CHAPTER 1

Introduction

1.1 The Standard Model of Cosmology: Its success

Standard model of cosmology (see for example [1, 2, 3]) is based on cosmological principle, which
states that the universe is spatially homogeneous and isotropic on large scales and it may be safely
assumed on scales $> 100 \text{Mpc}$. Such an assumption is described by the Robertson-Walker (RW)
metric which is given by

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right], \quad (1.1.1)$$

where $t$ is the cosmic time, $r, \theta, \phi$ are spatial co-ordinates, $a(t)$ is the scale factor of the universe and
$k = 0, \pm 1$ is the three space curvature parameter, corresponding to flat, closed and open universe
respectively. Now assuming that the universe is filled with perfect fluid, the energy-momentum
tensor is given by

$$T_{\mu\nu} = (p + \rho)v_\mu v_\nu + pg_{\mu\nu}, \quad (1.1.2)$$

where $v_\mu$ is the 4-velocity vector, $g_{\mu\nu}$ is the metric tensor, $p$ is the isotropic pressure and $\rho$
is the energy density of the perfect fluid. Now, the dynamics of the spacetime geometry is governed by
the Einstein’s equation which relates the geometry of the universe to its energy content as

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (1.1.3)$$

where, $G$ is universal gravitational constant and $G_{\mu\nu}$ is the Einstein’s tensor. In co-moving co-
dordinate system $v_\mu v^\mu = -1$, the corresponding field equations are

$$\frac{\ddot{a}}{a} + k \frac{\dot{a}}{a} = \frac{8\pi G}{3} \rho; \quad (1.1.4)$$

$$2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = -8\pi G p. \quad (1.1.5)$$

The continuity equation of the fluid, which is obtained from the Bianchi identity ($T^{\mu\nu;\rho} = 0$), is

$$\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = 0. \quad (1.1.6)$$

Combining equations (1.1.4) and (1.1.5), following relation is found

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p). \quad (1.1.7)$$
Note that continuity equation is not an independent equation but may be found under the combina-
tion of the field equations. Now, since there are three unknowns, viz., $a(t)$, $\rho(t)$ and $p(t)$ out of
two independent equations, a third relation is necessary. For instances, if the energy density
is dominated by one component, it is provided by an equation of state of the fluid, viz., $p = \omega \rho$,
where $\omega$ is the equation of state (EOS) parameter. In a given state, $\omega$ is constant, (e.g., $\omega = 0$
for dust and $\omega = \frac{1}{3}$ for radiation), but in general it is not. Now, it is clear that if strong energy
condition $(\rho + 3p) \geq 0$ i.e. $\omega \geq -\frac{1}{3}$ is satisfied, then universe will always be in decelerating phase
of expansion. Further, extrapolation of the expansion of the universe backwards in time using
general relativity yields an infinite density and temperature at a finite time in the past [4]. This
initial singularity commonly known as “Big Bang”.

The success of standard cosmology are the following. Firstly, Friedmann’s solution of Einstein’s
equation implies an expanding universe [5]. Finding luminosity distance $d_L$ vs redshift $z$
relationship of galaxies, expansion of the universe was experimentally confirmed by Hubble [6]. Secondly,
it predicts the existence of CMBR. It is supposed that almost three minutes after the big bang,
nucleosynthesis took place, i.e., hydrogen and helium nuclei formed from protons and neutrons.
Approximately, for the next $10^5$ years [7], the universe were in a radiation dominated phase in
which matter and radiation were strongly coupled through Thompson scattering. With the ex-
pansion the universe cools and at around 3000 K, protons and electrons combine to form neutral
hydrogen. At this era, known as recombination, photons are decoupled and free stream. Pho-
tons decoupled from matter from last scattering surface and could travel almost unhindered till
the present day, loosing energy as universe expands. This radiation is presently observed in the
microwave region, having a temperature of 2.73 K, known as CMBR [8]. Thirdly, the abundances
of the light atomic nuclei observed in the present universe agrees well with the predictions of this
model. The prediction gives the correct abundance ratio of $^{4}\text{He} / \text{H} \sim 0.25$, $^{2}\text{H} / \text{H} \sim 10^{-3}$, $^{3}\text{He} / \text{H} \sim 10^{-4}$
and $^{7}\text{Li} / \text{H} \sim 10^{-9}$ (by mass, not by number).

1.2 The very early universe

1.2.1 Singularity problem: need for quantum gravity

As already mentioned, extrapolation of the evolution of the universe backwards in time using
general relativity yields an infinite density and temperature at a finite time in the past [4]. This
singularity commonly known as “Big Bang”, refer to the early hot, dense phase itself, which can
be considered to be the “birth” of our universe and signals breakdown of the standard model.
Further the existence of Schwarzschild singularity signals that the theory of general relativity itself
must have to be replaced by some form of quantum theory of gravity close to the Planck epoch.
Stell [9] first produced a renormalized theory of gravity in 4-D taking into account higher order
curvature invariants terms in the action as $A = \int d^4 x \sqrt{g} \left[ \frac{R}{16\pi G} + R^2 + R_{\mu\nu}R^{\mu\nu} \right]$. However, the
theory contains ghost and so suffers from the problem of unitarity. So no renormalized theory of
gravity so far known to exist in 4-D. At this stage we would like to mention that Lovelock gravity
[10] alleviates the problem with unitarity, taking into account Gauss-Bonnet term, but at least in
five dimension. There is yet more complicated Lovelock terms, but those can be incorporated in
even higher dimension. Further, the main problem associated with Lovelock gravity is that it lacks
Hamiltonian formalism [11]. Hořava-Lifshitz gravity [12] on the other hand is a power counting
renormalized theory of gravity in 4-D, but it has been constructed at the expense of much cherished
Lorentz invariance. Recently, Koivistio and his collaborators [13] have also produced a renormalized
theory of gravity, but it is too complicated and beyond the scope present discussion. There had
been attempts to construct quantum theory of gravity from stringy description of fundamental
particles-the string theory. However, such theories have problems of their own. It is hoped that
a modified version of string theory or the theory of supergravity might possibly alleviate these
problems.

