CHAPTER 3

HIGH FIELD TRANSPORT PROPERTIES IN A GaN/AlGaN HETEROJUNCTION

Part of the work presented in this chapter has appeared in

3.1 Introduction

Semiconductor systems based on GaN have drawn attention as a potential candidates for the applications in high power, high mobility and optoelectronics devices [3.1]. GaN based HJs have advantages over the GaAs because of the higher inter-valley separation. Important differences between GaAs and GaN are: (i) GaN crystallizes more in the wurzite structure rather than the zinc blende structure of GaAs, (ii) it is more strongly polar than GaAs and (iii) its larger polar optical phonon energy is leading to larger drift velocities at high fields. Large saturation velocity, high breakdown voltage and thermal stability of GaN, make it an important material for microwave power devices [3.2-3.5]. In order to improve the functioning of the devices based on GaN/AlGaN heterostructures, a good understanding of their high field electron transport properties is important.

Field dependent drift velocity $V_d$ is one of the most important and widely studied high field transport properties. The large $V_d$ achieved in high fields is important as it determines the current density and high mobility of the carriers. Great deal of experimental work on field dependent drift velocity has been reported in steady state [3.6-3.14]. The Monte Carlo technique has been used for determining the velocity-field characteristics of these semiconductors [3.13-3.17]. Earlier, attempts have been made by Gokden et al [3.10] to calculate the high field drift velocity using the formalism (energy and momentum balance technique) of Ridley et al [3.18] considering electron scattering only by polar optical phonons. It is the dominant energy and momentum relaxation process at higher temperatures. Ridley et al [3.18] and Gokden et al [3.10] assume the drifted Maxwellian distribution for electron distribution function in an electric field and obtain analytical expressions for energy and momentum loss rates. But, in GaN/AlGaN heterostructures, typically
concentrations are large of the order of $10^{17} \text{ m}^{-2}$ leading to dominant electron-electron collisions. The electron distribution under the influence of electric field will be nearly drifted Fermi-Dirac (F-D) distribution function $f(E_k) = \left[ \exp \left\{ \beta (E_k - E_F) \right\} + 1 \right]^{-1}$, where $\beta = \left( k_B T_e \right)^{-1}$. This distribution function is characterized by the hot electron temperature $T_e$, drift momentum $h k_0$ and drift velocity $V_d = h k_0 / m$.

In high field region, a steady state of the carriers is always attainable because the rate of phonon emission or energy loss rate increases as the average energy of the carrier increases. If the electron-electron interaction is faster than the electron-phonon interactions then electrons thermalise rapidly establishing their own temperature $T_e$ which is higher than the lattice temperature $T$. Under thermal equilibrium with the lattice $T_e = T$. If $T_e$ slightly exceeds $T$, it is called as warm electron region. However, in high field region $T_e \gg T$ under non-equilibrium conditions referred as hot electron region. In this region, the distribution is well approximated by a Fermi-Dirac distribution at $T_e$ and it is called electron temperature model [3.19]. Momentum of the carriers is also randomized due to carrier-carrier collision and the net momentum of the carriers is developed. In the limit of rate of momentum exchange among the carriers being much greater than that between the carriers and the scattering centers, the distribution is characterized by drifted F-D in the electron temperature model. The use of drifted F-D distribution leads to the results which compare favorably with numerical solutions and is exact when electron-electron scattering dominates in energy and momentum exchanges. It is more essential and also appropriate to use this distribution function, in the present work, as we attempt to explain high field experimental data in a GaN/AlGaN HJ with large electron density over the large electron temperature range $T_e = 4.2-500 \text{ K}$. 

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Latest mobility studies in GaN HJs, as mentioned in section 2.3, have shown that there are new scattering mechanisms namely misfit deformation potential and misfit piezoelectric potential that limit momentum relaxation [3.20]. Hence, we expect these new scattering mechanisms to play the significant role in limiting the high field $V_d$ and mobility. We incorporate these mechanisms in momentum relaxation of the carriers in addition to the conventional scattering due to, interface roughness, remote impurity, dislocations and phonons (acoustic and optical).

In the present work, the momentum and energy balance technique [3.19] is employed using drifted F-D distribution function for the calculation of high field drift velocity. Energy relaxation is considered to be due to acoustic phonons via deformation potential and piezoelectric interactions and polar optical phonons taking account of hot phonon effect. Momentum relaxation is considered to be due to interface roughness (SR), misfit deformation potential (MDP), misfit piezoelectric potential (MPE), remote impurity (RI), dislocation via coulomb interaction (DC), acoustic and optical phonons. Consideration of all these mechanisms is very essential, because of the large $T_e$ range, in contrast with the scattering due to only polar optical phonons considered by Ridley et al [3.18] and Gokden et al [3.10]. These calculated results are compared with the available experimental data.

Hot electron energy relaxation is another important property intensively studied during last three decades in GaAs heterostructures (Ref.3.21 and references there in). Hot electron energy loss rate $P$ provides a better understanding of electron-phonon interaction than the conventional mobility studies. It is being addressed in GaN heterostructures in recent times [3.22-3.25]. Recently, Cheng et al [3.25] have reported low temperature ($T_e<20$ K) measurements of $P$ vs electron temperature $T_e$ for two GaN heterostructures to study electron-acoustic phonon interactions. These
authors attempt to explain experimental data of $P$ using formula in Bloch-Gruinesen (BG) regime (acoustic phonon energy $\sim k_B T \ll$ Fermi energy) with the $T_e^4$ power law dependence on $T_e$. However, the agreement between theory and the experimental results of Cheng et al [3.25] is poor in magnitude and behavior. In most of the work in low $T_e$, the BG regime power law is used to explain the $P$ vs $T_e$ data, which is not valid in larger $T_e$ region of BG regime. In view of this, we give $P$ calculation at low $T_e$ in a GaN/AlGaN HJ, by taking its full form of equations due to screened electron-acoustic phonon interaction and show considerable improvement in the agreement with the experimental results. We show that validity of BG regime is only for very low $T_e$. Also, calculation of $P$ is presented for $T_e$ up to 500 K and compared with the experimental data of Ref. [3.24].

