Chapter 1

Introduction

The recent years have seen an increased interest in control of uncertain systems. Sliding mode control (SMC) has evolved into an effective strategy for controlling plants with significant uncertainties and unmeasurable disturbances. The intent is to find a methodology that is in line with the practicality of actual plants.

1.1 Overview

Sliding Mode Control (SMC) is a special class of variable structure systems (VSS) that alters the dynamics of a system with a switching control (Emelyanov, 1970). The theory of sliding mode is based on the concept of changing the structure of controller in response to the changing state of system, to achieve a desired response (Itkis, 1976). The closed loop system can thus be made insensitive to system uncertainties and external disturbances. The increased interest is a result of robustness becoming an important requirement in modern control applications.

The invariance and robustness properties has resulted in SMC maturing into an effective strategy for controlling systems in the presence of uncertainties, external disturbances, and plant parameter variations (Fridman et al., 2011). SMC has now been developed into a general design method and extended to a wide spectrum of system types; predominantly in motion control (Sabanovic, 2011).
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The conventional SMC, however employs discontinuous control that results in undesirable chatter. This chatter causes excessive wear and tear of actuators and may excite unmodeled dynamics. Furthermore, the control can be designed only if bounds of uncertainty and disturbances are known. The conventional SMC is insensitive to only matched disturbances (Drazenovic, 1969), and also requires full state vector for implementation.

SMC has found a variety of applications in robotics, aerospace vehicles, electrical drives and automotive industry to name a few. There are however applications where; uncertainties and disturbances act in channels in which, a control input is not present. The magnetic levitation, under-actuated mechanical systems, systems having actuators in cascade, are all examples of such mismatched systems. When conventional SMC is applied to such systems, the controlled output is affected by uncertainties and disturbances even when the system is in sliding mode.

A control design using simultaneous estimation of states along with uncertainties and disturbances can effectively alleviate the problem. In this design, the effect of uncertainties is compensated by augmenting the controller designed for nominal system with the estimates. There are several techniques reported for uncertainty estimation like time delay control (TDC) (Youcef-Toumi and Ito, 1990), disturbance observer (DO) (Chen, 2004), extended state observer (ESO) (Han, 2009). The ESO enables estimation of states, with disturbances as an extended state. The uncertainty and disturbance estimator (UDE) (Zhong and Rees, 2004) is an effective technique for estimating slow varying uncertainties. The equivalent input disturbance (EID) method (She et al., 2008) is a recent addition to the literature.

The characteristics of discrete-SMC are different from continuous-SMC in that, they can undergo only quasi-sliding motion i.e. the state of the system can approach the switching surface but cannot generally stay on it. This is due to the fact that the control action can only be activated at sampling instants and control effort is constant over each sampling period (Misawa, 1997a). The $\delta$ operator can be used to formulate an unified design thereby resolving the dichotomy between results in continuous and discrete time control law; especially with regard to limiting properties as sampling time $T_s \to 0$ (Middleton and Goodwin, 1990). The $\delta$ operator is used for unification of sliding condition in (Ginoya et al., 2015b).
A new control method for uncertain dynamical systems is proposed and is shown in Fig. 1.1. It is based on a unique disturbance rejection concept leveraging the benefits of uncertainty and disturbance estimation. The proposed robust control scheme consists of an *Estimator* for estimating the uncertainty and disturbance, *Observer* for estimating the states, and *Control law* based on *Sliding Mode Control*. The proposed approach can be used for tracking as well as regulation problem.

![Figure 1.1: Configuration of proposed control](image)

This is motivated by the requirements of practical plant; since most plants and systems encountered in practice possess significant uncertainty and are subjected to varied disturbances. The robust control law is designed for both matched as well as mismatched uncertainties and the sliding surface is designed to preclude large initial control. The designed control laws are tested for representative plant configurations and validated for varied applications in motion and automotive control. The same is demonstrated through simulation, frequency response analysis and hardware tests for handling set-point changes, inertia and friction variations, plant uncertainties and external disturbances.