In the absence of a complete quantum theory of gravity, there were desperate attempts to remove
1.2. THE VERY EARLY UNIVERSE

the UV catastrophe appeared in cosmology, by constructing a quantum theory of cosmology-the quantum cosmology. Such an attempt lead to Whheler-deWitt (W-D) equation, which requires appropriate boundary condition to solve. Invoking Wormhole [14] boundary condition there was attempt to remove the initial singularity by a non-evolving throat of radius of the order of Planck length which connects two asymptotically flat universes in the Euclidean regime. Furthermore, microscopic wormholes might provide us with the mechanism that would solve the cosmological constant problem, while macroscopic wormholes might be responsible for the final stage of evaporation and complete disappearance of black hole. The problem with cosmological constant will be discussed later. However, here we just mention that it was hoped that wormholes might be the very cause of vanishing of cosmological constant. In the present dissertation we study some aspects of wormhole physics. Since all attempts to formulate a quantum theory of gravity require higher order curvature invariant terms like $R^2$ etc. So it is also required to make canonical formulation of higher order theory of gravity, which is surely non-trivial. This is another aspect of my study in the present dissertation. The very beauty of $R^2$ term is apparent as we gives a Hamiltonian structure to Lovelock action, which will also be discussed briefly.

1.2.2 Flatness, Horizon and structure formation problem: need for Inflation

- **Flatness problem**
  The flatness problem is analogous to cosmological fine-tuning problem of the universe. It arises from the observation that some of the initial conditions (e.g., the density of matter and energy) of the universe appear to be fine-tuned to very ‘special’ values, and that a small deviation from these values would have had massive effects on the nature of the universe at the current time.

  Data from the Wilkinson Microwave Anisotropy Probe (measuring CMB anisotropies) combined with that from the Sloan Digital Sky Survey and observations of type-Ia supernovae constrain the present value of the density parameter $\Omega$ to very close to 1. Slightest deviation in the value of $\Omega$ from 1 leads a huge difference from 1 at the very early universe. $\Omega = 1$ at $t = 14$ Gyr implies in view of standard model that $\Omega_{t=100s} = 1 \pm 10^{-11}$, and $\Omega_{\text{planck}} = 1 \pm 10^{-62}$, which is incredible. Therefore it must have been less than $10^{-62}$ at the Planck era. This leads cosmologists to question how the initial density came to be so closely fine-tuned to this ‘special’ value.

- **Horizon problem**
  The Cosmic microwave background radiation (CMBR), which is supposed to emit from the last scattering surface, is spatially homogeneous and isotropic. The homogeneity over the last scattering surface indicates causal connection among every points on the last scattering surface. But standard cosmology does not support this causal connections, since light signal travels a finite distance in a finite time and it can casually connect points within this finite distance which is called the horizon. In view of the standard model it has been found that the sky is splitted into $1.4 \times 10^4$ patches, which were never causally connected before emitting CMBR. So the puzzle, why the universe appears to be uniform beyond the horizon is called horizon problem.

- **Structure formation problem**
  The universe, as is now known from observations of the cosmic microwave background radiation, began in a hot, dense, nearly uniform state approximately 13.8 billion years ago [15]. However, looking at the sky today, we see structures on all scales, from stars and planets to galaxies and, on much larger scales still, galaxy clusters, and enormous voids between galaxies. It would have been generated from some seed of density perturbation, in the early universe. But standard model does not admit such perturbations.
• Associated problem

There is also an associated problem, required to be solved by the standard model, viz., monopole problem. A magnetic monopole is a hypothetical elementary particle in particle physics that is an isolated magnet with only one magnetic pole. In more technical terms, a magnetic monopole would have a net “magnetic charge”. The only plausible theory in elementary particle physics for how nuclei in the present universe were created in the big bang requires the use of what is called Grand Unified Theory (GUT). Our current understanding of elementary particle physics indicates that such theories should produce very massive particles called magnetic monopoles, and that there should be many such monopoles in the universe today. However, no one has ever found such a particle. Assuming GUT to be presently the correct theory, monopoles must have been smeared away in some manner during cosmological evolution. But the standard model has no answer to this problem. So the final problem is: where are the monopoles?

1.2.3 The Inflation

All the problems of standard model discussed above, but the singularity problem have been alleviated invoking one powerful concept: inflation. Alan Guth [16] for the first time proposed an intermediate phase of expansion of the universe, so called inflationary phase, at GUT epoch \(10^{-36}\) secs, before the standard Big Bang, of the very early universe. In the inflationary phase the universe underwent through a rapid exponential expansion, for a very short period of time, attributed by a scalar field which contributes in the energy-momentum tensor of the Einstein’s field equation. This is known as inflationary model of the universe.

Inflation works in a fairly simple manner. If the universe expands with a scale factor that grows more rapidly than the velocity of light (exponentially fast expansion), a very small region, initially in thermal equilibrium, can easily grow to encompass our entire visible universe at last scattering. The mutual thermalization of the ‘apparent decoupled regions’ is then of course no surprise. There is then no extraordinary homogeneity needed at the very beginning of the evolution of the universe. Two points which were casually connected in the very early expanding universe have fallen large apart so their past light cone do not intersect even they are expanded back to the last scattering surface. Thus horizon problem can be solved.