3.2 Theory

We consider a two-dimensional electron gas (2DEG) confined in a GaN/AlGaN HJ wurzite structure. We assume that only lowest subband is occupied with the electron wave function $\varphi(z)\exp(ikz)/\sqrt{A}$ (Eqn. 1.2.1.3) and the energy of the 2DEG is assumed to be parabolic in nature $E_k = \hbar^2 k^2/2m$ (Eqn. 1.2.1.11). Electron distribution is assumed to obey drifted F-D distribution in electron temperature model. In an applied electric field $F$, the momentum and energy balance equations are given by [3.19],

$$eF = -\sum_j \left\langle \frac{dP_F}{dt} \right\rangle_j \quad \text{and} \quad eFV_d = -\sum_j \left\langle \frac{dE_k}{dt} \right\rangle_j,$$

where $j$ stands for different scattering mechanisms. The expressions for $\left\langle dE_k/dt \right\rangle$ and $\left\langle dP_F/dt \right\rangle$ due to acoustic and optical phonons are obtained by following the standard procedure [3.19, 3.26]. Isotropic approximation is used for electron-acoustic phonon coupling.
3.2.1  Energy and momentum loss rates due to phonons

The average energy/momentum loss rate per carrier is calculated by finding the energy/momentum gained by the phonons from the carriers and dividing by the total number of carriers. Following the approach given in Ref. [3.19], we write,

$$\frac{dE_k}{dt} = P = -\frac{1}{N_e} \sum_Q \hbar \omega_Q \frac{\partial N_Q}{\partial t} \quad \text{and} \quad \frac{dP_F}{dt} = -\frac{1}{N_e} \sum_Q \hbar \omega_Q \frac{\partial N_Q}{\partial t}, \quad (3.2.1.1)$$

where $N_e$ is the total number of electrons which are participating, $\hbar \omega_Q$ is the phonon energy with wave vector $Q = (q_1^2 + q_z^2)^{1/2}$ and $(\partial N_Q / \partial t)$ is the rate of change of phonon occupation number $N_Q$. $Q_F$ is the phonon wave vector in the direction of electric field $F$ applied in the plane of the sample and it is given by $Q_F = (q + q_z) \cdot F/F = q \cos \varphi,$ where $\varphi$ is the angle between $q$ and $F$.

The rate of change of $N_Q$ is given by [3.26],

$$\frac{\partial N_Q}{\partial t} = 2 \frac{2\pi}{\hbar} \sum_{k} |M(Q)|^2 |F(q_z)|^2 e^{-2(q)} \{(N_Q + 1)f(k)[1 - f(k - q)]\delta(E_k - E_k + \hbar \omega_Q)$$

$$-N_Q f(k)[1 - f(k + q)]\delta(E_k - E_k - \hbar \omega_Q)\} \quad \text{(3.2.1.2)}$$

where $|M(Q)|^2$ is the electron-phonon matrix element.

In evaluating Eqn. (3.2.1.2), $f(E_k)$ is approximated to first order expansion

$$f(E_k) = f'(E_k) + (h\kappa V_\theta/k_B T_e)f''(E_k), \quad (3.2.1.3)$$

where $f'(E_k)$ is the F-D distribution at $T_e$.

3.2.2  Energy and momentum loss rates due to acoustic phonons

Substituting the matrix element (Eqn. 1.4.7.4) for the coupling of acoustic phonons via deformation potential in Eqn. (3.2.1.2), the expression for energy loss rate per electron $P_{dp}$ is obtained and it is given by [3.26],
This equation reduces to a simple power law in the BG regime and it is given by [3.27]

\[ p_{BG} = \frac{90 \xi(7) \Xi^2 \rho_F^2 (k_B T_0)^7}{\pi^2 n_s h^3 \sqrt{v_F^2 (q_F)^2 k_F}} \]  

(3.2.2.2)

where \( \xi(7) \) is the Riemann zeta function, \( k_F \) is Fermi wave vector and \( q_F = \frac{m e^2}{2 \pi \hbar h} \) is the screening wave vector.

The momentum loss rate \( \langle dP/dt \rangle_{dp} \) is found to be

\[
\begin{align*}
\langle dP/dt \rangle_{dp} = & -\frac{V_m \Xi^2 \rho_F^2 (k_B T_0)^7}{\pi^2 n_s h^3 \sqrt{v_F^2 (q_F)^2 k_F}} \int \frac{F(q_F)}{e^{\beta q_F}} dE_k \biggl\{ \\
& \times \left[ N_{q_0}^0 + 1 \left( 1 + \frac{\hbar \omega_{q_0}}{E_k} \right) f^0(E_k) \left[ 1 - f^{\prime}(E_k) \right] E_k - \gamma \right] \biggr\}^{1/2} dE_k
\end{align*}
\]

(3.2.2.3)

Here, \( i = l \), as only longitudinal acoustic phonons are involved in the scattering due to deformation potential coupling. In these equations \( n_s (=N_c / A) \) is the surface or 2D electron density, \( \Xi_d \) is the acoustic deformation potential constant, \( \rho_F \) is the mass density, \( |F(q_F)|^2 = (1 + q_F^2 / b^2)^3 \) is the overlap integral, \( b = 33 \pi m e^2 n_s / 2 e h^2 \) is the variational parameter determined by \( n_s \), \( e \) is charge of electron, \( e_i \) is static dielectric constant, \( N_{q_0}^0 = [\exp(\hbar \omega_{q_0} / k_B T) - 1]^{-1} \) is the acoustic phonon distribution function at lattice temperature \( T \), \( \gamma = (\hbar \omega_{q_0} - E_q)^2 / 4 E_q \) and \( \gamma_e = (\hbar \omega_{q_0} + E_q)^2 / 4 E_q \) and \( E_q = \hbar^2 q^2 / 2m \).