This work attempts to propose conceptual solutions to address the fundamental limitations in existing framework. It is envisaged that the proposed method lends itself well; in providing innovative solutions to practical problems. The central theme is to develop robust control algorithms for uncertain dynamical systems.
1.2 Concept of Sliding Mode Control

Consider a second-order system with a feedback control \( u \),
\[
\begin{align*}
\dot{x} &= ax + u \quad a > 0 \\
u &= -kx
\end{align*}
\]

The eigen-values of the closed-loop are,
\[
\lambda_{1,2} = \left(-a \pm \sqrt{a^2 - 4k}\right)/2
\]

If \( |k| = b \) and \( b > a^2/4 \), there are 2 structures corresponding to \( k < 0 \) or \( k > 0 \):

1. If \( k = b \), Fig. 1.2(a) shows the phase portrait of this structure.
2. If \( k = -b \), the phase portrait of the system is shown in Fig. 1.2(b).

Both these structures are unstable. However, note that in the second structure, there is a motion along the line corresponding to stable eigen value \( \dot{x} - \lambda_2x = 0 \), i.e. a motion which tends to the origin. Therefore, defining a switching function,
\[
s = \dot{x} + \lambda x
\]

and let the system switch on lines \( x = 0 \) and \( \sigma = 0 \); according to switching law,
\[
k = \begin{cases} 
  b & \text{if } s > 0 \\
  -b & \text{if } s < 0 
\end{cases}
\]

the resulting phase trajectory is shown in Fig. 1.2(c).
The two unstable structures replace each other on the line, hence all the trajectories are oriented towards the line and then asymptotically converge to origin. The motion along the line, which is not a trajectory of any structures is called sliding mode. The concept of SMC is illustrated in Fig. 1.3.

![Figure 1.3: Sliding mode concept](image)

The problem of SMC design involves selecting the parameters of each of the structures and define the switching logic. The motion of a SMC system includes two phases; the reaching phase and the sliding phase as shown in Fig. 1.3. During the reaching phase, the system state is pushed towards the switching surfaces. During this phase, the system response is sensitive to parameter uncertainties and disturbances. Thus, one would ideally like to shorten the duration or even eliminate the reaching phase.

One easy way to minimize the reaching phase and hence reaching time is to employ a larger control input (Utkin, 1992; Edwards and Spurgeon, 1999). This however, may cause extreme system sensitivity to unmodeled dynamics, actuator saturation and higher chattering as well. The robustness of the SMC can be improved by shortening the reaching phase or may be guaranteed during whole intervals of control action by eliminating the reaching phase (Ackermann and Utkin, 1998).
1.3 Uncertainty Estimation

There are several interesting techniques available for estimation of uncertainty. The uncertainty and disturbance estimator (UDE), disturbance observer (DO), and equivalent input disturbance (EID) are briefed in this section.

A generic uncertain plant can be described as,

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + B_d e(x, u, t) \\
y(t) &= Cx(t)
\end{align*}
\]  

(1.5)

where, \(x(t)\) is the state vector, \(u(t)\) is the control input, \(y(t)\) is the output of plant and \(e(x, u, t)\) is the lumped uncertainty.

\[
e(x, u, t) = \Delta A x(t) + \Delta B u(t) + v(t) + \zeta(x, t)
\]  

(1.6)

where, \(\Delta A\) is uncertainty in plant, \(\Delta B\) is uncertainty in input, \(v(t)\) is external disturbance and \(\zeta(x, t)\) represent nonlinearities in the plant.

1.3.1 Uncertainty and Disturbance Estimator (UDE)

The key idea in UDE based control is, to use a filter of appropriate band-width to estimate the lumped uncertainty; and to use the opposite of estimate in control to negate the effect of uncertainty (Zhong and Rees, 2004; Talole and Phadke, 2008). This method is based on an idea similar to TDC (Youcef-Toumi and Ito, 1990), but does not require derivative of system state and does not use time delayed signals. The lumped uncertainty \((e)\) can be estimated as,

\[
\dot{e} = G_f(s) e
\]  

(1.7)

where \(G_f(s)\) is a filter with sufficiently large bandwidth, of the form,

\[
G_f(s) = \frac{1}{1 + \tau s}
\]  

(1.8)

where \(\tau\) is a small positive constant.

