and $\gamma$ and equating the derivatives to zero we get

$$\frac{n}{\alpha} + n \log \beta - \sum_{1}^{r} n_k \log(t_k - \gamma + \beta) = 0 \quad (2.2.4)$$

$$\frac{n\alpha}{\beta} - (\alpha + 1) \sum_{1}^{r} n_k (t_k - \gamma + \beta)^{-1} = 0 \quad (2.2.5)$$

$$\alpha \sum_{1}^{r} n_k \log(t_k - \gamma + \beta)^{-1} = 0 \quad (2.2.6)$$
2.3 Waiting time distribution as a discrete variable:

Let $T$ be a waiting time (in terms of births) to get birth of a preferred sex. The probability distribution of the waiting time of birth for a preferred sex is based on the assumptions stated above that

$$P(t) = e^{-\lambda} (1 - e^{-\lambda})^t, t \geq 0$$

(2.3.1)

Where $T$ is the amount (in terms of births) of additional time needed for birth of a preferred sex to observe. Therefore we can write (2.3.1) as

$$P(T = t \mid X \geq \gamma) = P(T = t).$$

The $\lambda$ can be estimated by the method of maximum likelihood estimation (mle). Thus the log-likelihood function of (2.3.1) is

$$\log(L) = \sum_{t=0}^{d} n_t [-\lambda + t \log(1 - e^{-\lambda})],$$

$d$ is the maximum value observed. Differentiating Log-likelihood function with respect to $\lambda$ and equating the derivative to zero, finally we get

$$\text{mean } (\overline{X}) = \frac{1 - e^{-\hat{\lambda}}}{e^{-\hat{\lambda}}}$$

(2.3.2)
Model (2.3.1) takes infinite values, but in real situation, 't' takes a finite value 'd'(say). Thus model (2.3.1) can be written after right truncation

\[ P(t) = \frac{e^{-\lambda} (1 - e^{-\lambda})^t}{F(d)}, \quad t = 0, 1, 2, \ldots, d \quad (2.3.3) \]

Where \( F(d) = \sum_{t=0}^{d} e^{-\lambda} (1 - e^{-\lambda})^t \), \( d \) is the maximum value observed.

Thus the log-likelihood function of (2.3.3) is

\[ l = \sum_{r=0}^{d} n_r [-\lambda + t \log(1 - e^{-\lambda}) - \log F(d)] \quad (2.3.4) \]

Differentiating with respect to \( \lambda \) and equating the derivative to zero, finally we get

\[ \text{mean} = (1 - e^{-\lambda}) e^{-\lambda} \left( \frac{1 + 2(1 - e^{-\lambda}) + \ldots + d(1 - e^{-\lambda})^{d-1}}{F(d)} \right) \quad (2.3.5) \]

2.4 Application of the distributions:

For the validity of the model we have considered a sample of 'n' families (in particular women). Each woman is asked about the (their) preference of the child. Preferred sex is noted down at continuous point of time. Let \( f_T(t) \) be the probability function of
waiting time of birth for a preferred sex, where \( T \) is continuous (or discrete) random variable, representing the waiting time of birth for a preferred sex. Then the distribution function of \( T \) is

\[
F_T(t) = P[T \leq t] = \int_{t}^{\infty} f_T(t) \, dt
\]

Where \( t \) cannot be observed before \( \gamma \) and in case of discrete probability distribution, the maximum waiting time in terms of the number of births of preferred sex is observed at \('d'\). It is also known that

\[
P[a \leq X \leq b] = \int_{a}^{b} f_T(t) \, dt
\]

gives the probability that birth of a preferred sex will occur in the interval \((a,b]\). However the conditional probability that birth of a preferred sex will occur in the interval \((a,b]\) given that the individual does not get a birth up to \('a'\) is denoted by

\[
P[a \leq T \leq b \mid T > a] = \frac{P[a \leq T \leq b]}{1 - P[T \leq a]} = P[a \leq T \leq b]
\]

where \( P[T \leq a] = 0 \), \( a \) and \( b \) are the lower and upper age of the reproductive period.
Now assuming the value for $\gamma$, $\alpha(\gamma)$ and $\beta(\gamma)$ can be computed using (2.2.4) and (2.2.5), the corresponding likelihood, $L(\gamma)$ can be calculated. The value of $\gamma$ which maximizes $L(\gamma)$ is then found numerically (Johnson et al. 2004).