Since the universe is nearly flat today, it should have been even closer to critical density in early times. Inflation accomplishes this by blowing up the scale factor to huge proportions, like inflating a balloon to a larger volume erases the wrinkles and flattens the surface. An accelerating scale factor drives the density parameter \(\Omega\) towards 1 without any fine-tuning of initial conditions. Thus flatness problem is solved by the fact that the accelerated expansion ‘blows away’ the curvature.

If the universe were inflated right after the GUT-era, the monopole problem is trivially solved. All unwanted thermal relics (e.g., magnetic monopole, which may produce during symmetry breaking) are simply blown away by the dilution of the energy density caused by the inflation and untraceable at present.

Inflation is believed to be caused by a self-interacting quantum field. This field exhibits vacuum fluctuations and because the length scales of the fluctuations leave the Hubble scale during inflation, they are freezed and become classical. At Hubble scale re-entry, the fluctuations form the seeds for structure formation. Regions with a higher density accumulates matter and regions with a lower density loses some of their energy content to the higher density regions according to the Jeans-mechanism. A review on these issues may be found in [17] - [19].

Despite its success, Guth’s old inflationary model suffers from a problem of its own second order phase transition, called the graceful exit problem. The success of the standard model in explaining BBN and CMBR suggests that the universe must have started from a state of very hot dense plasma. However, inflation cools the universe down to a temperature at which nucleosynthesis would not have been possible. As a result, CMBR might not have been present and structures would not have been formed. This indicates, that inflation must have stopped, and by some means
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the temperature of the universe increased (reheating) to give way to the standard Big-Bang, which is now by no means a singularity but a hot thick soup of plasma. However, Guth’s inflation never ends.

This problem had been addressed in a number of models such as the new inflationary model [20, 21], chaotic inflationary model [22], extended inflationary model [23] - [25], hyperextended inflationary model [26], and in Starobinski’s model of inflation without phase transition [27]. However, all these models have some sort of demerits. The new inflationary and chaotic inflationary models require fine tuning of the effective potential parameter. This problem had been removed in the extended inflationary model where the first order phase transition yields variation in the gravitational constant as in Jordan-Brans-Dicke theories [28] - [30]. The extended inflationary model on the other hand finds it’s problem in setting a very small value of the Brans-Dicke parameter $\omega < 50$ so that the distortion in CMBR is negligible. This is against the present observed value of $\omega > 40000$ [31]. Nevertheless, we hope to find a successful inflationary scenario sooner or later. In the present dissertation we have not contributed in regard of inflation.

1.3 The early universe

1.3.1 Structure formation problem again: need for dark matter

From our previous discussion it appeared that this problem has been resolved since seeds of perturbations which were formed in the very early universe, went outside the horizon due to inflating universe and freezed. As inflation stopped these seeds had a reentry within the horizon, which forms structures today. Now the question is how the standard model helped these seeds to grow and form the structures we observe today? It requires some exotic kind of matter viz., “dark matter” (DM) which could form a potential well to accumulate baryonic matter. The name “dark” is due to its non-interacting behaviour with the other. Since dark matter interacts only with the gravitational field, it plays a key role in the structure formation. The gravitational Jeans instability allows compact structures to form and is not opposed by any force such as radiation pressure in the case of dark matter. As a result, dark matter begins to collapse into a complex network of dark matter Halos, well before ordinary baryonic matter, which is impeded by pressure force. Without dark matter the epoch of Galaxy formation would have occurred at a substantially later stage, than observed.

There are also some other evidences for the existence of dark matter, e.g., the evidence of the peculiar orbital velocities of galaxies in cluster, observed evidence from “Gravitational Lensing”, observations such as peculiar rotational curves of typical spiral galaxies which show that most of the stars in a spiral galaxy, orbit roughly at the same speed. This implied that the mass densities of spiral galaxies are uniform even beyond the center where most of the stars located. These observations also support the existence of dark matter. There are mainly two candidates of dark matter (Hot and cold). Hot DM are relativistic DM like neutrinos with mass of order a few eV and cold DM are non-relativistic DM like neutralinos having masses in the range of 10 to 1000 GeV. A third candidate, called the Warm Dark Matter (WDM) having speed intermediate between those of HDM and CDM has mass of order 1 to 10 keV.

Many experiments are now underway to try and detect the dark matter particles. It is believed that some components of dark matter particles interact through the weak nuclear force with ordinary matter. These hypothetical dark matter particles are therefore called Weakly Interacting Massive Particles (WIMPs) e.g., neutralino. WIMPs are predicted by supersymmetric extensions to the standard model of particle physics. Other exotic particles hypothesised as CDM candidates are the axions, WIMPzillas, photinos and gravitinos.

So, we see that baryons, which is the only ingredient of standard model can not solve the problem. Further, the measured total density parameter at present epoch is close to 1, while that of baryonic matter is 0.04. So rest should be in the form of some other matter, may be the dark matter. Nevertheless, dark matter may be incorporated within the standard model as yet another
component of energy momentum tensor which acts as dust. So in principle, it is not required to modify the standard model to resolve the issue of structure formation. However, in the present dissertation a chapter has been included to expatiate the fact that creation of dark matter from gravitational field might solve the problem of late time cosmic acceleration without invoking dark energy at all, which we discuss next.