The accuracy of estimation can be improved with a higher-order filter.
1.3.2 Disturbance Observer (DO)

Disturbance observer is an estimation scheme that utilizes an auxiliary dynamical system to estimate the uncertainties and disturbances acting on the system. The disturbance estimates are then used in the control law to compensate the effect of uncertainty.

\[
\begin{align*}
\dot{\hat{e}} &= p + lx \\
\dot{\hat{p}} &= -l(Ax + Bu + Bd\hat{e})
\end{align*}
\]

where \(p\) is an auxiliary variable and \(l\) is user chosen constant.

The accuracy of estimation can be improved by estimating the uncertainties and its derivatives with a higher-order DO.

1.3.3 Equivalent Input Disturbance (EID)

The EID is a disturbance on the control input channel that produces the same effect on controlled output as actual disturbances (She et al., 2008).

Consider a uncertain plant as,

\[
\begin{align*}
\dot{x}_1 &= x_2 + d_1 \\
\dot{x}_2 &= x_3 + d_2 \\
\dot{x}_3 &= u
\end{align*}
\]

and

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= u + d_e
\end{align*}
\]

The plant with EID can be written as,

\[
\begin{align*}
\dot{x}_1 &= x_2 + d_1 \\
\dot{x}_2 &= x_3 + d_2 \\
\dot{x}_3 &= u + d_e
\end{align*}
\]

Figure 1.4: General uncertain plant

Figure 1.5: General plant with EID
Consider \( u = \sin \omega_u t \), \( d_1 = \sin \omega_{d_1} t \) and \( d_2 = \sin \omega_{d_2} t \)

Therefore, for the original plant (equation (1.10), Fig. 1.4),

\[
\begin{align*}
\dot{x}_1 & = -\cos \omega_u t + \cos \omega_{d_2} t - \sin \omega_{d_1} t \\
\dot{x}_2 & = -\sin \omega_u t + \cos \omega_{d_2} t + \sin \omega_{d_1} t \\
\dot{x}_3 & = \cos \omega_u t - \omega_{d_1} \cos \omega_{d_1} t + \sin \omega_{d_2} t \\
\end{align*}
\]

(1.12)

The output for the plant with EID (equation (1.11), Fig. 1.5) can be derived as,

\[
\begin{align*}
\dot{u} & = \sin \omega_u t \\
\dot{d}_c & = \dot{d}_1 + \dot{d}_2 \\
& = -\omega_{d_1}^2 \sin \omega_{d_1} t + \omega_{d_2} \sin \omega_{d_2} t \\
\dot{u} + \dot{d}_c & = \sin \omega_u t - \omega_{d_1}^2 \sin \omega_{d_1} t + \omega_{d_2} \sin \omega_{d_2} t \\
\dot{x}_3 & = \cos \omega_u t - \omega_{d_1} \cos \omega_{d_1} t + \sin \omega_{d_2} t \\
\dot{x}_2 & = -\sin \omega_u t + \sin \omega_{d_1} t + \cos \omega_{d_2} t \\
\dot{x}_1 & = -\cos \omega_u t + \cos \omega_{d_1} t + \sin \omega_{d_2} t \\
\dot{y} & = -\cos \omega_u t + \cos \omega_{d_1} t + \sin \omega_{d_2} t \\
\end{align*}
\]

(1.13)

The equations (1.12) and (1.13) clearly depict that the output in both the cases is same. This implies that instead of estimating \( d_1 \) and \( d_2 \), the EID \( (d_c) \) can be estimated to compensate the effect of \( d_1 \) and \( d_2 \) on the output. The analysis also demonstrates that a full-order observer is necessary for estimation of EID.
1.4 Literature Review

Variable structure control (VSC) with sliding mode first made its appearance in early sixties in the erstwhile Soviet Union (Emelyanov, 1970; Itkis, 1976). Since then, significant interest on variable structure systems and sliding mode control has been generated in the control community. Early utilization of this SMC approach can be found in (Utkin, 1977; Decarlo et al., 1988; Hung et al., 1993). One of the most interesting aspects of sliding mode is the discontinuous nature of control action. The primary function of each of the nonlinear feedback channels is to switch between two distinctively different system structures, so that a new type of motion called Sliding Mode exists in a manifold (Young et al., 1999). A SMC system may be regarded as a combination of subsystems, each with fixed structure and each operating in a specified region of state space. With the help of SMC, it is possible to combine the useful properties of each of the structures. A SMC would then possess new properties not present in any of the subsystems used; e.g. an asymptotically stable system may consist of two structures neither of which is asymptotically stable (Utkin, 1977). This results in a system performance which is robust to parameter variations and disturbances.

