CHAPTER 4

MULTI-ATTRIBUTE DECISION MAKING APPROACHES

4.1 INTRODUCTION

Since today’s globalized market requires cost effective quality products, the manufacturing systems and processes have become more complex. Hence, concurrent approaches are required to solve such complex problems. There are many multi-attribute decision making approaches like AHP, ANP and TOPSIS, which are used by many researchers to solve various problems in various fields. Applications of these tools are detailed in literature review. In this chapter, basic features, methodology, pros and cons of these approaches are briefed.

4.2 ANALYTICAL HIERARCHY PROCESS

Analytical Hierarchy Process is a Multi-Attribute Decision Making (MADM) approach that involves structuring multiple choice criteria into a hierarchy, assessing the relative importance of these criteria, comparing alternatives for each criterion, and determining an overall ranking of the alternatives. This approach developed by Thomas L. Saaty (1977, 1980), is one of the most widely used MADM techniques in decision-making field. Saaty described case applications ranging from the choice of a school for his son to the planning of transportation systems for the Sudan. It has been widely used in corporate planning, portfolio selection, and benefit/cost analysis.
4.2.1 Methodology

The AHP consists of the following steps:

1. To identify the goal and available alternatives
   In this step, the goal of the problem has to be defined and the alternative choices available are also to be identified from the literature and from the survey.

2. To define factors/attributes/criteria
   Factors affecting the goal are to be identified from the group discussion and from the literature.

3. To structure problem into hierarchical levels
   After identifying the criteria, the entire problem has to be structured according to the hierarchical order. Figure 4.1 shows the typical hierarchical structure in which the goal is at level 0. Level 1 shows the various factors affecting the goal and level 2 shows the available alternatives.

4. To make pair wise comparison between the criteria
   Once the problem has been decomposed and the hierarchy is constructed, prioritization procedure starts in order to determine priority vector of the factors with respect to the goal and priority vector of choice with respect to factors. To determine the priority vector, the pair wise comparison between the factors and the choices are taken into account according to Saaty’s rule. The pair wise judgment starts from the second level and finishes in the lowest level (choices/alternatives). In each level, the criteria are compared pair wise according to their levels of influence and based on the specified criteria in the higher level. In AHP, multiple pair wise comparisons are made based on a standardized comparison scale of nine levels as shown in table 4.1.
Figure 4.1 Hierarchical structure of the problem

Table 4.1 Nine-point intensity of importance scale and its description

<table>
<thead>
<tr>
<th>Definition</th>
<th>Intensity of importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equally important</td>
<td>1</td>
</tr>
<tr>
<td>Moderately more important</td>
<td>3</td>
</tr>
<tr>
<td>Strongly more important</td>
<td>5</td>
</tr>
<tr>
<td>Very strongly more important</td>
<td>7</td>
</tr>
<tr>
<td>Extremely more important</td>
<td>9</td>
</tr>
<tr>
<td>Intermediate values</td>
<td>2, 4, 6, 8</td>
</tr>
</tbody>
</table>

5. In this (third) step, the priority vector is obtained from normalized Eigen vector of the pair wise comparison matrix. To obtain the priority vector; the following calculations are carried out:
   a. Summing up each column of the comparison matrix.
   b. Dividing each element of the matrix with the sum of its column, this is the normalized relative weight. The sum of the entire normalized column is 1.
c. The normalized principal Eigen Vector can be obtained by averaging across the rows. Since it is normalized, the sum of all elements in priority vector is also 1.

6. In this step, consistency ratio (CR) is calculated. The consistency ratio is proposed by Saaty, which is a comparison between consistency index (CI) and random consistency index. To find that, it is required to find the principal Eigen value and consistency index. Principal Eigen value $\lambda_{\text{max}}$ is obtained from the summation of products between each element of Eigen Vector and sum of the columns of the normalized matrix. Values of CI and CR are calculated by the formulae given below:

$$\text{CI} = \frac{\lambda_{\text{max}} - n}{n-1} \quad (4.1)$$

$$\text{CR} = \frac{\text{CI}}{RI} \quad (4.2)$$

where, RI is the average random consistency index, which is proposed by Saaty (1980) and is given in the table 4.2.

<table>
<thead>
<tr>
<th>N</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
<td>0.9</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>

The final consistency ratio (CR) concludes whether the evaluations are sufficiently consistent, and is defined as the ratio of the CI and the random index. Saaty’s condition for consistency is CR < 0.1.