1.4 The late universe

1.4.1 The Celebrated Observations: need for dark energy

I. Cosmic Microwave Background Radiation

About 3,80,000 years after the Big Bang, when universe cooled to a temperature of about $T \approx 3000K$, Thomson scattering between electrons and protons in the cosmic plasma stopped as it was no longer favourable. This allowed protons and electrons to form neutral atoms in a process known as recombination [7]. These atoms could no longer absorb the thermal radiation. This event in turn set the photons free from their interaction with matter and hence they began to free-stream throughout the universe from the surface of last scattering at a redshift $z \sim 1100$ and so the universe became transparent. These photons form the so called Cosmic Microwave Background Radiation (CMBR) as Big-Bang remnant in the microwave part of the spectrum, since the wavelength of this primordial radiation has now been stretched as a result of energy loss due to the expansion of the universe. The CMBR was first predicted by George Gamow and his collaborators [32] and it was accidentally detected by Penzias and Wilson [8], who measured the temperature of this radiation to be $T = 3.5K$. Later Cosmic Background Explorer (COBE) satellite found the temperature of the CMBR to be $T = 2.7K$ together with spatial temperature anisotropies in the CMBR [33]. These anisotropies, which are thought to have been generated by small density perturbations, are filled with valuable information about the cosmological parameters. After COBE, the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [34] - [36] measured these anisotropies with higher accuracy and today the Planck satellite [37] is pushing the accuracy of these measurements to the limit, hence providing us with ever so precise cosmological parameters. Figure 1.1 shows the CMBR temperature fluctuation sky map as observed by Planck satellite.
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Figure 1.2: The sizes of the spots of temperature variation in the CMB sky will be the same as the original size, smaller or larger than the actual size in flat, open and closed universes respectively. This results from the fact that light rays follow geodesics. Credits: CMB Map Lab

Further, the curvature of the space-time manifold in the universe is determined using light rays, in the same way as one can use the total sum of the interior angles of triangles to work out the curvature in 2-dimensional flat, spherical and hyperbolic spaces. This is because in GR light rays follow geodesic paths. We can use the CMBR for this task. The basic principle is that one can compare the observed sizes of hot and cold patches in the CMB sky created through temperature anisotropies with the theoretically predicted sizes for the universe with different curvatures. Figure 1.2 shows a schematic representation of the argument. The observed value of the curvature is found in this way to be very close to zero and hence we believe that we are living in a flat universe.

Another outcome is that the energy density of radiation can be calculated from the temperature of the CMBR. The dominant portion of radiation in the universe in all wavelengths is found in the CMBR which is the relic radiation left over from the Big-Bang and fills up the whole universe. Taking the temperature of the CMBR to be $T = 2.7K$, the energy density of radiation has been found to be around $\Omega_0^r \simeq 10^{-5}$. Therefore we can see that it is much smaller compared to the matter density today.

II. Type Ia Supernovae

Type Ia supernovae (SNe Ia) are ideal astronomical objects for distance determination. These objects are created through the explosion of accreting white dwarf stars when they reach the Chandrasekhar limit. Since they all have the same progenitors and are triggered through a consistent underlying mechanism, so they may be assumed to have constant absolute magnitude and hence may be used as standard candles. Indeed it was the observations of SNe Ia that confirmed the acceleration of the universe by two independent teams of Riess et al. [38] and Perlmutter et al. [39].

The analysis of SNe Ia data are made by plotting its observed distance modulus $\mu(=m-M)$ against redshift $z$ in Hubble diagram and comparing this light curve with the curve obtained from theoretically predicted values. Here, $m$ and $M$ are the apparent and absolute magnitude of SNe Ia. A physical quantity called luminosity distance $d_L$ is defined as

$$d_L = \sqrt{\frac{L}{4\pi F}}, \quad (1.4.1)$$

where $L$ and $F$ are the apparent and the absolute luminosity respectively. Finally, one obtains
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Figure 1.3: Comparison between various flat models and the observational data. The observational data points, shown with error-bars, are obtained from the “gold” sample of Riess et al. [38]. The most recent points, obtained from HST, are shown in red. Figure from [40].

the relation

$$\mu = 5 \log_{10} \left( \frac{d_L}{Mpc} \right) + 25.$$  \hfill (1.4.2)

Now to obtain a best fit with the experimental luminosity-distance versus redshift curve, vacuum energy is invoked. The Hubble parameter then is expressed in the following convenient form

$$\left( \frac{H}{H_0} \right)^2 = \sum_i \Omega^0_i (1 + z)^{3(1+\omega_i)} = \Omega^0_r (1 + z)^4 + \Omega^0_m (1 + z)^3 + \Omega^0_\Lambda + \Omega^0_k (1 + z)^2,$$  \hfill (1.4.3)

where $\Omega^0_r, \Omega^0_m, \Omega^0_\Lambda$ and $\Omega^0_k$ are the density parameters corresponding to radiation, matter, vacuum energy and curvature at present epoch respectively. Here, $\omega_i$ is the corresponding equation of state (EOS) parameter. Clearly, $\sum_i \Omega^0_i = 1$ which is also supported by observational data. Hence the luminosity distance is given by

$$d_L = \frac{1 + z}{H_0} \int_0^z \frac{dz'}{\sum_i \Omega^0_i (1 + z')^{3(1+\omega_i)}}.$$  \hfill (1.4.4)

It has been found that the best fit of the experimental curve with the theoretical curve in a two component flat universe requires vacuum energy density $\Omega^0_\Lambda = 0.7$ and matter energy density $\Omega^0_m = 0.3$ [38, 39]. Figure 1.3 shows a plot of distance modulus vs. redshift [40]. So the universe is mostly filled with (70%) dark energy.