The motion of a SMC system include two phases; the reaching phase and sliding phase. During the reaching phase, the system states move towards the switching plane. The robustness of the SMC can be improved by shortening the reaching phase or may be guaranteed during whole intervals of control action by eliminating the reaching phase (Ackermann and Utkin, 1998). The concept of SMC has been extended to discrete case in (Gao et al., 1995; Misawa, 1997a) and the \( \delta \)-operator has been introduced in (Middleton and Goodwin, 1990).

VSC systems are switching feedback control systems known to be insensitive to matched uncertainties (Drazenovic, 1969). The high frequency switching results in chattering phenomenon (Utkin and Lee, 2006), which is due to presence of finite time delays for control computation and limitations of physical elements like actuators. The chatter can be mitigated by using a smooth approximation of discontinuous function (Slotine and Sastry, 1983; Burton and Zinobar, 1986), and boundary layer control with continuous control inside the boundary.
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The conventional SMC also requires full state vector for implementation and as such, an observer to estimate the states becomes imperative. There has been considerable interest in developing robust observers for uncertain dynamical systems. The classical Luenberger observer fails when the output is sensed in presence of model uncertainties and/or sensor noise. As such, observer designs in the presence of disturbances, dynamic uncertainties and nonlinearities pose great challenges in practical applications.

Slotine designed observer using sliding surfaces and proved that sliding-mode observers offer advantages similar to sliding controllers (Slotine et al., 1987). A related approach applied to a more restricted class of problem is taken by Zak and Walcott. A lyapunov based approach is used for system with matched and bounded uncertainties (Walcott and Zak, 1988). An observer based on TDC is proposed in (Chang et al., 1997). A combined state and perturbation observer using the time delay observer (Kwon and Chung, 2003) and proportional integral observer (Chang, 2006) are available. A SMC design using state and extended disturbance observer (Ginoya et al., 2015a) for mismatched uncertain systems is an interesting addition to the literature.

The conventional SMC can be designed only if bounds of uncertainty and disturbances are known, however they are not easy to find. The bounds can be determined adaptively (Slotine and Coetsee, 1986; Yoo and Chung, 1992b), however it is limited to only structured uncertainties. An alternative approach could be estimation of uncertainty and disturbances, which relaxes the requirement of knowing the bounds. The UDE (Zhong and Rees, 2004) is one such technique that uses a low-pass filter of appropriate order and band-width for estimation. This method is based on an idea similar to the TDC (Yousef-Toumi and Ito, 1990), but does not require the derivative of system states and does not use time delayed signals. The SMC using UDE enforces sliding mode without using discontinuous control, and without requiring the knowledge of uncertainties or their bounds. The UDE has been applied to SMC of linear systems (Talole and Phadke, 2008), systems with state delays (Stobart et al., 2011; Kuperman and Zhong, 2011) and input-output linearization (Talole and Phadke, 2009). A variety of applications using UDE are reported as in (Phadke and Talole, 2012; Kolhe et al., 2013).
The problem of matching conditions can be solved for a certain class of systems using the back-stepping technique (Krstic et al., 1995). A technique called as multiple-sliding surface control (Huang and Chen, 2004) handles the problem in a similar way. The issue of mismatched uncertainty in nonlinear systems is also handled using robust estimation techniques like DO (Yang et al., 2011a, 2012a) and generalized ESO (Li et al., 2012). A SMC design employing DO to compensate mismatched uncertainties and its higher order derivatives is an interesting solution. The design uses a novel sliding surface, which includes the estimate of unmatched disturbances (Ginoya et al., 2014). The strategy substantially alleviates the problem of chatter in control; in addition to counteracting the effect of mismatched uncertainties. Several other techniques like adaptive control, dynamic sliding surface based control, and integral SMC are also available for compensating mismatched uncertainties.

