CONVENTIONAL OPTIMIZING METHODS

There are number of methods available for optimization. However, the transformer equivalent optimization problem has only two variables, one objective and one equality constraint. The following commonly available methods are applied for the transformer equivalent optimization problem.

LAGRANGE MULTIPLIER METHOD

(Singiresu 1996) Lagrange found a method for optimizing a constrained problem. The objective function and constraint equation for the transformer is already formed. The number of variables to be optimized for optimum design of a required objective is only two. The problem in general may be stated as:

\[
\text{Find } X = (x_1, x_2) \tag{A3.1}
\]

which minimizes \( f(X) \) subject to

\[
g(x_1, x_2) = 0 \tag{A3.2}
\]

an equality constraint. For this problem, the necessary condition for the existence of an extreme point at \( X = X^* \) are

\[
\left. \left( \frac{\partial f}{\partial x_1} - \frac{\partial f}{\partial x_2} \frac{\partial g}{\partial x_2} \right) \right|_{x_1^*, x_2^*} = 0 \tag{A3.3}
\]
By defining a quantity \( \lambda \), called the Lagrange multiplier, as

\[
\lambda = \left( \frac{\partial f}{\partial x_2} \right)_{x_1^*, x_2^*} \quad \text{(A3.4)}
\]

Equation (A3.13) can be either expressed as

\[
\left( \frac{\partial f}{\partial x_1} + \lambda \frac{\partial g}{\partial x_1} \right)_{x_1^*, x_2^*} = 0 \quad \text{(A3.5)}
\]

or

\[
\left( \frac{\partial f}{\partial x_2} + \lambda \frac{\partial g}{\partial x_2} \right)_{x_1^*, x_2^*} = 0 \quad \text{(A3.6)}
\]

In addition, the constraint equation has to be satisfied at the extreme point, that is,

\[
g(x_1, x_2)_{x_1^*, x_2^*} = 0 \quad \text{(A3.7)}
\]

Thus Equations (A3.15), (A3.16) and (A3.17) represent the necessary conditions for the point or variable \((x_1^*, x_2^*)\) to be an extreme point.

The necessary conditions given by above equations are more commonly generated by constructing a function \(L\), known as the Lagrange function, as

\[
L(x_1, x_2, \lambda) = f(x_1, x_2) + \lambda g(x_1, x_2) \quad \text{(A3.8)}
\]
By treating $L$ as a function of three variables, $x_1, x_2$ and $\lambda$, the necessary conditions for its extreme are given by

$$\frac{\partial L}{\partial x_1}(x_1, x_2, \lambda) = \frac{\partial f}{\partial x_1}(x_1, x_2) + \lambda \frac{\partial g}{\partial x_1}(x_1, x_2) = 0 \quad (A3.9)$$

$$\frac{\partial L}{\partial x_2}(x_1, x_2, \lambda) = \frac{\partial f}{\partial x_2}(x_1, x_2) + \lambda \frac{\partial g}{\partial x_2}(x_1, x_2) = 0 \quad (A3.10)$$

and

$$\frac{\partial L}{\partial \lambda}(x_1, x_2, \lambda) = g(x_1, x_2) = 0 \quad (A3.11)$$

**RANDOM JUMPING METHOD**

This method is an iterative method used for optimization problems. This method is used to solve an unconstrained minimization problem. It is true that rarely a practical design problem would be unconstrained; still, it is important for the following reasons:

1. The constraints do not have significant influence in certain design problems.

2. Some of the powerful and robust methods of solving constrained minimization problems require the use of unconstrained minimization techniques.

3. The study of unconstrained minimization techniques provides the basic understanding necessary for the study of constrained minimization methods.

4. The unconstrained minimization methods can be used to solve certain complex engineering analysis problems.

All the unconstrained minimization methods are iterative in nature and hence they start from an initial trial solution and proceed toward the
minimum point in a sequential manner. The general iterative scheme is shown in Figure A3.1 as a flow diagram.

It is important to note that all the unconstrained minimization methods (1) require an initial point \( X_i \) to start the iterative procedure, and (2) differ from one another only in the method of generating the new point \( X_{i+1} \) (from \( X_i \)) and in testing the point \( X_{i+1} \) for optimality.

![Flow diagram for general iterative scheme of optimization](image)

**Figure A3.1 General iterative scheme of optimization**

The constrained problem of transformer is actually made into an unconstrained problem by suitably assigning range of values for the inner
variables. These ranges have lower and upper bound respectively. Further Random jumping method is used as a comparator in this work to assess optimum values of the variables.

Let the bounds be \( l_i \) and \( u_i \) for each design variables, \( x_i, i=1,2,...,n \), for generating the random values of \( x_i \):

\[
l_i \leq x_i \leq u_i, \ i=1,2,...,n
\]  

(A3.9)

In the random jumping method, we generate sets of \( n \) random numbers, \((R_1, R_2, ..., R_n)\) that are uniformly distributed between 0 and 1.

Each set of these numbers, is used to find a point, \( X \), inside the hypercube defined by Equation (A3.10) as

\[
X = \begin{bmatrix}
    x_1 \\
    x_2 \\
    \vdots \\
    x_n
\end{bmatrix} = \begin{bmatrix}
    l_1 + R_1 (u_1 - l_1) \\
    l_2 + R_2 (u_2 - l_2) \\
    \vdots \\
    l_n + R_n (u_n - l_n)
\end{bmatrix}
\]  

(A3.12)

and the value of the function is evaluated at this point \( X \). By generating a large number of random points \( X \) and evaluating the value of the objective function at each of these points, the smallest value of \( f(X) \) is obtained, which is the desired minimum point or the optimum point. The interesting feature of this method in this work is that a comparison is made actually with the known value of the constraint equation and the optimum values are assessed from this method.

This method is an iterative method. This method is also used for the transformer problem and results are presented and compared.