Since the observed amount of Baryons present in the universe is only 4%, while SNe Ia data reveals that the matter contribution is about 30%, so, naturally there should be about 26% of some exotic kind of matter which interacts only gravitationally with the others. These type of matter, as already mentioned, is called the “dark matter”.

$$\Omega_m = 0, \Omega_A = 1$$
$$\Omega_m = 0.31, \Omega_A = 0.69$$
$$\Omega_m = 1, \Omega_A = 0$$
III. Baryon Acoustic Oscillations

Another important observational discovery is the Baryon Acoustic Oscillations (BAO) which arise from the same density perturbations that cause the anisotropies in the CMB temperature. Prior to recombination era, photons and baryons were tightly coupled to each other through Thomson scattering. In that epoch perturbations resulted in the creation of gravitational instability in the “photon-baryon” fluid and the collision-less dark matter. These gravitational instabilities continued to grow in the dark matter part but the baryons could not collapse under the force of gravity as the radiation pressure of photons would oppose this. This is basically the mechanism that holds a star together but in the case of a star the two opposite forces are balanced. In the early universe plasma however there is an imbalance between the forces leading to oscillations in the photon-baryon fluid like sound waves in spherical shells. After recombination photons free-stream, and these acoustic oscillations leave their imprint both on the CMBR and the distribution of matter. Of course the oscillations occur at many different wavelengths but there is a characteristic resonant wavelength, which we can measure. This distance scale has grown with the universe’s expansion which means that we observe it in the distribution of galaxies today at about 100 Mpc. This length scale translates into about 1 degree between the hot and cold patches of the CMB sky. Putting these two information together one can thus use this natural “standard ruler” to trace the universe’s expansion history back to the time when the CMBR was emitted.

The BAO was first convincingly detected by two international teams, viz., the Two degree Field Galaxy Redshift Survey (2dFGRS) [41] and the Sloan Digital Sky Survey (SDSS) groups [42]. Figure 1.4 shows an SDSS map of galaxy distribution. WMAP and SDSS maps show the characteristic scale set by the BAO in the early universe. So, BAO observations confirms that dark matter are present in the universe.

The Constraints on various cosmological parameters from different observations have been tabulated in table-1 (Courtesy: Amanullah et al [43]). Figure 1.5 and 1.6 shows the argument of table-1 (Courtesy: Amanullah et al [43]). An important feature of figure 1.6 is that for good fit the EOS...
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Figure 1.5: 68.3%, 95.4% and 99.7% confidence regions in the \((\Omega_M, \Omega_{\Lambda})\) plane from SNe combined with the constraints from BAO and CMB. Cosmological constant dark energy \((\omega = -1)\) has been assumed. (Courtesy: Amanullah et al [43]).

Parameter \(\omega\) may even be less than \(-1\), which leads to Phantom energy.

**Table - 1**

<table>
<thead>
<tr>
<th>Fit</th>
<th>(\Omega_M)</th>
<th>(\Omega_k)</th>
<th>(\omega)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNe+BAO+CMB</td>
<td>0.277 (\pm 0.014)</td>
<td>0 (fixed)</td>
<td>(-1.009 \pm 0.004)</td>
</tr>
<tr>
<td>SNe+BAO+CMB</td>
<td>0.278 (\pm 0.014)</td>
<td>-0.004 (\pm 0.004)</td>
<td>(-1) (fixed)</td>
</tr>
<tr>
<td>SNe+BAO+CMB</td>
<td>0.281 (\pm 0.015)</td>
<td>-0.005 (\pm 0.007)</td>
<td>(-1.026 \pm 0.050)</td>
</tr>
</tbody>
</table>

So, various observation suggest that the universe is very nearly flat with a total density equal to the critical density and there appeared a deficiency in the total energy budget of the universe. The observations of matter had corresponded to only about 1/3 of the critical density. Therefore, cosmologists reasoned that the rest 70% must be some kind of yet unknown vacuum energy. But this vacuum energy is non-interacting and dubbed as dark energy.

The origin of dark energy is still unknown. However contribution of dark energy in some form must be incorporated to explain late time accelerated expansion of the universe. Here, in this section we shall briefly discuss some of the possible candidates of dark energy.

1.4.2 Cosmological constant (\(\Lambda\)CDM) model

In cosmology, the cosmological constant (\(\Lambda\)) was originally introduced by Albert Einstein to “hold back gravity” ensure a static universe, which was then philosophically accepted view. Later, Einstein abandoned the idea, stating it to be his “greatest blunder” after Hubble’s discovery of
expanding universe. Later cosmological constant was revived by field theorists as vacuum energy density of the universe. After the discovery of the accelerating universe from distant supernovae, the cosmic microwave background and large galaxy redshift surveys, it has been confirmed that the mass-energy density of the universe includes around 70% in dark energy. The cosmological constant is the simplest possible candidate for dark energy since it is constant in both space and time.

In $\Lambda$CDM model, the contribution of the dark energy is attributed with the energy-momentum tensor in the Einstein equation so that the general Friedmann equation reads

$$\left(\frac{H}{H_0}\right)^2 = \Omega_r^0(1+z)^4 + \Omega_m^0(1+z)^3 + \Omega_\Lambda^0,$$  \hspace{1cm} (1.4.5)

for an almost flat present universe. Here $\Omega$ stands for the density parameter and suffix zero stands for the present epoch. It has been found that SNe Ia data in Hubble diagram is best fitted with the theoretical curve of $\Lambda$CDM model taking $\Omega_r^0 = 8 \times 10^{-5}, \Omega_m^0 = 0.26 + \Omega_{\Lambda}^0 = 0.74$ in the background of FRW metric. But the problem associated with this model is $\Lambda$ itself. The vacuum energy density, as calculated by the field theorists, is some $10^{120}$ order of magnitude greater than the cosmological constant ($\Lambda$) required by the cosmologists to explain late time cosmic acceleration. This is known as cosmological constant problem. Another way of stating the problem is that the observed renormalized cosmological constant is at least 120 orders of magnitude smaller than the quantum corrections, thus requiring an enormous fine-tuning of the bare cosmological constant.