The Equivalent Input Disturbance (EID) approach is an interesting way to tackle mismatched systems. An EID is a disturbance on the control input channel that produces the same effect on controlled output as actual disturbances do. An EID always exists for a controllable and observable plant with no zeros on imaginary axis (She et al., 2008). Several extensions of EID to MIMO (She and Xin, 2007), under-actuated (She et al., 2012), systems; as also applications in motion control, power systems (Hu et al., 2013), automotive control systems (She et al., 2007), mechatronics (She et al., 2011), etc have been reported. However the technique merits additional investigation especially in the context of higher-order filter. The body of work in the literature on EID, also does not include SMC based control, DO based estimation or uncertainty in plant parameters.

The robust sliding mode control (SMC) design augmented by estimates of uncertainty and disturbance can be employed in a variety of applications like motion control, robotics, magnetic levitation (Yang et al., 2011b; Lin et al., 2007b), active steering control (Ding and Taheri, 2010; Rajamani, 2011; Zhang and Wang, 2015), anti-lock braking systems (Mirzaeinejad and Mirzaei, 2010; Rajamani et al., 2012; Pasillas-Lépine et al., 2012).

The state-of-the-art gives an overview of the body of work in the literature and gives insights into what more needs to be done for interesting solutions.
1.5 Motivation

SMC is an effective strategy for controlling systems with significant uncertainties and unmeasurable disturbances. However there are certain issues, concerns and restrictions that merit attention.

- The control is discontinuous, which results in undesirable chatter. The chatter causes excessive wear and tear of actuators and may excite unmodeled dynamics.

- The sliding mode can be enforced only if bounds of uncertainty and disturbances are known. However, the bounds are not always easy to find. The rate of change of uncertainty is also a concern.

- The conventional SMC design has capabilities to compensate disturbances of only matched type, i.e. the disturbance entering the same channel as input. This poses severe limitations on the applicability of SMC.

- A majority of SMC strategies are based on state feedback. However, it is of common knowledge that though most practical systems are observable, all the system states are seldom measurable. Therefore, the sliding mode control algorithms may not be implementable in many cases. The problem is further compounded in the presence of uncertainties and disturbances.

- An accurate mathematical model may not be always available in case of most of the plants, i.e. uncertainty, un-modeled dynamics and external disturbances exist. As such, uncertainty estimation is imperative for disturbance rejection.

This work aims at designing robust sliding mode control strategy that mitigates the restrictions imposed in the existing design. The problem of nonlinear systems with matched as well as mismatched uncertainty is considered. The objective is to have simultaneous state and uncertainty estimation. The estimation methods used are UDE, DO and EID. The thesis also aims at validating the designs in motion and automotive control application.
1.6 Objectives

The central theme is to develop robust control algorithms for uncertain dynamical systems in varied domains.

- Design a boundary layer SMC law using UDE for mitigating chatter.
- Design a SMC law for nonlinear systems by estimating the states and
  - matched uncertainties using UDE.
  - matched and mismatched uncertainties using EID and filter.
  - matched and mismatched uncertainties using EID and DO.
- Design a discrete SMC algorithm with estimation of states and uncertainties.

1.7 Methodology

The core aspects of this work is divided into five distinct phases, which broadly encapsulate the stages of;

- Redefining the control problem
- Convergence and robustness analysis
- Stability analysis
- Performance evaluation
- Application validation

The objectives are realized using a approach comprising of,

- Theoretical Formulation
  Derivation of control law, Derivation of stability proof and stability analysis,
  Convergence and Robustness analysis
- Numerical Simulation
- Application Validation
1.8 Main Contributions

The algorithms proposed in this treatise may be summarized as,

- A new boundary layer sliding mode control strategy is designed for chatter reduction. The control scheme uses a discontinuous control outside the boundary layer and switches over to uncertainty and disturbance estimator (UDE) based control inside. The problem of large initial control underlying the method of UDE, is also addressed with a modified sliding surface. The overall stability of the system is proved and the results are verified on an illustrative example and application to flexible joint system. The results show that the proposed method exhibits much better control performance than the baseline SMC using ‘sat’ function, for reduced chattering.