The temperature dependent static dielectric function in the random phase
approximation is given by $\varepsilon(q, T_e) = 1 + (q_s/q) F(q) \Pi(q, T_e)[1 - G(q)]$ [3.28], where $q_s = (2m^2/\varepsilon_s \hbar^2)$ is the screening parameter, $\Pi(q, T_e)$ is given by Eqn. (8) of [3.29], $[1 - G(q)] = 1 - q/(q^2 + k_F^2)^{1/2}$ is the local field correction with $k_F = (2m\pi \alpha)^{1/2}$ and $F(q) = [3(q/b)^2 + 9(q/b) + 8]/8((q/b) + 1)^3$ is the screening form factor.

Similarly, for screened piezoelectric scattering, expression for energy loss rate is given by [3.21]

$$P_{\text{zz}} = \frac{\varepsilon^2 m^{1/2}}{2^{3/2} \pi^2 n_F^2 \hbar^2} \sum_{\rho = 0}^{\infty} \int dq \int dq_z \frac{A_{\rho}(q, q_z) |F(q_z)|^2}{\varepsilon^2(q)} \exp \left( \frac{\hbar \omega_{q} \left( 1 - \frac{1}{T_e} \right)}{k_s} - 1 \right) \right)$$

$$\times \int_{E_k}^{E_k + \hbar \omega_{q}} \frac{f(0)(E_k + \hbar \omega_{q})}{\sqrt{E_k - \gamma_{\text{ai}}}} dE_k.$$ (3.2.2.4)

For longitudinal phonons $A_{\rho}(q, q_z) = \left((\varepsilon_{33} q_z^2 + (\varepsilon_{33} + 2\varepsilon_{15}) q q_z^2) / Q^2 \varepsilon_s \right)^2$ and for transverse phonons $A_{\rho}(q, q_z) = \left((-\varepsilon_{33} - \varepsilon_{15}) q q_z^2 - \varepsilon_{15} q z / Q^2 \varepsilon_s \right)^2$ [3.25]. Here, $\varepsilon_{33}$, $\varepsilon_{31}$ and $\varepsilon_{15}$ are piezoelectric coefficients.

Expression for energy loss rate in BG regime, due to screened piezoelectric scattering, for LA and TA modes in the form of power law are, respectively, given by [3.25]

$$p_{\text{BG}, \text{LA}} = \frac{3 \varepsilon^2 (5\varepsilon^2 m^2 (k_F T_e)^5}{32 \pi \hbar^2} v_{\text{t}}^2 \rho(q) \varepsilon_s^2 k_F^3 [5e_{33}^2 + 6e_{33}(e_{31} + 2e_{15}) + 5(e_{31} + 2e_{15})^2], \quad (3.2.2.5)$$

and

$$p_{\text{BG}, \text{TA}} = \frac{3 \varepsilon^2 (5\varepsilon^2 m^2 (k_F T_e)^5}{32 \pi \hbar^2} v_{\text{t}}^2 \rho(q) \varepsilon_s^2 k_F^3 [35e_{15}^2 + 10e_{15}(e_{33} - e_{31} - e_{15}) + 3(e_{33} - e_{31} - e_{15})^2]. \quad (3.2.2.6)$$

The momentum loss rate is found to be
\[
\left\langle \frac{dP_r}{dt} \right\rangle_{pr} = -\frac{V_q m^{3/2} e^2}{2^{1/2} \pi^2 n_e \hbar \rho(k_B T_e)} \sum_{r \in \nu_1} \int_0^{\infty} \int_0^{\infty} \frac{A(q, q_z) F(q_z) q^2}{E(q) Q} dq \, dq_z \\
\times \left\{ \int_{\tau_e}^{\infty} \left[ \left( 1 + \frac{\hbar \omega_0}{E_q} \right) f^0(E_k) \left[ 1 - f^0(E_k - \hbar \omega_0) \right] \right] \frac{E_k - \gamma_s}{\sqrt{E_k^2 - \gamma_s}} dE_k + \int_{\tau_e}^{\infty} \left[ \left( 1 - \frac{\hbar \omega_0}{E_q} \right) f^0(E_k) \left[ 1 - f^0(E_k + \hbar \omega_0) \right] \right] \frac{E_k - \gamma_s}{\sqrt{E_k^2 - \gamma_s}} dE_k \right\}.
\]

(3.2.2.7)

### 3.2.3 Energy and momentum loss rates due polar optical phonons

At high $T_e$ electrons lose their energy to the lattice by emitting polar optical phonons. Consequently, the population of phonons increases over its thermodynamic value and it is described by plank distribution but with the temperature higher than the lattice temperature. This is described by hot phonon distribution. We take this hot phonon effect in calculation of energy and momentum loss rates due to polar optical phonons.

