CHAPTER 6

AUTO AND CROSS POWER SPECTRUM ANALYSIS OF TEMPERATURE IN THE MIDDLE ATMOSPHERE AND SUNSPOT NUMBER
6.1. INTRODUCTION

The main objective of the study as made in Chapters 3, 4 and 5 of the thesis was to investigate the influence of a 11-year solar cycle on temperature variations in the middle atmosphere. It is a well-known fact that along with the long period temperature variations, many short period fluctuations like quasi-biennial, annual, semi-annual, etc., oscillations are present at various levels of the middle atmosphere. These short period oscillations were, however, suppressed in the earlier study by taking the annual mean values of data. The aim of the present study in this Chapter is to evaluate these short period oscillations present in the monthly mean temperature data at various altitudes of the middle atmosphere and sunspot number data for a time series of about 144 months. Power spectrum analysis is used for this purpose, which gives a measure of the distribution of variance in a time series over a continuous domain of all possible wavelengths, ranging from an infinite wavelength to the shortest wavelength that can be resolved by any scheme of harmonic analysis.

In the case of auto power spectrum, the estimated spectra of the two series, viz., temperature and sunspot number, give information only about the oscillations in the
two individual series. Similarities between the spectra of these two series such as peaks at similar frequencies may raise the possibility that the series are related. To investigate the possibilities to find a solar-related temperature variations in the middle atmosphere computation of the cross spectrum of the two series were made in this Chapter. This technique is, however, useful to find any short period oscillations in temperature at various levels in the middle atmosphere associated with the solar activity.

Thus by employing both the auto and cross power spectrum analyses of temperature and sunspot number, one can not only find their individual oscillations at various periodicities, but also to investigate the relationship between these two series at various frequency intervals.

6.2. DATA AND ANALYSIS

Monthly mean temperature data at 10 km altitude intervals of the middle atmosphere (20 to 80 km) obtained from the two meteorological rocket launching sites, viz., Thumba (8°N) and Volgograd (49°N) have been chosen for the present analysis. Both the temperature and sunspot number data were taken from 1971 January to 1982 December, i.e., for a period of total N = 144 months. The techniques used to compute the auto power spectrum and the cross spectrum are discussed in detail in Chapter 2 of this thesis. A detailed computer programme for the same is also given in Appendix-B.
The spectra for both sunspot number and temperature are based on a choice of maximum lag \( m = 14 \) months, which is a reasonably small percentage (i.e., about 10\% of the total number of data) of the total length of each record while at the same time large enough to provide a rather good spectral resolution. The evaluation of each of these spectra should begin with an examination of the serial co-variance values of the series that are obtained in the course of the calculation of the spectra. By estimating the serial correlation for the first three lags and their relationships, it has been found that the conditions of Markov's red noise are satisfied. Therefore, we assume that the series contain simple Markov persistence, and the tentative 'null' continuum \( S_k \) is calculated in each altitude levels of the middle atmosphere.

The confidence limits are derived by computing the degrees of freedom \( \gamma \) for each spectral estimate in these spectra, with \( N = 144 \) months and \( m = 14 \) months. By using the expression (2.14) \( \gamma \) has been calculated to have a value of 20.07. The 95 per cent points of the \( \chi^2/\gamma \) distribution for this value of \( \gamma \) are found to be 1.57 from the statistical table (Hald, 1952), which correspond to the probability equivalent to 95 percent significance point for one-tailed test of spectral peak. This represent the factor by which the ordinate value of the null continuum at each harmonic in the spectrum are multiplied to locate the 95\% confidence limits of the spectrum.
The cross spectra and the coherence factors between sunspot number and temperature are estimated as explained in Chapter 2. Both the cospectrum and the quadrature spectrum were considered for the study. The data for this analysis are same as those used for the estimation of auto power spectrum analysis. Student's t-test is applied to the estimated coherence factor to find their significance.

6.3 RESULTS AND DISCUSSION

6.3.1 Power spectrum of sunspot number

The power spectrum of monthly mean Zürich sunspot number for the period between 1971 January to 1982 December is shown in Fig. 6.1. The dotted curve in this figure indicates the 95% confidence limit. It can be seen that no peaks of this spectrum exceeded the 95% confidence limit of the chosen null continuum. Hence it may conclude that the sunspot number series is probably random, in the period considered for the present study.