The value which satisfies the following equation is then taken as the value of $\beta$

\[ \hat{\beta} = \frac{n^2}{\hat{\psi}(\beta, \gamma)}, \]

Where $\hat{\psi}(\beta, \gamma) = (n + \sum_{i} n_k \log(t_k - \gamma + \hat{\beta}) - n \log \hat{\beta})$

Then by using equation (2.2.4) the value of $\alpha(\beta, \gamma)$ is obtained.

The cross-sectional data is used to fit the waiting time distribution as a continuous variable and discrete variable.
Table 1: Comparison of Observed and Expected Number of women who got birth of a preferred sex.

<table>
<thead>
<tr>
<th>Waiting time (in years)</th>
<th>Observed number of women</th>
<th>Expected number of women</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 - 17</td>
<td>26</td>
<td>25</td>
</tr>
<tr>
<td>17 - 19</td>
<td>42</td>
<td>41</td>
</tr>
<tr>
<td>19 - 21</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>21 - 23</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>23 - 25</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>25 - 27</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>27 - 29</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>29 - 31</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>31 - 33</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>33 - 35</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>35 +</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
X^2 = 0.465^{\text{ns}}
\]

\[
X^2(5\%) = 7.815
\]

\(\text{ns}\) indicates non-significant at 5% level of significance.
Table 2: Comparison of Observed values and Expected values.

<table>
<thead>
<tr>
<th>Preferred sex for the first time</th>
<th>Observed number of Women</th>
<th>Expected number of women</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\lambda = 0.637$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Model 2.3.1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\lambda = 0.686$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Model 2.3.3)</td>
</tr>
<tr>
<td>0</td>
<td>77</td>
<td>83</td>
</tr>
<tr>
<td>1</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5 +</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

$\chi^2$  
$\chi^2_{3}(5\%)$

4.18<sub>ns</sub>  
7.815

3.32<sub>ns</sub>  
7.815

<sub>ns</sub> indicates non-significant at 5% level of significance.

Model (2.2.2), the waiting time distribution for the preferred sex, gives the best fit for this particular data set. Models (2.3.1) and (2.3.3), the waiting time distributions for the preferred sex in terms of the number of births, also give the best fit.

2.5 Development of probability model for consecutive preferred sex:

Consider the Dandekar's modified Poisson process (Biswas 1988)

$$F(t, x|\lambda) = \exp\{-\lambda(t-\Theta x)\} \sum_{r=0}^{x} \frac{\lambda(t-\Theta x)^r}{r!} \quad (2.5.1)$$
Which gives the probability distribution of the number of conceptions or births in time \((0, t]\) given that each conception has intensity \(\lambda\), conform to Gamma distribution with density of the form

\[
f(\lambda) = \frac{\beta^a e^{-\beta \lambda} \lambda^{k-1}}{\alpha}, \quad 0 \leq \lambda \leq \infty \quad (2.5.2)
\]

Suppose that conceptions are not observed before time \(\gamma\), \((\gamma, t]\), then modified form of (2.5.1) can be written as

\[
F(x, t|\lambda) = \exp\{-\lambda(t-\gamma)\} \sum_{r=0}^{x} \frac{\lambda(t-\gamma)^r}{r!}, \quad t > \gamma, \quad (2.5.3)
\]

