1. **INTRODUCTION**

Ecology is a branch of biology that deals with the distribution and abundance of living organisms. A group of individuals of the same species occupying a particular space is defined as a population. Members of a population rely on the same resources, are influenced by similar environmental factors, and have a high likelihood of interacting with and breeding with one another. On the other hand, a community is an assemblage of two or more populations of different species occupying the same geographical area. The interaction of a community of organisms with their physical environment forms an ecosystem.

An ecosystem model is an abstract, usually mathematical, representation of an ecological system, which is a framework for formalizing our understanding of a biological system. Ecological models usually take the form of sets of simultaneous differential equations as the most popular tool to solve dynamical problems. Once it has been established that its solutions are in good agreement with experimental results, a mathematical model can become a vehicle for biologists for future biological experiments.

Coral reefs are among the most biologically diverse ecosystems, critical to the survival of tropical marine ecosystems and hence of local people. Coral reefs develop in shallow, warm water, usually near land and mostly in the tropics. [1]. The sun is the initial source of energy for this ecosystem. Coral polyps do not photosynthesize, but have a symbiotic relationship with single-celled organisms called Zooxanthellae living inside coral polyp tissues [2]. When the symbiotic Zooxanthellae are released from coral polyps, the coral loses its pigment, leading to a bleached or completely white appearance. Coral reefs are quite vulnerable to disturbances, both natural and anthropogenic. Out of many possible reasons of coral bleaching, we focus our attention on the roles of macro-algae, invasive lionfish (*Pterois volitans*), overfish-
ing of herbivores and anthropogenic chemicals from agricultural run-off in coral bleaching [3, 4].

Role of macroalgae on coral bleaching

Algae are well known to provide structural framework, physical consolidation, food, contribution to reef sediment, primary productivity, nitrogen-fixation and high species diversity in coral reefs. Despite their recognized roles in the natural function of coral reefs, macro-algal cover on reefs is commonly and increasingly related to coral reef decline [5]. As observed by researchers, reefs have a tendency to exist in alternate coral or algae-dominated states [6]. The shifts are driven partly by the competition for light and space between corals and algae. Macro-algae may compete with corals by basal encroachment, shading or abrasion and allelopathic chemical defenses [7, 8]. Faster growing macro-algae often overgrow corals, depriving them of essential sunlight and causing their decline. Also, hydrophobic allelochemicals released by some algae species disrupts endosymbiosis between Zooxanthellae and coral polyps, resulting bleaching and death of corals [9].

Role of herbivore-harvesting on coral bleaching

In coral reefs, herbivores play a critical role in regulating the competitive relationship between macroalgae and corals. Coral reefs throughout the world have suffered substantial declines in coral cover and species diversity due to loss of herbivorous reef-fish [10]. The grazing of macro-algae by herbivores contributes to the resilience of the coral-dominated reef. There is substantial evidence that herbivore removal by harvesting has resulted in increased algal growth on coral reefs at the expense of living coral cover. This resulted in a phase shift from coral-dominated reef to algal reef with proliferation of macro-algae, resulting coral bleaching [6].

Role of anthropogenic chemicals on coral bleaching

Marine pollutants that originate from agricultural run-off have potential toxic effect on reef-building corals. Among the anthropogenic chemicals, insecticide from agricultural run-off causes the most serious problems because they are designed
1. Introduction

Specifically to kill organisms. Species, under the presence of a toxin can exhibit trophic-mediated positive or negative effects on other species in the community, without any direct effect on itself. For instance, exposure to a pesticide may have no detectable direct effects on a particular organism, but may directly reduce the density or alter the behavior of its prey, competitors, or predators, leading to profound detrimental indirect effects of its community [11]. Exposure to a toxin can reduce the density or alter the behavior of organisms in coral reef, leading to profound detrimental effects on coral reef ecosystem [12].