- A robust sliding mode control (SMC) strategy for an uncertain nonlinear system subjected to time varying disturbance is proposed. The class of system considered includes state dependent nonlinearity in the input vector (in addition to the plant matrix). The control scheme uses uncertainty and disturbance estimator (UDE) to estimate the lumped uncertainty and the accuracy of estimation is improved with a higher-order filter. The control law is made implementable by estimating the states as well, to give a robust observer. The proposed control enforces sliding without using discontinuous control and without requiring the knowledge of uncertainties or their bounds. The overall boundedness is proved. The effectiveness of the proposed strategy is verified for model following and robust performance; and validated for an inverted pendulum system.

- A SMC strategy for nonlinear systems with mismatched uncertainties is proposed. A nonlinear model of the plant is considered with state-dependent uncertainty. The mismatched uncertainties are estimated using EID method. An observer is designed for estimation of EID. The effect of higher-order filter for improving the accuracy of estimation; by estimating disturbance and its derivatives is proved. The theory is generalized to a \( k^{th} \)-order filter. The conventional sliding surface is modified to improve system performance without causing a large increase in initial control; mitigating the effects of chatter.
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The overall stability is proved using Lyapunov theory. The results of designed strategy are verified by application to an anti-lock braking systems (ABS) control case-study. The ABS is considered to include actuator dynamics and the slip regulation is ensured for different friction models.

- A robust control strategy for systems with matched and mismatched uncertainties using EID approach is proposed. A nonlinear model of the plant is considered with state-dependent uncertainty. The SMC law is augmented with estimate of EID and the same is obtained using a DO. As earlier, the observed states are used for estimation. The design of extended DO to accommodate a large class of mismatched disturbances is also shown, which estimates disturbance and its derivatives. The overall stability is proved using Lyapunov theory. The results are verified by application to an active steering control problem.

- The concept of discrete sliding mode using $\delta$-operator is introduced. The $\delta$-operator is used to formulate an unified design thereby resolving the dichotomy between results in continuous and discrete time control law. The design is robustified by designing a discrete observer that estimates states, uncertainty and disturbance. The analysis with a 2nd order filter is shown to prove the improvement in estimation. The overall stability is proved in the usual way and the results are verified for an industrial motion case-study.

The core idea underlying all the aforementioned work is the estimation of states, uncertainty and disturbance for robust sliding mode control. The system considered are all nonlinear with matched as well as mismatched uncertainty. The various designs proposed, address the limitations of the conventional SMC design. The sliding variable ($\sigma$), estimation error ($\tilde{e}$) and state estimation error ($\tilde{x}$) are ultimately bounded in all cases in the sense of Corless and Leitmann (1981).

The simplicity of control design supplemented by estimation capability of methods like UDE, DO and EID makes this approach an attractive proposition. The applicability of designed control to a wide range of systems (as demonstrated in the work), leads the author to believe that uncertainty estimation based sliding mode control techniques shall be a major topic of interest in robust control.
1.9 Organization of Thesis

The thesis is divided into seven chapters. Chapter 1 gives a road-map of the thesis and answers the fundamental questions of what, why and how. A brief review of literature is also included. The issues and concerns in conventional SMC are then addressed in successive chapters.

Chapter 2 deals with the design and validation of boundary-layer SMC law using UDE, for chatter mitigation. The UDE is further used to synthesize a observer-controller structure for nonlinear uncertain systems in chapter 3. The class of system considered includes state dependent nonlinearity in the input vector as well as plant matrix and also external disturbance. Chapter 4 is concerned with SMC of mismatched uncertain systems using EID method. The effect of higher-order filter for improving the accuracy of estimation is proved.

The work is further extended for robust control of systems with matched and mismatched uncertainties in Chapter 5. The EID is estimated using DO and this estimate is supplemented in the control law. The estimation is generalized to a $n^{th}$-order DO. The results of both these EID approaches are verified for automotive applications. Chapter 6 introduces the concept of discrete sliding mode control using $\delta$-operator. The control law is made implementable by designing a discrete observer that estimates states, uncertainty and disturbance.

Throughout chapter 2 to 6, some essential results of other researchers are briefly introduced, discussed and referenced. The conclusions and recommendations for future work are presented in the final chapter of the thesis.