6.3.2 Power spectra of temperatures in the middle atmosphere over Thumba

Fig. 6.2 shows the power spectra of temperatures at various altitude levels of the middle atmosphere over the tropical station, Thumba. The red noise continuum and the associated 95% confidence limits are illustrated as continuous and dotted curves, respectively, in each of the spectra.
FIG. 6.1 POWER SPECTRUM OF SUNSPOT NUMBER FOR THE PERIOD 1971 - 1982 [MAXIMUM LAG = 14 MONTHS]
FIG. 6.2 POWER SPECTRA OF TEMPERATURE AT VARIOUS HEIGHTS OVER THUMBA
[MAXIMUM LAG = 14 MONTHS]
Comparing the actual spectra with the null continuum and its confidence limits at each levels of the middle atmosphere the following results are obtained.

i) In the spectrum at 20 km, no peaks were found to exceed the 95% confidence limit of the chosen null continuum. However, the second harmonic, i.e., the wavelength of about 14 months was found to be dominant at this height.

ii) Both the spectra at 30 and 40 km heights showed that the fifth harmonic of the estimated spectra exceeds the 95% confidence limit. The fifth harmonic represents a band of wavelength slightly less than 6 months, showed that the semi-annual oscillation is stronger in the middle stratosphere over the tropics. Except this particular spectral band, no other part of the spectra were to exceeded the 95% confidence limit.

iii) At 50 and 60 km, no significant oscillations were found to exist. Eventhough the zeroth and the first harmonics of the spectral estimate at 60 km showed higher peaks, they could not exceed the 95% confidence limit of the chosen null continuum.

iv) The zeroth and the first harmonics of the spectrum at 70 km were found to be significant. They represent the long period and 28 month period oscillations in temperature.
This indicates that the long period and quasi-biennial oscillations in temperature are present in the tropical middle mesosphere.

v) At 80 km, no part of the spectrum found to exceed the 95% confidence limit.

In general, it can be seen that the semi-annual oscillations in temperature were present in the middle stratosphere, while long period as well as quasi-biennial temperature oscillations were existed in the tropical middle mesosphere.

6.3.3 Power spectra of temperatures in the middle atmosphere over Volgograd

The power spectra of temperatures at different levels in the middle atmosphere over the mid-latitude station, Volgograd, is illustrated in Fig. 6.3. The results obtained by a comparative study of the actual spectrum, with the corresponding null continuum and its 95% confidence limit are described as follows.

i) At all levels of the stratosphere, i.e., from 20 to 50 km, the second and third harmonics of the spectra were found to over-shoot the 95% confidence limit. These two harmonics represent the wavelength band of the 14 month and 9 month periodicities, respectively, and showed the strong existence of annual oscillation in temperature. These oscillations were not at all significant in the tropical
FIG. 6.3 POWER SPECTRA OF TEMPERATURE AT VARIOUS HEIGHTS OVER VOLGOGRAD [MAXIMUM LAG = 14 MONTHS]
stratosphere where the semi-annual oscillations were significant (see Fig. 6.2). Except the second and third harmonics no other part of the spectra in the entire stratospheric levels of the mid-latitude region were significant.

ii) The peaks of the spectrum at the fifth and the seventh harmonics were found to exceed the 95% confidence limit at 60 km. This, however, shows a unique feature as compared with other levels of the atmosphere. The estimate for the fifth and the seventh harmonics represents a band of wavelengths 5.6 and 4 months, respectively, and no doubt should be interpreted as weak manifestation of the bi-annual and tri-annual oscillations in the mid-latitude temperature at 60 km altitude.

iii) No part of the spectrum at 70 km were found to exceed the 95% confidence limit. However, the second and third harmonics of the spectrum showed higher peaks, but none not exceeded the significant limit.

iv) At 80 km, the second and third harmonics of the spectrum exceeded the confidence limit, and thus indicating the presence of annual oscillations in temperature in the upper mesosphere over Volgograd.

The conclusion may be

In brief, it can conclude that except in the lower mesosphere, all other altitude levels of the mid-latitude
Middle atmosphere were strongly influenced by the annual oscillations in temperature. At 60 km, the annual oscillations became insignificant, but the bi-annual and tri-annual oscillations became significant. It is also interesting to note that the quasi-biennial oscillation of temperature was totally absent in the mid-latitude middle mesosphere. The longer period oscillations were also not detected in the mid-latitude region.

6.3.4 Cross spectra of sunspot number and temperature at Thumba

Fig. 6.4 represents the estimated cross spectra of sunspot number and temperatures at various layers of the middle atmosphere over Thumba. The real part of the cross spectrum, which measures the extent to which these oscillations have the same phase in the two series (or with opposite sign, i.e., with a phase shift of a half cycle), called as the cospectrum is shown in continuous lines and the imaginary part of the spectrum, which measures the extent to which there are oscillations with a phase difference of a quarter cycle (in either direction), called as the quadrature spectrum, is shown as discontinuous lines in Fig. 6.4. Following results are drawn from this analysis.

i) Both the real and imaginary parts of the cross spectra were negative in the middle and lower stratosphere.
FIG. 6.4 CROSS SPECTRA BETWEEN SUNSPOT NUMBER AND TEMPERATURE AT THUMBA
ii) Large variations in the magnitude of both the cospectrum and the quadrature spectrum were found in most of the harmonics of the cross spectrum at 40 km. This is a speciality of the spectrum at 40 km height to those of the spectra at other levels. At 40 km level, the quadrature spectrum have a higher magnitude than that of the cospectrum.

iii) In mesospheric levels, the directions of the cospectrum and the quadrature spectrum became positive and have a higher magnitude in the zeroth and first harmonics. Except these two peaks, no other part of the spectrum have a significant change.