The expressions for energy loss rate due to polar optical phonons is given by

\[ P_{po} = 2^{1/2} m^{3/2} e^2 \left( \hbar \omega_0 \right)^2 \int_0^{\infty} \int_0^{\infty} \frac{F(q_z)^2}{Q^2} \left[ \left( N_Q (\omega_0) + 1 \right) \exp \left( -\frac{\hbar \omega_0}{k_B T_e} \right) - N_Q (\omega_0) \right] \frac{E_k}{\sqrt{E_k^2 - \gamma_s}} dE_k, \]

(3.2.3.1)

and momentum loss rate is shown to be,

\[
\left\langle \frac{dP_T}{dt} \right\rangle_{po} = -\frac{V_q m^{3/2} e^2}{2^{1/2} \pi^2 n_e \hbar^2 \epsilon' (k_B T_e)} \int_0^{\infty} \int_0^{\infty} \frac{F(q_z)^2 q^2}{Q^2} dq \, dq_z \\
\times \left\{ \int_{\tau_e}^{\infty} \left[ \left( 1 + \frac{\hbar \omega_0}{E_q} \right) f^0(E_k) \left[ 1 - f^0(E_k - \hbar \omega_0) \right] \right] \frac{E_k - \gamma_s}{\sqrt{E_k^2 - \gamma_s}} dE_k + \int_{\tau_e}^{\infty} \left[ \left( 1 - \frac{\hbar \omega_0}{E_q} \right) f^0(E_k) \left[ 1 - f^0(E_k + \hbar \omega_0) \right] \right] \frac{E_k - \gamma_s}{\sqrt{E_k^2 - \gamma_s}} dE_k \right\}.
\]

(3.2.3.2)
where $\omega_q = \omega_o$ is wave vector independent frequency of polar optical phonons, $\varepsilon' = (1/\varepsilon_\infty - 1/\varepsilon_o)^{-1}$ with $\varepsilon_o$ being the high frequency dielectric constant, $\gamma_a = (\hbar \omega_0 - E_q)^2/4E_q$ and $\gamma_c = (\hbar \omega_0 + E_q)^2/4E_q$. In the above equations $N_Q(\omega_0)$ is the hot phonon distribution function obtained by solving the phonon Boltzmann equation in the relaxation time approximation as follows.

The phonon-Boltzmann equation describing the rate of change of phonon occupancy, $N_Q(\omega_0)$, in steady state, is given by,

$$\left( \frac{dN_Q(\omega_0)}{dt} \right)_{\text{Coll}} + \left( \frac{dN_Q(\omega_0)}{dt} \right)_{\text{el-ph}} = \left( \frac{dN_Q(\omega_0)}{dt} \right)_{\text{Total}} = 0. \quad (3.2.3.3)$$

In relaxation time approximation,

$$\left( \frac{dN_Q(\omega_0)}{dt} \right)_{\text{Coll}} = - \frac{N_Q(\omega_0) - N_Q^0(\omega_0)}{\tau_p}, \quad (3.2.3.4)$$

where $N_Q^0(\omega_0)$ is equilibrium phonon distribution function given by Bose distribution and $\tau_p$ is the phonon relaxation time. Using Eqns. (3.2.1.2) and (3.2.3.4) in Eqn. (3.2.3.3), the solution of the phonon Boltzmann equation is obtained. It is given by,

$$N_Q(\omega_0) = \frac{N_Q^0 + \tau_p \Gamma(Q) \exp \left( -\frac{\hbar \omega_0}{k_b T_p} \right)}{1 + \tau_p \Gamma(Q) \left[ 1 - \exp \left( -\frac{\hbar \omega_0}{k_b T_p} \right) \right]}, \quad (3.2.3.5)$$

with

$$\Gamma(Q) = 2^{3/2} m^{1/2} e^2 (\hbar \omega_0) |F(q)|^2 \int_{\tau_c}^{\infty} \frac{f^0(E_k) [1 - f^0(E_k + \hbar \omega_0)] dE_k}{\sqrt{E_k - \gamma_a}} \left[ \right. \left. q \right], \quad (3.2.3.6)$$

where $<z> = 3/b$ is the mean well width weighted by the density distribution $\varphi(z)$. Ridley [3.18] obtains similar equation but for Maxwellian distribution.
3.2.4 Momentum loss rates due to other (non-phonon) scattering mechanisms

Besides the momentum loss rates due to phonons, we have considered momentum relaxation due to surface roughness, dislocations via coulomb field, remote impurity, roughness induced new mechanisms [3.20] such as misfit deformation potential and misfit piezoelectric potential scattering. We express the momentum loss rates due to all these mechanisms in terms of their respective average momentum relaxation times \( \langle \tau \rangle \) [3.18], i.e.

\[
\langle dP_e/dt \rangle_j = m V_e \langle \tau \rangle_j,
\]

where \( j \) stands for different scattering mechanisms (other than electron-phonon scattering) considered here. The relaxation time \( \tau_j \) due to a disorder \( j \) is expressed in terms of the autocorrelation function [3.20],

\[
\frac{1}{\tau_j(E_k)} = \frac{1}{\langle 2\pi \rangle} \frac{q^2}{h E_k} \int_0^{2\pi} dq \theta \frac{q^2}{(4k^2 - q^2)^{1/2}} \epsilon^2(q)
\]

where \( \langle U_j(q)^2 \rangle \) is the autocorrelation function of the disorder.

Relaxation times for different disorder (non-phonon) scattering mechanisms are given in section 2.3.1, of Chapter 2, namely Eqn. (2.3.1.1) for SR, Eqn. (2.3.1.2) for DC, Eqn. (2.3.1.3) for MDP and Eqn. (2.3.1.4) for MPE scattering, respectively.