7. Overall composite weight of each alternative is computed in this step. It is just normalization of linear combination of multiplication between weight matrix of criteria and priority vector matrix.

$$\text{Over all composite matrix of alternative} = \text{Weight matrix of criteria } \times \text{ priority matrix} \quad (4.3)$$
The following generalized simple example explains the entire AHP. The hierarchical network for the example is given in figure 4.1. In the given example, A, B, and C are the factors and X, Y, and Z are the choices.

1. Priority vector matrix with respect to goal is arrived and is given below:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0.33</td>
<td>0.25</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>1</td>
<td>.5</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A</th>
<th>0.125</th>
<th>0.1</th>
<th>0.143</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.375</td>
<td>0.3</td>
<td>0.286</td>
</tr>
<tr>
<td>C</td>
<td>0.5</td>
<td>0.6</td>
<td>0.571</td>
</tr>
<tr>
<td>Sum</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Priority vector of the factors

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>EV</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.125</td>
<td>0.1</td>
<td>0.143</td>
<td>0.368</td>
<td>0.123</td>
</tr>
<tr>
<td>B</td>
<td>0.375</td>
<td>0.3</td>
<td>0.286</td>
<td>0.961</td>
<td>0.320</td>
</tr>
<tr>
<td>C</td>
<td>0.5</td>
<td>0.6</td>
<td>0.571</td>
<td>1.671</td>
<td>0.557</td>
</tr>
</tbody>
</table>

2. The same procedure is applied to determine the priority vector of choices with respect to the factors which are given below:

Priority vector of the choice with respect to factors A, B, C

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.123</td>
<td>0.087</td>
<td>0.265</td>
</tr>
<tr>
<td>B</td>
<td>0.320</td>
<td>0.274</td>
<td>0.655</td>
</tr>
<tr>
<td>C</td>
<td>0.557</td>
<td>0.639</td>
<td>0.080</td>
</tr>
</tbody>
</table>
3. To obtain the overall priority of the choices, it is required to multiply the priority choice vector of 3x3 with vector priorities of the factor with respect to goal (3x1)

\[
\begin{array}{ccc}
X & Y & Z \\
A & 0.123 & 0.087 & 0.265 \\
B & 0.320 & 0.274 & 0.655 \\
C & 0.557 & 0.639 & 0.080 \\
\end{array}
\times
\begin{array}{c}
PV \\
A & 0.123 \\
B & 0.320 \\
C & 0.557 \\
\end{array}
=
\begin{array}{c}
PV \\
X & 0.305 \\
Y & 0.491 \\
Z & 0.334 \\
\end{array}
\]

According to the result arrived, priority choice as Y, Z and X has the highest priority

**Strengths**

- The advantages of AHP over other multi-criteria methods are its flexibility, intuitive appeal to the decision makers and its ability to check inconsistencies. Generally, users find the pair wise comparison form of data input straightforward and convenient.

- Additionally, the AHP method has the distinct advantage that it decomposes a decision problem into its constituent parts and builds hierarchies of criteria. Here, the importance of each element (criterion) becomes clear.

- AHP helps to capture both subjective and objective evaluation measures. While providing a useful mechanism for checking the consistency of the evaluation measures and alternatives, AHP reduces bias in decision making.

- The AHP method supports group decision-making through consensus by calculating the geometric mean of the individual pair wise comparisons.
• AHP is uniquely positioned to help model situations of uncertainty and risk, since it is capable of deriving scales where measures ordinarily do not exist.

Weaknesses

• Many researchers have observed some cases in which ranking irregularities can occur when the AHP or some of its variants are used. This rank reversal is likely to occur e.g. when a copy or a near copy of an existing option is added to the set of alternatives that are being evaluated.

• The AHP–method can be considered as a complete aggregation method of the additive type. The problem with such aggregation is that compensation between good scores on some criteria and bad scores on other criteria can occur. Detailed, and often important, information can be lost by such aggregation.

• With AHP, the decision problem is decomposed into a number of subsystems, within which and between which a substantial number of pairwise comparisons need to be completed. This approach has the disadvantage that the number of pairwise comparisons to be made may become very large (n (n−1)/2), and thus becomes a lengthy task.