The computed coherence factors between the sunspot number and temperature at the middle atmosphere is illustrated in Table 6.1, which enables one to find the significance of the estimated cross spectrum. It can be seen that the significant values of coherence factor varied with time and height in the stratosphere. In mesospheric levels (especially in the middle mesosphere), highly significant coherent factors in its zeroth and first harmonics of the spectrum have been observed. This certainly explains the existence of a definite solar related temperature variations, of long period as well as quasi-biennial period, in the middle mesosphere over the tropical region.

6.3.5 Cross spectra of sunspot number and temperature at Volgograd

The cross spectra of sunspot number and temperature at various height levels of the mid-latitude region are
Table 6.1
Estimated coherence factors between sunspot number and temperatures at various levels in the middle atmosphere over Thumba and Volgograd

<table>
<thead>
<tr>
<th>Harmonics (Cycles/month)</th>
<th>Thumba</th>
<th>Volgograd</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20 km 30 km 40 km 50 km 60 km 70 km 80 km</td>
<td>20 km 30 km 40 km 50 km 60 km 70 km 80 km</td>
</tr>
<tr>
<td>0</td>
<td>0.05 0.47* 0.00 0.08 0.85* 0.63* 0.32* 0.65* 0.24 0.73* 0.01 0.81* 0.83* 0.03</td>
<td>0.39* 0.35* 0.38* 0.53* 0.64* 0.20 0.42*</td>
</tr>
<tr>
<td>1</td>
<td>0.12 0.23 0.02 0.11 0.69* 0.50* 0.25 0.11 0.05 0.18 0.03 0.55* 0.44* 0.01</td>
<td>0.15 0.13 0.38* 0.24 0.15</td>
</tr>
<tr>
<td>2</td>
<td>0.00 0.22 0.05 0.67* 0.29* 0.23 0.05 0.39* 0.35* 0.38* 0.53* 0.64* 0.20 0.42*</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.08 0.06 0.32* 0.22 0.06 0.07 0.12 0.24 0.27 0.24 0.22 0.02 0.21 0.25*</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.21 0.22 0.17 0.17 0.02 0.01 0.08 0.08 0.15 0.09 0.13 0.38* 0.24 0.15</td>
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</tr>
<tr>
<td>5</td>
<td>0.31* 0.14 0.07 0.12 0.01 0.10 0.18 0.00 0.12 0.02 0.21 0.13 0.04 0.16</td>
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</tr>
<tr>
<td>6</td>
<td>0.09 0.07 0.03 0.10 0.07 0.10 0.09 0.02 0.02 0.04 0.06 0.05 0.01 0.11</td>
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</tr>
<tr>
<td>7</td>
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<tr>
<td>8</td>
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<tr>
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<tr>
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<tr>
<td>13</td>
<td>0.05 0.06 0.04 0.07 0.01 0.04 0.09 0.03 0.02 0.04 0.03 0.01 0.04 0.09</td>
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<tr>
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<td></td>
</tr>
</tbody>
</table>

* Highly significant
+ Marginally significant
illustrated in Fig. 6.5. The results seen from the Fig. 6.5 are as follows.

i) At all levels in the stratosphere, the cospectrum is negative in direction while the quadrature spectrum is positive with peaks at the third and fourth harmonics.

ii) Near the stratopause, i.e., at 50 km, the cospectrum is negative and decreasing uniformly its magnitude with increasing frequency. Similarly, the quadrature spectrum which is highly positive initially were also found to be decreasing its magnitude. After the seventh harmonics, the cospectrum became positive, while the quadrature spectrum became negative.

iii) Both the real and imaginary parts of the spectrum showed positive values in the middle mesosphere, with peaks at the zeroth and first harmonics.

iv) Near the mesopause, i.e., at 80 km, the nature of the cospectrum and the quadrature spectrum is just opposite to what observed at 50 km. Here at 80 km, the cospectrum is positive and decreasing gradually, while the quadrature spectrum is negative and decreasing its magnitude with increasing frequency.

Both the real and imaginary parts of the cross spectrum showed positive and definite higher peaks in its first two
FIG. 6.5 CROSS SPECTRA BETWEEN SUNSPOT NUMBER TEMPERATURE AT VOLGOGRAD

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FIG. 6.5 CROSS SPECTRA BETWEEN SUNSPOT NUMBER TEMPERATURE AT VOLGOGRAD
harmonics in the middle mesosphere. This shows that the solar-related temperature variations have a significant effect at these two periodicities in the middle mesosphere over the mid-latitude site.

The coherence factor showed (Table 6.1) significant values in the entire levels of the middle atmosphere at its second harmonics (i.e., the 14 month periodicity). In some of the stratospheric levels, the zeroth harmonics were also found to have significant coherence factors. In the middle mesosphere, the first three harmonics of the cross spectra showed higher significant coherence factors. This, however, explains the presence of long term effect of solar activity on temperatures in the middle mesosphere.