Additionally we include momentum relaxation due to remote impurity as we are covering large \( T_e \) region. Relaxation time due to remote impurity scattering is given by [1.49]

\[
\frac{1}{\tau_{ri}(E_k)} = \frac{32\pi e^4 b^6 N_{hi}^{1/3}}{3\hbar e^2 E_k} \int_0^{2\pi} dq (4k^2 - q^2)^{1/2} \epsilon^2(q)
\]

The average of the momentum relaxation time of each mechanism is given by Eqn. (2.2.2).
The total \( \tau \), due to all the mechanisms combined, is calculated using Matthiessen's rule

\[
\frac{1}{\tau} = \frac{1}{\tau_{\text{SR}}} + \frac{1}{\tau_{\text{DE}}} + \frac{1}{\tau_{\text{MPE}}} + \frac{1}{\tau_{\text{MDP}}} + \frac{1}{\tau_{\text{RF}}}.
\] (3.2.4.4)

### 3.3 Results and discussion

We present the numerical results of the drift velocity, electron temperature, energy and momentum loss rates in a GaN/AlGaN heterojunction by solving the coupled momentum and energy balance equations obtained in the previous section. The material parameters used are: \( m = 0.22 \, m_0, \, m_z = 0.18 \, m_0, \, \hbar \omega_0 = 92 \, \text{meV}, \, \varepsilon = 5.47, \, \epsilon_f = 9.5, \, \rho = 6.15 \times 10^3 \, \text{kgm}^{-3}, \, v_t = 6.56 \times 10^3 \, \text{ms}^{-1}, \, v_i = 2.68 \times 10^3 \, \text{ms}^{-1}, \, \Xi_d = 8.5 \, \text{eV}, \, N_{\text{dis}} = 1 \times 10^{13} \, \text{m}^{-2}, \, N_{\text{ri}} = 1 \times 10^{23} \, \text{m}^{-3}, \, e_{15} = -0.3 \, \text{C/m}^2, \, e_{31} = -0.49 \, \text{C/m}^2 \) and \( e_{33} = 0.73 \, \text{C/m}^2 \).

In Fig. 1, numerical calculation are presented for drift velocity \( V_d \) vs electric field \( F \) for the sample of Vitusevich et al. [3.12] with \( n_s = 1.05 \times 10^{17} \, \text{m}^{-2} \) at \( T = 4.2 \, \text{K} \) along with their experimental data. We have chosen \( \Delta = 5 \, \text{Å}, \, \Lambda = 80 \, \text{Å} \) and \( \Xi_d = 8.5 \, \text{eV} \) as fitting parameters to obtain the observed low field mobility \( \mu = 1640 \, \text{cm}^2/\text{Vs} \) [3.12]. \( V_d \) is obtained by solving the coupled energy and momentum loss rates due to all the possible scattering mechanisms. In the low field region \( F < 10^5 \, \text{V/m} \) \( V_d \) increases linearly with the increasing \( F \) obeying Ohm's law, where the low temperature scattering mechanisms such as SR, MPE and MDP are dominant in momentum relaxation. A good agreement is obtained in the low field region with the experimental data. The mobility limiting dominant scattering mechanisms in this region, for the parameters chosen, are MPE, MDP and SR (as seen in Fig. 8). In the high field region \( F > 10^5 \, \text{V/m} \), emission of polar optical phonons takes place causing larger energy and momentum loss rate leading to almost saturation drift velocity. In
our calculation the phonon relaxation time $\tau_p$ is treated as a parameter. The experimental value for $V_d$ saturates at $F = 6 \times 10^7$ V/m. The calculated $V_d$ are shown for $\tau_p = 0$ ps (dashed curve) and $\tau_p = 0.025$ ps (solid curve). We observe that the curve with $\tau_p = 0.025$ ps gives marginally better agreement with the experimental data in the high field region.

**Figure 1.** Drift velocity $V_d$ vs electric field $F$ for the sample of Ref.3.12 with $n_s = 1.05 \times 10^7$ m$^{-2}$ at lattice temperature $T = 4.2$ K. Closed squares represents the experimental data [3.12]. Continuous and dashed curve are, due to all the mechanisms combined, with ($\tau_p = 0.025$ ps) and without ($\tau_p = 0$ ps) hot phonon effect, respectively. In the inset mobility $\mu$, due to all the mechanisms combined, is shown as function of $F$.

In the inset of Fig. 1, drift mobility is shown as a function of electric field for the same parameters with $\tau_p = 0.025$ ps. We observe that in the low field region ($F < 10^5$ V/m) mobility remains constant at a value of 1640 cm$^2$/Vs and decreases as the field increases. For example, for $F = 6 \times 10^6$ V/m, $\mu = 450$ cm$^2$/Vs.
Figure 2. Drift velocity $V_d$ vs electric field $F$ for the sample of Ref. 3.12 with $n_s = 1.05 \times 10^{17}$ m$^{-2}$ at $T = 4.2$ K, only due to polar optical phonon scattering with $\tau_p = 0.025$ ps. Continuous (dotted) curve is due to drifted Fermi-Dirac (Maxwellian) distribution function. In the inset dashed curve is due to only polar optical phonon scattering with the drifted Maxwellian distribution function and continuous curve is due to all the mechanisms combined with the drifted Fermi-Dirac distribution function. Both the curves are with $\tau_p = 0.025$ ps. Closed squares represents the experimental data [3.12].