Further, large theoretical value of vacuum energy would preclude the formation of the present structure of the universe. This was realized in the 80’s and there had been attempt to make it vanish by invoking Euclidean wormholes [14]. We have some contribution in this field which has been included in the present dissertation. Nevertheless, present approach [44] to solve the problem of both the cosmological constant and the dark energy, considering slowly varying cosmological constant w.r.t time. Alternatively a dynamical model of cosmological constant is also realized from scalar field theory.

Figure 1.6: 68.3%, 95.4% and 99.7% confidence regions of the $(\Omega_M, \omega)$ plane from SNe combined with the constraints from BAO and CMB. Zero curvature and constant $\omega$ have been assumed. (Courtesy: Amanullah et al [43]).
1.4.3 Scalar-field models of dark energy

The cosmological constant corresponds to a fluid with a constant equation of state $\omega = -1$. However, other models for which the state parameter is dynamical are also supported by different observations. In these models the state parameter evolves and its present value is close to -1 or even less. So one can consider a situation in which the equation of state of dark energy changes with time. A dynamical EOS parameter has the advantage to give a possible solution for the dark energy problem alleviating the coincidence problem and the fine tuning problem. The question why the dark energy possesses such a very small value at present time, is called the coincidence problem. Now, if we find a model with an evolution such that the EOS parameter of dark energy becomes dominant at late times independently of the initial conditions we have an answer to the coincidence problem. Note that dark energy could not have been dominant in the early universe, because in that case structures like galaxies could not have been formed. Therefore it is convenient to search for models with a tracker behavior, in which the dark energy density closely tracks the radiation density until very recently. After this epoch the scalar field has to start behaving as dark energy, eventually dominating the universe. Secondly we have a solution for the fine tuning problem, if the field, which creates an EOS parameter, naturally arises from particle physics and gives exactly the density of the critical density at late times. These considerations motivate a search for a dynamical dark energy model caused by some exotic field. So far, a wide variety of scalar-field dark energy models have been proposed in the literature. We shall briefly discussed some of the important ones in this section.

Quintessence model

Quintessence is a hypothetical form of dark energy postulated as an explanation of the observation of an accelerating rate of expansion of the universe. Many models of quintessence have a tracker behavior, which partly solves the cosmological constant problem. In these models, the quintessence field has a density which closely tracks (but is less than) the radiation density until matter-radiation equality, which triggers quintessence to start having characteristics similar to dark energy, eventually dominating other forms of energy density of the universe. In Quintessence model [45] - [48] scalar field is minimally coupled to gravity. The Einstein-Hilbert (E-H) action is then modified as

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} \phi_{,\mu} \phi^{,\mu} - V(\phi) \right] + S_m, \quad (1.4.6)$$

where $S_m$ corresponds to matter contribution. Now, in a spatially flat FRW spacetime the field equations are

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \quad (1.4.7)$$

$$3 \frac{\dot{a}^2}{a^2} = 8\pi G \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \quad (1.4.8)$$

and

$$2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G \left( \frac{1}{2} \dot{\phi}^2 - V(\phi) \right). \quad (1.4.9)$$

From the above two equations EOS parameter is given by

$$w_\phi = \frac{p}{\rho} = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)} \quad (1.4.10)$$

which shows that if the scalar field initially evolves very rapidly then the universe evolves through a decelerated phase at early era and after that due to slow evolution of the scalar field universe finally enters into late time accelerated phase of expansion. However, despite attempts in different forms,
Quintessence model has been found to suffer from coincidence problem. This problem has been addressed in K-essence model, where field dependent kinetic energy term had been introduced.

**K-essence model**

The name K-essence is an abbreviation of Kinetic Quintessence. In K-essence model, the late time acceleration is obtained depending on the kinetic energy of the scalar field \[49\] - \[51\]. Here the general action which is the function of scalar field \(\phi\) and \(X(= -\frac{1}{2}\phi,\mu\phi^\mu)\), is given by

\[
S = \int d^4x\sqrt{-g} \left[ \frac{R}{16\pi G} + p(\phi, X) \right] + S_m, \tag{1.4.11}
\]

where \(p(\phi, X)\) is the Lagrangian density corresponding to pressure density of the scalar field. After some mathematical approximation and calculation one finds

\[
p(\phi, X) = f(\phi)(-X + X^2), \tag{1.4.12}
\]

the energy density

\[
\rho = f(\phi)(-X + 3X^2) \tag{1.4.13}
\]

and the EOS parameter

\[
w_\phi = \frac{1 - X}{1 - 3X}. \tag{1.4.14}
\]

Here the kinetic term \(X\) determines expansion. For \(\frac{1}{2} < X < \frac{2}{3}\), the value of EOS parameter is \(-1 < w < -\frac{1}{3}\). But, the fine-tuning problem is not solved here since we need to fine tune \(f(\phi)\) in order to get the value of the energy density of the present universe.