• Another important disadvantage of the AHP method is the artificial limitation of the use of the 9–point scale. Sometimes, the decision maker might find difficulty to compare. For example, whether one alternative is 6 or 7 times more important than the other.
4.3 ANALYTIC NETWORK PROCESS (ANP)

ANP is also introduced by Saaty (1980). Many problems cannot be structured hierarchically when the interaction of higher level element with lower level element and this interdependency must be taken into account. ANP provides solution even when the problem cannot be structured hierarchically and ANP takes the account of interaction between the cluster and between the elements of cluster. The figure 4.2 shows modeling of a problem through network model.

The network model shows the interdependency between the cluster and between the elements of the cluster.

4.3.1 Methodology

ANP is nothing but an up-gradation of the AHP, in which criteria are compared and linked to each other. The links are represented by bi-directional arrows. ANP also uses the same comparison scale, which is recommended by Satty (1980), when comparing two criteria. The methodology consists of the following steps:
Step 1: Pair wise comparison matrix between the criteria

Pair wise comparison matrix has been developed between the criteria with respect to goal as in AHP.

Step 2: Pairwise comparison matrices for interdependencies

In this step, pair wise comparisons are made to determine the interdependencies among the criteria. Consistency ratio for each pairwise comparison is also checked and it should be less than 10%( as in AHP).

Step 3: Development of super matrix for interdependencies

In this step, super matrix is developed for interdependencies that exist among the criteria. To develop the super matrix, the values are taken from the priority matrices. The number of pair wise comparison matrices determines the size of the super matrix. This super matrix is raised to a certain level to obtain the converged super matrix, so that all the values in the row converge to same value. The converged super matrix is also called as limited super matrix.

Step 4: Selection of the best alternative

Desirability indices define the best alternative. The values show the relative importance of the alternatives in supporting the goal. The desirability index, $D_i$, for the alternative $I$ is defined as

$$
D_i = \sum_{i=1}^{J} \sum_{k=1}^{K} P_j X A_{ij}^k X S_{ikj}
$$

(4.4)

where $P_j$ is the relative importance weight of the criteria.

$A_{ij}^k$ is the relative importance weight of criteria $k$ of dimension $j$ and superscript $i$ that show the interdependency relationships.

$S_{ikj}$ is the relative impact of the alternative $i$ on criteria $k$ of goal (principle) $j$

Note: In Chapter 7, all the steps of ANP are explained with illustration.
Table 4.3 Comparison between AHP and ANP

<table>
<thead>
<tr>
<th></th>
<th>AHP</th>
<th>ANP</th>
</tr>
</thead>
<tbody>
<tr>
<td>It requires decomposition of problems into hierarchical structure</td>
<td>Does not require any hierarchical decomposition</td>
<td></td>
</tr>
<tr>
<td>Interdependence and outer-dependence are not taken into account</td>
<td>Interdependence and outer-dependence are taken into account</td>
<td></td>
</tr>
<tr>
<td>Complex problems cannot be solved</td>
<td>Complex structured problems can be solved</td>
<td></td>
</tr>
<tr>
<td>Ratio scale priority vectors derived from the comparison matrices are synthesized linearly</td>
<td>Ratio scale priority vectors derived from the comparison matrices are not synthesized linearly as in AHP</td>
<td></td>
</tr>
<tr>
<td>No such supermatrix</td>
<td>Improved “supermatrix” technique is provided to synthesize</td>
<td></td>
</tr>
<tr>
<td>Computation is simple</td>
<td>Computation is not simple and requires computer assistance</td>
<td></td>
</tr>
</tbody>
</table>

Advantages

- ANP considers the interdependencies as well as outer dependencies.
- ANP provides better results than AHP
- It does not require any hierarchical structuring of the problem.

Limitations

- When the number of attributes is more, the manual calculation becomes difficult.
- Development of super matrix requires more skill.
- The entire process requires computer processing.
4.4 TOPSIS

It is one of the multi-criteria decision making approaches which considers qualitative and quantitative attributes. In this approach, beneficial and non-beneficial attributes are considered and in this method two artificial alternatives are hypothesized.

They are:

- Ideal alternative: the one which has the best level for all attributes considered.
- Negative ideal alternative: the one which has the worst attributes values.

TOPSIS selects the alternative that is closest to the ideal solution and farthest from the negative ideal solution.