Therefore the unconditional Poisson process is obtained by

\[
F(t, x) = \int_{0}^{\infty} F(t, x|\lambda)f(\lambda)d\lambda
\]

\[
= \int_{0}^{\infty} e^{\lambda(t-\gamma)} \sum_{r=0}^{x} \frac{\lambda(t-\gamma)^r}{r!} \frac{\beta^a e^{-\beta \lambda} \lambda^{k-1}}{\alpha} d\lambda
\]

\[
= \sum_{r=0}^{x} e^{\lambda(t-\gamma)} \frac{(\lambda(t-\gamma))^r}{r!} \frac{\beta^a e^{-\beta \lambda} \lambda^{k-1}}{\alpha} d\lambda
\]

\[
= \sum_{r=0}^{x} \frac{\beta^a(t-\gamma)^r}{r!} \frac{1}{\alpha} \int_{0}^{\infty} e^{-\lambda(t+\beta-\gamma)} \lambda^{r+\alpha-1} d\lambda
\]

\[
= \sum_{r=0}^{x} \frac{\beta^a(t-\gamma)^r}{r!} \frac{(r + \alpha)}{\alpha} \frac{1}{(t + \beta - \gamma)^{r+\alpha}} \quad (2.5.4)
\]
Suppose we write equation (2.5.4) as

\[
F(t, n) = \sum_{r=0}^{n} \frac{\beta^r(t-\gamma)^r}{(r-1)\alpha(r+1)\alpha(t+\beta-\gamma)^{r+\alpha}}
\]  

(2.5.4(i))

\[= P[X \leq n|t] \]

\[1 - F(t, n) = P[X > n|t] \]

\[1 - F(t, n) = P[T_n \leq t] \]  

(2.5.5)

Where \(T_n\) represents the waiting time for the \(n^{th}\) birth of preferred sex. Therefore we have

\[P[t \leq T_n \leq t + \delta t|t] = \frac{d}{dt} [1 - F(t, n)]\]

\[\frac{d}{dt} [1-F(t,n)] = f_n(t_n) \]

\[= \frac{d}{dt} \left[ 1 - \sum_{r=0}^{n} \frac{\beta^r(t-\gamma)^r}{(r-1)\alpha(r+1)\alpha(t+\beta-\gamma)^{r+\alpha}} \right] \]  

(2.5.6)

If \(n=0\), then the waiting time distribution for the first conception or birth, which is not observed before \(\gamma\) is given by,

\[\frac{d}{dt} [1-F(t_0,0)] = f_0(t_0) \]

\[= \frac{d}{dt} \left[ 1 - \frac{\beta^\alpha \alpha}{\alpha(t_0 + \beta-\gamma)^\alpha} \right] \]
\[ f_1(t_1) = \frac{\alpha \beta^\alpha}{(t_0 + \beta - \gamma)^{\alpha+1}} \]  

(2.5.7)

Which is the waiting time distribution of birth of first preferred sex.

Similarly the waiting time of birth for various orders of preferred sex

If \( n=1 \), Then equation (2.5.6) becomes,

\[ f_1(t_1) = \frac{d}{dt} \left[ \sum_{r=0}^{\alpha} \beta^\alpha (t_1 - \gamma)^r (r + \alpha) \frac{\alpha (r + 1) (t_1 + \beta - \gamma)^{\alpha+1}}{(t_1 + \beta - \gamma)^{\alpha+1}} \right] \]

\[ = \frac{d}{dt} \left[ \sum_{r=0}^{\alpha} \beta^\alpha (t_1 - \gamma)^r (r + \alpha) \frac{\alpha (r + 1) (t_1 + \beta - \gamma)^{\alpha+1}}{(t_1 + \beta - \gamma)^{\alpha+1}} \right] \]

\[ = \frac{\beta^\alpha}{(t_1 + \beta - \gamma)^{\alpha+1}} \frac{\alpha}{(t_1 + \beta - \gamma)\alpha} - \frac{\beta^\alpha}{(t_1 + \beta - \gamma)^{\alpha+1}} \frac{(\alpha - 1)}{(t_1 + \beta - \gamma)^{\alpha+1}} \]