To bring out the difference between the results due to drifted Maxwellian and Fermi-Dirac distribution functions drift velocity $V_d$ vs electric field $F$ is shown in Fig. 2 only for polar optical phonon scattering with hot phonon effect for $\tau_p = 0.025$ ps The use of drifted Maxwellian distribution function is found to underestimate $V_d$ by about three times at low fields ($F < 10^5$ V/m) and by nearly two times at high fields ($F \approx 10^6$ V/m). Inset shows $V_d$ vs $F$ using drifted F-D distribution in which dotted curve is due to only polar optical phonon scattering considered in
the momentum and energy balance equations and continuous curve is due to all the possible mechanisms. We observe the need for inclusion of all possible mechanisms to explain the experimental data over larger $T_e$ range, unlike the earlier calculations [3.10, 3.12]. In the present sample of GaN/AlGaN HJ concentration is very high of the order of $10^{17}$ m$^{-2}$, and the use of drifted F-D distribution is justifiable than the Maxwellian distribution.

![Figure 3](image_url)

**Figure 3.** Drift velocity $V_d$ vs electric field $F$ for the sample of Ref. 3.10 with $n_s = 1.5 \times 10^{17}$ m$^{-2}$ at $T = 77$ K. Closed squares represents the experimental data [3.10]. Curves are due to all the mechanisms combined. Dashed curve is for $\tau_p = 0$ ps, solid curve is for $\tau_p = 0.025$ ps and dotted curve is for $\tau_p = 5$ ps.

Additionally, in Fig. 3, we compare the calculated values of $V_d$ vs $F$ with the experimental data of Gokden et al. [3.10] for the sample with $n_s = 1.5 \times 10^{17}$ m$^{-2}$ at $T = 77$ K. The material parameters used in this calculation are same as those used in the
calculations of Fig. 1. Gokden et al. [3.10] have calculated $V_d$ vs $F$ taking account of non-drifting hot phonons with Maxwellian distribution for electrons and have not made any quantitative comparison between the experimental data and their calculated values. However, our calculations using drifted Fermi-Dirac distribution function give a good agreement with the experimental data. Comparing the curves with $\tau_p = 0.025$ ps (solid curve) and $\tau_p = 0$ ps (dashed curve), we again observe that, the agreement between experimental data and calculated values is improves marginally. However, we see that, for $\tau_p = 5$ ps, at $F = 1 \times 10^6$ V/m, $V_d$ is reduced by about three times.

We have shown electron temperature $T_e$ vs electric field $F$ for the sample of Vitusevich et al. [3.12] along with their experimental data in Fig. 4. A reasonably good agreement is obtained with the experimental data of Ref. [3.12] choosing $\tau_p = 0.025$ ps (solid curve). By comparing it with the dashed curve ($\tau_p = 0$ ps), we find that $T_e$ is enhanced due to hot phonon effect. For example, for $F = 2 \times 10^5$ V/m, $T_e = 390$ ($310$) K for $\tau_p = 0.025$ ($\tau_p = 0$) ps. The enhancement of $T_e$, due to hot phonon effect, is found to increase with the increasing $F$. This increase in $T_e$ may be attributed to the reduced power loss by the electrons due to hot phonons. In Fig. 6 of Ref. [3.12] extrapolation of $T_e$ to high electric field $10 \times 10^6$ V/m shows that $T_e$ does not exceed 650 K, where as from our calculation for the field about $4 \times 10^6$ V/m $T_e$ is 600 K. Matulionis et al [3.30] have carried out measurements of the noise temperature, where for $F = 9 \times 10^5$ V/m the noise temperature is about 1150 K, and from our calculations for a high field of $6.4 \times 10^6$ V/m the electron temperature is about 1000K.
Figure 4. Electron temperature $T_e$ vs electric field $F$ for the sample of Ref. 12 with $n_s = 1.05 \times 10^{17}$ m$^{-2}$ at $T=4.2$ K. Closed squares represents the experimental data [3.12]. Continuous and dashed curves are due to all the mechanisms combined with ($\tau_p = 0.025$ ps.) and without ($\tau_p = 0$ ps) hot phonon effect, respectively.

In Fig. 5 we compare the calculated $P$ with the experimental data [3.12] in the range $T_e = 200 - 550$ K. The $P$ calculations are carried out with $\tau_p = 0$ ps and $\tau_p = 0.025$ ps. We observe that the calculated energy loss rate with hot phonon effect with $\tau_p = 0.025$ ps (solid curve) gives a reasonably good agreement with the experimental data in the entire temperature range considered. The hot phonon effect is found to reduce the energy loss rate by about two times at $T_e = 600$ K. In contrast, it is to be noted that, in Ref. [3.12], electron-polar optical phonon scattering time $\tau$ is used as a fitting parameter in the standard expression $<dE/dt>_p = (\hbar \omega_0/\tau) \exp(-\hbar \omega_0/k_B T_e)$, to obtain a fit to the experimental data.
Figure 5. Energy loss rate $P$ vs electron temperature $T_e$ for the sample of Ref. 3.12 with $n_s = 1.05 \times 10^{17}$ m$^{-2}$ at $T = 4.2$ K. Closed squares represent the experimental data [3.12]. Curves are due to acoustic and polar optical phonons combined. Continuous and dashed curves are with ($t_p = 0.025$ ps) and without ($t_p = 0$ ps) hot phonon effect, respectively.