TOPSIS uses weighted score method which can be written as

$$W_i = \sum_{j=1}^{i} A_j S_j$$  \hspace{1cm} (4.5)

where $S_i$ - weight of choice $i$ and $W_j$ - weight for criterion $j$

4.4.1 Methodology

Step 1: To determine the objective and identify the significant evaluation criteria. Identification of criteria is based on the literature and group discussion.

Step 2: To construct a decision matrix based on identified criteria. Each row of the decision matrix is allocated to one alternative and each column to one criterion. Therefore, an element, $m_{ij}$ of the decision
matrix indicates the performance of \(i^{th}\) alternative with respect to \(j^{th}\) criterion.

**Step 3:** To obtain the normalized decision matrix, \(D_{ij}\) using the following equation:

\[
D_{ij} = m_{ij} / \left( \sum_{j=1}^{M} [M_{ij}^{2}]^{0.5} \right) 
\]  

(4.6)

**Step 4:**

a) To decide on the relative importance of each criteria with respect to the objective by constructing a pair-wise comparison matrix using the scale of relative importance of AHP. In this matrix, the values for diagonal elements are always assigned as 1. The remaining values of relative importance in the pair-wise comparison matrix are decided from 1 to 9 depending on the requirements. Assuming there are \(M\) criteria, the pair-wise comparison of \(i^{th}\) criterion with respect to \(j^{th}\) one yields a square matrix \(A1\), where \(a_{ij}\) denotes the comparative importance of \(i^{th}\) criterion with respect to \(j^{th}\) one. In this matrix, \(a_{ij} = 1\) when \(i = j\) and \(a_{i}=1/a_{ji}\).

b) To find the relative normalized weight \((w_{j})\) of each criterion by (i) calculating the geometric mean of \(i^{th}\) row, and (ii) normalizing the geometric mean of rows in the comparison matrix. This can be represented using the following equations:

\[
GM_{i} = \left[ \prod_{j=1}^{M} a_{ij} \right]^{1/M} 
\]  

(4.7)

\[
W_{i} = GM_{i} / \sum_{j=1}^{M} GM_{j} 
\]  

(4.8)

c) To calculate the matrices, \(A_{3}\) and \(A_{4}\) such that \(A_{3} = A_{1}xA_{2}\) and \(A_{4} = A_{3}/A_{2}\), where

\(A_{2} = [w_{1}, w_{2}, ..., w_{M}]^{T}\).

d) To determine the maximum Eigen value \((\lambda_{max})\), which is the average of matrix \(A4\).
e) To calculate the consistency index as \( CI = (\lambda_{\text{max}} - M)/(M - 1) \). The smaller the value of \( CI \), which is the deviation from consistency.

f) To calculate the consistency ratio, \( CR = CI/RI \), where \( RI \) is the random index value obtained by different orders of the pair-wise comparison matrices. Usually, a \( CR \) of 0.1 or less is considered as acceptable, indicating the unbiased judgments made by the decision makers.

Step 5: To obtain the weighted normalized matrix, \( V_{ij} \).

\[
V_{ij} = W_j D_{ij}
\]  
(4.9)

Step 6: To obtain the ideal (best) and the negative ideal (worst) solutions using the following equations:

\[
V^+ = \left\{ \left( \frac{\max_i V_{ij} / j \in j} {\sum_{i} \min_{i'} V_{ij} / j \in j'} \right), i = 1,2,..N \right\}
\]  
(4.10)

\[
V^- = \left\{ \left( \frac{\min_i V_{ij} / j \in j} {\sum_{i} \max_{i'} V_{ij} / j \in j'} \right), i = 1,2,..N \right\}
\]  
(4.11)

where, \( J = (j = 1,2,...,M)/j \) is associated with beneficial attributes and \( J' = (j = 1,2,...,M)/j \) is associated with non-beneficial attributes.

Step 7: To obtain the separation measures. The separations of each alternative from the ideal and the negative ideal solutions are calculated by the corresponding Euclidean distances, as given in the following equations:

\[
S^+ = \left\{ \sum_{j=1}^{M} (V_{ij} - V^+)^2 \right\}, i = 1,2,...N
\]  
(4.12)

\[
S^- = \left\{ \sum_{j=1}^{M} (V_{ij} - V^-)^2 \right\}, i = 1,2,...N
\]  
(4.13)
Step 8: The relative closeness of a particular alternative to the ideal solution is computed as follows:

\[ P_i = \frac{S_i^-}{S_i^- + S_i^+} \]

(4.14)

Alternative whose relative closeness value less is selected as best one.