\[ = \beta^\alpha \left[ \frac{\alpha}{(t_1 + \beta - \gamma)^{\alpha+1}} - \frac{\alpha(t_1 - \gamma)}{(t_1 + \beta - \gamma)^{\alpha+1}} \right] \]

\[ = \alpha \beta^\alpha \left[ \frac{1}{(t_1 + \beta - \gamma)^{\alpha+1}} \right. \]

\[ \left. - \frac{(t_1 + \beta - \gamma)^{\alpha+1} - (\alpha + 1)(t_1 - \gamma)(t_1 + \beta - \gamma)^{\alpha}}{(t_1 + \beta - \gamma)^{\alpha+2}} \right] \]

\[ = \alpha \beta^\alpha \frac{(\alpha + 1)(t_1 - \gamma)}{(t_1 + \beta - \gamma)^{\alpha+1}} \]

\[ = \frac{\alpha (\alpha + 1) \beta^\alpha (t_1 - \gamma)}{(t_1 + \beta - \gamma)^{\alpha+2}} \text{, } \gamma \leq t_1 < \infty, \quad (2.5.8) \]
Similarly, if \( n=2 \), in equation (2.5.6) we have

\[
 f_2(t_2) = \frac{d}{dt} \left[ 1 - \sum_{r=0}^{2} \frac{\beta^\alpha (t_2 - \gamma)^r}{\alpha (r - 1)} \frac{(r + \alpha)}{(t_2 + \beta - \gamma)^{(r+\alpha)}} \right]
\]

\[
 = \alpha \beta^\alpha \frac{(t_2 + \beta - \gamma)^{\alpha+1}}{(t_2 + \beta - \gamma)^{\alpha+1}} \left[ \frac{(t_2 + \beta - \gamma)(\alpha + 1)(t_2 - \gamma)}{(t_2 + \beta - \gamma)^{\alpha+1}} \right] - \frac{\alpha(\alpha + 1)\beta^\alpha (t_2 - \gamma)}{(t_2 + \beta - \gamma)^{\alpha+2}} \left[ \frac{(t_2 - \gamma)(t_2 + \beta - \gamma) - (\alpha + 2)(t_2 - \gamma)^2}{(t_2 + \beta - \gamma)^{\alpha+2}} \right]
\]

\[
 = \frac{\alpha(\alpha + 1)\beta^\alpha (t_2 - \gamma)}{(t_2 + \beta - \gamma)^{\alpha+2}} \left[ \frac{(\alpha + 2)(t_2 - \gamma)}{(t_2 + \beta - \gamma)^{\alpha+2}} \right] = \frac{\alpha(\alpha + 1)(\alpha + 2)\beta^\alpha (t_2 - \gamma)^2}{2(t_2 + \beta - \gamma)^{\alpha+3}}; \gamma \leq t_2 < \infty; \quad (2.5.9)
\]

If \( n=3 \), then (2.5.6) becomes,

\[
 f_3(t_3) = \frac{d}{dt} \left[ 1 - \sum_{r=0}^{3} \frac{\beta^\alpha (t_3 - \gamma)^r}{r\alpha \gamma (r - 1)} \frac{\gamma (r + \alpha)}{(t_3 + \beta - \gamma)^{(r+\alpha)}} \right]
\]

\[
 = \frac{\alpha(\alpha + 1)(\alpha + 2)(\alpha + 3)\beta^\alpha (t_3 - \gamma)^3}{6(t_3 + \beta - \gamma)^{\alpha+4}}; \gamma \leq t_3 < \infty; \quad (2.5.10)
\]

Therefore the waiting time for \( n \)th birth of preferred sex is,

\[
 f_{n-1}(t_{n-1}) = \frac{\alpha(\alpha + 1) ... (\alpha + n - 1)\beta^\alpha (t_{n-1} - \gamma)^{n-1}}{(n-1)! \gamma (t_{n-1} + \beta - \gamma)^{\alpha+n}}; \gamma \leq t_{n-1} < \infty
\]