**Phantom model**

In the Quintessence and K-essence model \(w \geq -1\). However the recent observational data indicates that the value of EOS parameter is \(-1.24 \leq w \leq -0.96\) [52]. The dark energy corresponding to a region where \(w < -1\) is called phantom dark energy. In a simple model phantom dark energy can be explained by introducing a scalar field having negative kinetic energy [53]. Such a field may be motivated from S-brane constructions in string theory [54]. Further there are some specific models in Brane world or Brans-Dicke scalar-tensor gravity that also lead to phantom dark energy [55] - [57]. The corresponding action is given by

\[
S = \int d^4x\sqrt{-g} \left[ \frac{R}{16\pi G} + \frac{1}{2}\phi,\mu\phi^\mu - V(\phi) \right] + S_m. \tag{1.4.15}
\]

In the background of FRW metric the energy density and pressure density are

\[
\rho = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad p = -\frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{1.4.16}
\]

The EOS parameter is given by

\[
w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 - V(\phi)}. \tag{1.4.17}
\]

So for \(\frac{1}{2}\dot{\phi}^2 < V(\phi)\), we have \(w_\phi < -1\). But in this model if the universe contains only dust and phantom matter, then at an epoch when the phantom energy dominates over the matter energy, the scale factor diverges in a finite time and the expansion rate of the universe reaches to an infinite value in finite time leading to a future singularity [53, 58]. This situation is called Big-Rip. However in some phantom models, future Big-Rip singularity is avoided by considering canonical form of the kinetic energy term [55, 56] and by suitable choice of the potential term or through a double transition (cite my work and some others). Further, phantom fields generally suffer from
severe ultra-violet quantum instabilities. Since the energy density of a phantom field is unbounded from below, the vacuum becomes unstable against the production of ghosts and normal fields [59].

Tachyon field

Tachyon is a hypothetical superluminal unstable particle with a negative squared mass. Perhaps the most famous example of a tachyon is the Higgs boson of the Standard model of particle physics. In its uncondensed phase, the Higgs field has a negative mass squared, and is therefore a tachyon. By a very small disturbance the tachyon will roll down its potential and it induces tachyon condensation, i.e. the tachyon gets a real mass. In a certain class of string theories, there is a possibility that a rolling tachyon condensates. Sen [60, 61] showed that the decay of D-branes produces a pressureless gas that resembles classical dust. A rolling tachyon has an interesting equation of state whose parameter smoothly interpolates between $-1$ and 0 [62]. The cosmological consequence is that, as long as the tachyon rolls down to the minimum of its potential, the universe expands. The potential of a tachyon can have different forms [63] - [68].

An effective action for the tachyon is

$$S = -\int d^4x V(\phi)\sqrt{-\text{det}(g_{ab} + \partial_a\phi\partial_b\phi)}, \quad (1.4.18)$$

where $V(\phi)$ is the tachyon potential. Corresponding energy momentum tensor has the form

$$T_{\mu\nu} = \frac{V(\phi)\partial_\mu\phi\partial_\nu\phi}{\sqrt{1 + g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi}} - g_{\mu\nu}V(\phi)\sqrt{1 + g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi}. \quad (1.4.19)$$

In a flat FRW background the energy density and the pressure density are given by

$$\rho = -T^0_0 = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \quad \text{and} \quad p = T^i_i = -V(\phi)\sqrt{1 - \dot{\phi}^2}. \quad (1.4.20)$$

So, the equation of state of the tachyon is

$$\omega = \frac{p}{\rho} = \dot{\phi}^2 - 1. \quad (1.4.21)$$

Hence an accelerated expansion occurs for $\dot{\phi}^2 < \frac{2}{3}$. However, there is doubt in the presence of a tachyon field at the late universe.

Chaplygin gas

So far we have discussed a couple of scalar-field models of dark energy where the equation of state is $\omega = \frac{p}{\rho}$. There exist another interesting class of dark energy models involving a fluid known as a Chaplygin gas [69]. In simplest form this fluid has the following specific equation of state:

$$p = \frac{A}{\rho}, \quad (1.4.22)$$

where $A$ is a positive constant. The equation of state for the Chaplygin gas can be derived from the Nambu-Goto action for a D-brane moving in the D + 1 dimensional bulk [70, 71]. Now, using the equation of state (1.4.22), continuity equation yields

$$\rho = \sqrt{A + \frac{B}{a^6}}, \quad (1.4.23)$$

where $B$ is a constant. The following asymptotic behavior is realized

$$\rho \sim \frac{\sqrt{B}}{a^3} \quad \text{for} \quad a \ll \left(\frac{B}{A}\right)^{1/6}, \quad (1.4.24)$$
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\[
\rho \sim -p \sim \sqrt{A} \quad \text{for} \quad a \gg \left(\frac{B}{A}\right)^{1/6}. \tag{1.4.25}
\]

So, at early times when \( a \) is small, the gas behaves as a pressureless dust. However it behaves as a cosmological constant at late times, which leads to an accelerated expansion.

Chaplygin gas provides an interesting possibility for the unification of dark energy and dark matter. However Chaplygin gas models have undergone strong observational constraint from CMB anisotropies [72] - [74]. This comes from the fact that the Jeans instability of perturbations in Chaplygin gas models behaves similarly to cold dark matter fluctuations in the dust-dominant stage but disappears in the acceleration stage. The combined effect of the suppression of perturbations and the presence of a non-zero Jeans length gives rise to a strong integrated Sachs-Wolfe (ISW) effect, which leads to the loss of power in CMB anisotropies. This situation can be alleviated in the generalized Chaplygin gas model [75] with \( p = A/\rho^\alpha \), where \( 0 < \alpha < 1 \). However, even in this case the parameter \( \alpha \) is rather severely constrained, i.e., \( 0 \leq \alpha < 0.2 \) at the 95% confidence level [73].

Though the dark energy models discussed above have field theoretic support and have been introduced to explain cosmological evolution but none of the existing dark energy model is fully satisfactory. Firstly, for a viable scalar-tensor cosmological model the scalar mode has to obey the Chameleon mechanism [76] - [81]. Secondly, these type of exotic scalar fields are very difficult to detect.