To see the relative importance of different phonon scattering mechanisms, $P$ is shown in the range $T_e = 10$ - 300 K in Fig. 6 at $T = 4.2$ K for $n_s = 1.05 \times 10^{17}$ m$^{-2}$ [3.12]. The combined and separate contributions due to acoustic phonons via deformation potential and piezoelectric interactions and polar optical phonons with hot phonon effect are shown. Observed behavior is similar to the one in GaAs/GaAlAs heterojunctions [3.21]. At very low temperature ($T_e < 15$ K) the contribution from piezoelectric interaction via transverse mode is dominant (curve c), attributing to the large piezoelectric coupling and smaller velocity of the transverse acoustic phonons in GaN/AlGaN HJ. In the range $T_e = 20$ - 80 K contribution from acoustic phonon via deformation potential coupling is dominant.
(curve a). For smaller $T_e$, $P$ due to acoustic phonons increases rapidly and approaches $\sim (T_e - T)$ dependence at higher $T_e$. $P$ due to polar optical phonons is calculated for $\tau_p = 5$ ps (curve e) and $\tau_p = 0$ ps (curve d). Hot phonon effect for $\tau_p = 5$ ps at $T_e = 300$ K is found to reduce the $P$ by nearly two orders of magnitude. We observe that $P$ due to emission of optical phonons is dominant for $T_e > 90$ K (150 K) for $\tau_p = 0$ ps (5 ps). It is to be noted that in GaAs/GaAlAs HJs the energy loss rate is dominated by polar optical phonons for $T_e > 40$ K [3.21]. This difference is attributed to the fact that polar optical phonon energy (92 meV) in GaN/AlGaN HJs is nearly 2.5 times that of GaAs/GaAlAs HJs (36 meV).

![Figure 6](image-url) Energy loss rate $P$ vs electron temperature $T_e$ with $n_e = 1.05 \times 10^{17}$ m$^{-2}$ at $T = 4.2$ K for different phonon scattering mechanisms. Curve a is due to acoustic deformation potential scattering, curve b(c) is due to piezoelectric scattering for longitudinal (transverse) mode, curve d(e) is due to polar optical phonons without (with $\tau_p = 0.025$ ps) hot phonon effect and curve f(g) is due to all the mechanisms combined without (with $\tau_p = 0.025$ ps) hot phonon effect.
Figure 7. Low temperature energy loss rate $P$ vs electron temperature $T_e$ with $n_s = 3.41 \times 10^{12}$ cm$^{-2}$ at $T = 0.28$ K for the sample A of Ref. [3.25].

In the following we analyze the low $T_e$ ($< 12$ K) data of $P$. We have numerically calculated $P$ vs $T_e$ with $\Sigma_d = 8.3$eV for Al$_{0.15}$Ga$_{0.85}$N/AlN/GaN and Al$_{0.83}$In$_{0.17}$N/AlN/GaN samples of Ref. [3.25]. Cheng et al. [3.25] have explained their data of $P$ vs $T_e$ by using the expressions in the BG regime in the dirty limit ($k l_e < 1$, $l_e$ is the electron mean free path). However, the carrier mobilities of both the samples are high and they are in the clean limit. Here we attempt to explain the experimental data $P$ vs $T_e$ of Cheng et al. [3.25] using the full form of Eqns. (3.2.2.1) and (3.2.2.4) which are in the clean limit.

Fig. 7 shows $P$ vs $T_e$ for sample A of Cheng et al [25] (0.862 K < $T_e$ < 10.4 K) with $n_e = 3.41 \times 10^{12}$ cm$^{-2}$. It is found that $P$ due to transverse acoustic phonons via piezoelectric coupling is dominant over the contribution from longitudinal acoustic phonons over the entire range of $T_e$. We observe that for about $T_e < 3$ K the BG
formulae in clean limit \((kl_e >> 1)\) [3.25, 3.27], for piezoelectric scattering, coincides with the results of Eqn. (3.2.2.4), noting that contribution from deformation potential coupling is negligible in this region. However, for about \(T_e > 3\) K, \(P\) is overestimated by BG regime Eqns. (3.2.2.5) and (3.2.2.6). For eg. \(P\) due to BG regime equations is greater than \(P\) due to detailed Eqn. (3.2.2.4) by three times at \(T_e = 6\) K and about an order of magnitude at 10 K. The calculated \(P\) due to Eqns. (3.2.2.1) and (3.2.2.4) improves agreement with the experimental data, although there is not very good agreement. Reason for this disagreement to this extent is not yet known.

In Fig 8. \(P\) vs \(T_e\) (1.8K\(<\)\(T_e\)<16.2K) is shown for the sample B of Ref. [3.25]. Again, as found in sample A, the calculations from Eqns. (3.2.2.1) and (3.2.2.4) are closer to the experimental data than the \(P\) values due to BG regime equations.

**Figure 8.** Low temperature energy loss rate \(P\) vs electron temperature \(T_e\) with \(n_e=10.26\times10^{12}\) cm\(^2\) at \(T = 1.5\) K for sample B of Ref. [3.25].
Figure 9. Energy loss rate $P$ due to polar optical phonons vs electron temperature $T_e$ for $n_s = 1.05 \times 10^{17}$ m$^{-2}$ at $T = 4.2$ K shown for different $\tau_p$'s. Continuous curve is for $\tau_p = 0$ ps, dashed curve is for $\tau_p = 0.025$ ps, dotted curve is for $\tau_p = 0.5$ ps, dash-dotted curve is for $\tau_p = 2.0$ ps and dash-double dotted curve is for $\tau_p = 5.0$ ps.

The energy loss rate due to polar optical phonons as a function of $T_e$ is shown in Fig. 9. The hot phonon effect on $P$ is shown for four different values of $\tau_p$. We observe that $P$ decreases significantly with increase of $\tau_p$. For instance at $T_e = 300$ K, $P$ reduces nearly three times for $\tau_p = 0.025$ ps (dashed curve), nearly by an order of magnitude for $\tau_p = 0.5$ ps (dotted curve), about fifty times for $\tau_p = 2$ ps (dash-dotted curve) and nearly by two orders of magnitude for $\tau_p = 5$ ps (dash-double dotted curve). Hot phonon effect increases with the increase of $T_e$ because of emission of large number of phonons.
Figure 10. Mobility $\mu$ vs electron temperature $T_e$ for $n_e = 1.05 \times 10^{17}$ m$^{-2}$ for different scattering mechanisms. Curve a is due to acoustic deformation potential scattering, curve b(c) is due to piezoelectric scattering for longitudinal (transverse) mode, curve d is due to polar optical phonons with hot phonon effect ($\tau_p = 0.025$ ps), curve e is due to surface roughness, curve f is due to misfit deformation potential, curve g is due to misfit piezoelectric potential, curve h due to dislocations via coulomb field, curve i is due to remote impurity scattering and curve j due to all the mechanisms combined with hot phonon effect ($\tau_p = 0.025$ ps).