Role of an invasive predator on coral bleaching

The invasion and establishment of *Pterois volitans*, which has been most heavily sighted in the Atlantic pose a major new threat to south Atlantic and Caribbean coral reefs [13]. *Pterois volitans* have the potential to disrupt coral reef community population structure and dynamics. Not only are they voracious predators that

![Fig. 1.1: Distribution of *Pterois volitans* (in red) in the Atlantic Ocean](image)

out-compete many other species for food resources, but they also have few known
natural predators of their own. It has been observed that in the presence of predatory *Pterois volitans*, there is a rapid loss of herbivorous fish in coral reef ecosystems [14, 15]. Aside from the rapid and immediate mortality of marine life, the loss of herbivorous fish also sets the stage for macro-algae to potentially overwhelm the coral reefs and disrupt the delicate ecological balance in which they exist leading to coral bleaching [17, 18]. If left unchecked, researchers contend, *Pterois Volitans* could spell disaster for the coral reef ecosystems [19, 20]. Studies have shown when *Parrotfish* are prevented from feeding along the area of the reef, the coral is ‘smothered’ to death by the growth of algal mats. The grazing activity of *Parrotfish* keeps this in check [21, 22]. According to NOAA (National Oceanic and Atmospheric Administration), commercial harvesting of adult *Pterois volitans* is required to reduce the growth of *Pterois volitans* to mitigate their impact on coral reef ecosystems [23].

Nevertheless, despite the attention towards this issue, the mechanism for the occurrence of coral bleaching and its possible control strategy are not yet well established and required special attention. Hence mathematical modelling is necessary. The dissertation is focused on mathematical modeling of coral reef ecosystem to prevent its degradation.

In Chapter II, an analysis is made using a four-dimensional mathematical model where there is a constant supply of algae with Parrotfish at the first trophic level and *Pterois volitans* at the remaining two trophic levels. The adult predator, *Pterois volitans* at the third trophic level, exhibits a distinct cannibalistic attitude to its juveniles at the second trophic level. We examined how the interactions of *Parrot-
fish, juvenile and adult *Pterois Volitans* determine predator prey dynamics. Our analysis leads to different thresholds in terms of the model parameters acting as conditions under which the species associated with the system cannot thrive even in the absence of predation. Local stability of the system is obtained when one or more of the predators go extinct. Under appropriate circumstances a positive rest point of the system is obtained. Computer simulations have been carried out to illustrate different analytical findings. Our results indicate that if the concentration of algae is very low, all the species in the three trophic levels would go extinct. Increase of the concentration of algae initially increases the concentration of all the species, but at a high concentration of algae, juvenile *Pterois volitans* goes extinct.

Chapter III deals with the problem of lethal effects of insecticides from agricultural run-off on coral-reef ecosystem. Insecticides at realistic concentrations have a direct lethal impact on zooplankton, but no direct impact on phytoplankton. However, bioconcentration of insecticides by phytoplankton increases the persistence of insecticides in aquatic ecosystems and cause negative effects at higher trophic levels. We study a food chain with a constant supply of input limiting nutrient and an insecticide, carbaryl. At the first trophic level phytoplankton, which is resistant to carbaryl, is growing on the supplied input nutrient. Zooplankton, *Daphnia* is introduced in the system at the second trophic level which are growing on phytoplankton. Carbaryl is lethal to *Daphnia*. On analyzing our model we observe that with low concentration of input nutrient, phytoplankton and *Daphnia* will coexist in the system if the input carbaryl concentration is lowered. Also, high concentration of input nutrient leads to an oscillatory coexistence of all the species. In this case by increasing the washout rate of *Daphnia*, all the species coexist. It is observed that higher amount of carbaryl leads to the extinction of *Daphnia* without any ill effect on phytoplankton. In such a case, decrease of dilution rates of both phytoplankton and *Daphnia* restores the system to a stable coexistence of all the species.