1.4.4 Modified \( F(R) \) theory of gravity

We have realized that though Einstein gravity is well tested in the solar system it cannot alone explain the phenomena in very high as well as in low curvature regions. Modification in Einsteins theory is therefore another strong possibility to accommodate the phenomena of early universe as well as the phenomena in cosmological scale at late time universe. An alternative approach [82] - [88] is a phenomenological modification of Einstein gravity to obtain an effective contribution of dark energy. The geometrical modifications can arise from quantum effects such as higher curvature corrections to the Einstein Hilbert action. This approach is known as modified theory of gravity.

The simple and most discussed modified gravity is \( f(R) \) gravity based on fundamental theory of physics. In this model Ricci curvature scalar is replaced by a more general functional form \( F(R) \) in the Einstein-Hilbert action. As already discussed earlier that E-H action is not renormalizable. In order to get renormalized theory gravity, it is necessary to include higher order curvature terms in the E-H action [89, 9]. Further in the recent past it has also been shown that the effective low energy gravitational action admits higher order curvature invariant terms, when quantum correction or string/M theory are taken into account [90].

It has been shown in the literature that E-H action with \( R^{-1} \) term can explain the late time acceleration [91] - [94] of the universe without any kind of exotic matter field. However, \( R^{-1} \) term fails to produce Newtonian gravity in the weak energy limit and so it is not consistent with the solar test [95]. Further, it also shows unavoidable instabilities within matter in the weak gravity limit [96] and fails to explain big bang nucleosynthesis (BBN) [97]. In fact \( R^{1+\delta} \) theory of gravity suffered initial setback under synthesis of light elements, shift of the horizon size at matter-radiation equality and perihelion-precession observation of Mercury [98]. All these data together puts up severe constraint on \( \delta \), viz., \( 0 < \delta < 7.2 \times 10^{-19} \). Further, solar system also puts up a severe constraints on alternative theories of gravity [99, 100]. Particularly, for an action

\[
A = \int \sqrt{-g}d^4x R^n, \tag{1.4.26}
\]

Newtonian gravitational field is recovered for \( n = 1 \). Any other value of \( n \), which appreciably differs from 1 is ruled out from light bending data in the sun limb and planetary periods [99]. The
problem was alleviated [101] by considering an action in the form

$$A = \int \left[ \beta R^m + \alpha R + \gamma R^{-n} \right] \sqrt{-g} d^4x,$$  

(1.4.27)

$m > 0, n > 0$, which passes solar test and therefore is suitable to explain the cosmological evolution right from the inflationary era through to late time accelerated epoch. At the initial stage, $R^m$ term dominates and a de-Sitter solution is realizable for $m = 2$ in particular, explaining inflationary epoch without invoking phase transition [27, 102]. In the middle, the linear term dominates giving way to the standard BBN and structure formation and finally $R^{-n}$ term dominates and late stage of accelerated cosmological expansion is realized, without invoking dark energy. However, $R^{-n}$ term is not distinguished at all, since neither it is generated by one-loop quantum gravitational corrections nor from any other physical consequence. Rather, it was considered just to invoke late time accelerated expansion.

Therefore, it is required to find some other scalar invariant term, suitable to explain late time cosmic acceleration, having Newtonian analogue in the weak field limit to pass the solar test. Such a theory, dubbed as $F(R)$ theory of gravity [103, 104] is much more attractive, promising and intriguing than scalar field models.

In $F(R)$ gravity there are two variational principles that can be applied to derive the field equations, namely standard metric variation and Palatini variation. In the later the metric and connection are assumed to be independent variables. Interesting feature of $F(R)$ gravity is that it is dynamically equivalent to Brans-Dicke gravity with a potential both in metric formalism when BD coupling function $\omega_0 = 0$ [95, 105, 106] and in Palatini formalism when BD coupling function $\omega_0 = -\frac{3}{2}$ [86, 106]. This equivalence ensures the validity of Ehlers Geren Sachs (EGS) theorem [107] for $F(R)$ gravity [108]. In the metric $f(R)$ gravity a massive scalar mode appears [109], which is dynamical unlike GR where only massless graviton is found. In the weak field limit the massive scalar mode of metric $F(R)$ gravity, in some models [91, 110, 111] becomes light enough, $m \sim 10^{-33}$ eV, through Chameleon mechanism at cosmological curvature causing late time acceleration of the universe [112] - [114]. In this theory Big-Bang Nucleosynthesis is well constrained in some models [115, 116] and sufficiently long matter era is obtained [115] to allow the primordial density perturbation generated during inflation to grow and become the present observed structure of the universe. The growth and evolution of local scalar perturbations in metric $F(R)$ gravity reproduce GR at high curvature [83, 117]. It is found that $F''(R) > 0$ is required for scalar perturbations and also to avoid matter instability of the theory. In some models [118] of $F(R)$ gravity evolution encounter a singularity other than Big-Rip singularity.

### 1.5 Present orientation

So far we have discussed various problems of standard model of cosmology in different era. Some celebrated observations and their outcomes have also been discussed. Some non-standard model, their approaches to solve the puzzle, and their shortcomings have also been discussed. Now, we are going to discuss our attempts to resolve the issue. In the following chapter some aspects of $F(R)$ theory have been discussed. In chapter-3, it has been shown that late time acceleration may be achieved without dark energy via particle (CDM) creation mechanism. Canonical formulation of the curvature-squared action in the presence of lapse function has been presented in chapter-4. A detailed study of quantum and semiclassical Euclidean wormholes for Einstein’s theory with a minimally coupled scalar field has been performed for a class of potentials in chapter-5. Finally, we end up with conclusion in chapter-6.