The relative importance of scattering mechanisms limiting the mobility in high field region is shown in Fig.10 in the range $T_e = 10 - 500$ K by plotting $\mu$ vs $T_e$ for $n_e = 1.05 \times 10^{17}$ m$^{-2}$. The scattering due to MPE is dominating upto $T_e$ about 200 K, with close significant contribution from SR and MDP mechanisms. For $T_e > 200$ K, the polar optical phonon scattering (with hot phonon effect) is found to dominate momentum relaxation. As discussed earlier the parameters are chosen such that the observed low field mobility at $T_e = 4.2$ K is obtained.
In Fig. 11, we have shown the momentum loss rate \( <dP_F/dt> \) as a function of \( T_e \), due to all the scattering mechanisms combined, for the sample with \( n_s = 1.05 \times 10^{17} \text{ m}^{-2} \) at \( T = 4.2 \text{ K} \). It is calculated by writing the total \( <dP_F/dt> = V_d f_i(T_e) \), in which \( V_d \) is obtained again by solving the coupled equations of energy and momentum loss rates and \( f_i(T_e) \) is the coefficient of \( V_d \) due to combination of all the scattering mechanisms considered. \( <dP_F/dt> \) is presented for four different values of \( \tau_p \) and we find that hot phonon effect reduces \( <dP_F/dt> \) also. For example, at \( T_e = 300 \text{ K} \), for \( \tau_p = 0.025, 2 \) and \( 5 \) ps, the reduction is by a factor of about 1.5, 5.5 and 7.5, respectively. It is to be noted that hot phonon effect is found to reduce \( <dP_F/dt> \) of one-dimensional hot electron gas in quantum well wires also [3.31].

\[
\text{Figure 11. Momentum loss rate } <dP_F/dt> \text{ due to all the scattering mechanisms combined vs } \text{electron temperature } T_e \text{ for } n_s = 1.05 \times 10^{17} \text{ m}^{-2} \text{ at } T = 4.2 \text{ K shown for four different } \tau_p \text{'s. Continuous curve is for } \tau_p = 0 \text{ ps, dashed curve is for } \tau_p = 0.025 \text{ ps, dotted curve is for } \tau_p = 2.0 \text{ ps and dash-dotted curve is for } \tau_p = 5.0 \text{ ps.} \]
At this point we would like to note that, although hot phonon effect is significant on energy and momentum loss rates (Figs. 9 and 11) and electron temperature (Fig. 5), but the drift velocity is relatively less affected (Fig. 1).

The hot phonon distribution $N_q(\omega_0)$ is shown as function of $q/b$ in Fig. 12, at $T = 4.2$ K for $T_e = 100, 200$ and $300$ K with $\tau_p = 0.025$ ps taking $q_z/b = 0.01$. Inset in Fig. 10 is for $\tau_p = 5$ ps. $N_q(\omega_0)$ is larger for large $T_e$, because large number of phonons are emitted at higher $T_e$. This in turn reduces the power loss. Also, we find the phonon distribution to increase with the increasing $\tau_p$, at a given $T_e$, as phonons will decay slowly.

![Figure 12](image_url)

**Figure 12.** Hot polar optical phonon distribution $N_q(\omega_0)$ vs normalized in plane wave vector $q/b$ at $T = 4.2$ K for $\tau_p = 0.025$ ps. Curves are shown for three different electron temperatures $T_e = 100$ (dotted), 200 (dashed) and 300 (Solid) K. Inset is for $\tau_p = 5$ ps.
It is to be emphasized that, we have used all the possible mechanisms responsible for momentum and energy relaxations to explain the observed high field data with many parameters being involved. These parameters are set depending upon significance of these mechanisms in different temperature regions while explaining the data. Besides, we note that in GaAlAs/GaAs heterojunctions the use of Fang-Howard trial wave function, which is widely used in the literature giving analytical expressions for overlap integral, leads to underestimation of phonon emission rates [3.32]. We believe that such underestimation will result into varying the parameters involved in the electron-phonon scattering to explain the experimental data. To reduce further complexity, the Fang-Howard variational function is used in the present work.

In summary, the drift velocity, electron temperature, electron energy and momentum loss rates of a 2DEG are calculated in a wurtzite GaN/AlGaN HJ for electric fields up to $F = 10^7$ V/m employing energy and momentum balance technique, assuming drifted Fermi-Dirac (F-D) distribution function for electrons. The calculated drift velocity, energy loss rate and electron temperature are compared with the experimental data [3.10, 3.12] and a good agreement is obtained with due consideration of hot phonon effect. An attempt is also made to explain the recent low temperature experimental data of [3.25], in wurtzite GaN/AlGaN heterostructures, using the full form of expressions for electron energy loss rate $P$ due to acoustic phonons via deformation and piezoelectric potentials. We have demonstrated the inadequacies of using BG formula in the low temperature regime. The effect of using drifted F-D distribution, contrary to the drifted Maxwellian distribution function used in the earlier calculations, is brought out.
References:


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