In Chapter IV, we have proposed and analyzed a model consisting of dissolved limiting nutrients, toxin producing phytoplankton and herbivorous fish where the growth of herbivorous fish reduces due to the effect of ciguatoxin (CTX) released by phytoplankton species. Our analysis leads to different thresholds which are ex-
pressible in terms of model parameters and determine the existence and stability of various states of the system. Conditions for co-existence or extinction of populations are derived. On analyzing the model it is observed that lowering the concentration of nutrient leads to the extinction of all the species. Also, the system cannot tolerate high concentration of limiting nutrient. In this case all the organisms in the system become coexistent if the death rate of herbivorous fish is reduced. Moreover, higher allocation of the consumption of phytoplankton on producing CTX gradually increases CTX in the system as long as the growth of phytoplankton is not hampered. Increasing the amount of allocation beyond a certain threshold leads to the extinction of all the organisms in the system.

In Chapter V we study allelopathic effects of plasmid-bearing and plasmid-free bacteria of reef-ecosystem in a selective media. We consider a model of competition between plasmid-bearing and plasmid-free bacteria for two complementary nutrients in a chemostat. We assume that the plasmid-bearing bacterium produces an allelopathic agent at the cost of its reproductive abilities which is lethal to plasmid-free bacteria. Our analysis leads to different thresholds in terms of the model parameters acting as conditions under which the organisms associated with the system cannot thrive even in the absence of competition. Local stability of the system is obtained in the absence of one or both the organisms. Also, global stability of the system is obtained in the presence of both the organisms. On analyzing the model it is observed that if the concentrations of nutrients are very low, both the bacteria in the chemostat would go to extinction. Increase of the dilution rate leads to the extinction of the plasmid-bearing bacteria in the chemostat. Furthermore, if the plasmid-bearing bacteria stops producing a toxin, the plasmid-free bacteria will take over the culture leading to the extinction of plasmid-bearing bacteria.

In Chapter VI, we investigate coral-macroalgal phase shift in presence of harvesting of herbivorous reef-fish by means of a continuous time model in a food chain. We study a model with two interacting organisms, macro-algae and corals. Herbivorous Parrotfish is growing in the system feeding mostly on macro-algae and a small amount of corals. Parrotfish are harvested with a non-constant harvesting policy. Mathematical results like boundedness, stability of equilibria, permanence
of the system have been established. Extensive numerical studies are carried out using experimental data of macroalgae and corals [77] to illustrate theoretically observed results. Study reveals that harvesting of herbivores lead to phase shifts from coral-dominated to macro-algae dominated regime. Also, it is observed that in absence of harvesting, corals can recover even after extensive mortality. This shows that herbivores play a critical role in facilitating recovery of coral reefs.

In Chapter VII, a three dimensional stage structured predator-prey nonautonomous model is proposed and analyzed to study the effect of predation and cannibalism of adult \textit{Pterois Volitans} with its non-constant harvesting. We assume that the reproduction of adult \textit{Pterois Volitans} after predating Parrotfish is not instantaneous but will be mediated by some discrete time delay required for egg deposition, embryo development, and hatching. Conditions for local asymptotic stability of steady states are derived. The length of the delay preserving the stability is also estimated. Moreover, it is shown that the system undergoes a Hopf bifurcation when the time lag due to reproduction crosses certain critical value. On analyzing the model it is observed the system exhibits dynamic instability due to greater reproduction time delay. Increase of the maximal rate of harvesting of adult \textit{Pterois Volitans} helps the system to attain its stability. High levels of maximal uptake rate of adult \textit{Pterois Volitans} on Parrotfish induce oscillation around the positive leading to dynamic instability. This represents the phenomenon of ecological imbalance due to the presence of the invasive \textit{Pterois Volitans} in coral reef ecosystem. This dynamic instability can be controlled by increasing the maximal rate of harvesting of adult \textit{Pterois Volitans}.

In chapter VIII, we have presented an outline of future scope for extension and development of the present work.