CHAPTER-1

Introduction

1.1 Concept of Magnetohydrodynamics and its Applications

Magnetohydrodynamics is the union of three widely separated disciplines, namely, electrodynamics, fluid dynamics and thermodynamics. It deals with the flow of electrically conducting fluids, namely, plasmas, liquid metals, salt water or electrolytes etc., in the presence of electric and magnetic fields. Magnetohydrodynamic phenomena are outcome of mutual interaction between magnetic field and electrically conducting fluid flowing across it i.e. electric current is generated when an electrically conducting fluid flows in the presence of a magnetic field and the resulting current and magnetic field together produce a force, called Lorentz force, which resists fluid motion. The electric current also generates its own magnetic field, called induced magnetic field, which perturbs the original magnetic field. An opposing or pumping force on the fluid can be produced by applying an electric field perpendicular to the magnetic field. Science of Magnetohydrodynamics i.e. MHD is the detailed studies of the phenomena, which occur in nature and are also produced in engineering devices in the presence of electric and magnetic fields. Geophysicists encounter MHD phenomena in the interaction of conducting fluids and magnetic fields that are present in and around heavenly bodies. It is useful in astrophysics because much of the universe is filled with spaced charged particles which are permeated by magnetic fields. Engineers employ MHD principles in designing of heat exchangers, pumps, generators, accelerators, flow-meters, communications and radar systems; in solving space vehicle
propulsion, control and re-entry problems and in developing confinement schemes for controlled fusion.

Magnetic fields pervade interstellar space and aid the formation of stars by removing excess angular momentum from collapsing interstellar clouds. Sun spots and solar flares are magnetic in origin, Sun spots being caused by buoyant magnetic flux tubes, perhaps $10^4$ Km in diameter and $10^5$ Km long, erupting from the surface of the Sun. The terrestrial magnetic field is now known to be maintained by fluid motion in the core of the Earth. MHD is also intrinsic part of controlled thermo-nuclear fusion. Here plasma temperatures of around $10^8$ K must be maintained and magnetic forces are used to confine the hot plasma away from the reactor walls. In the metallurgical industries, magnetic fields are routinely used to heat, pump, stir and levitate liquid metals. Perhaps the earliest application of MHD is the electromagnetic pumping. The electromagnetic pump has found its ideal application in fast breeding nuclear reactors where it is used to pump liquid sodium coolant through the reactor core. Perhaps the most widespread application of MHD in engineering is the use of electromagnetic stirring. This is routinely used in casting operations to homogenize the liquid zone of a partially solidified ingot. The resulting motion has a profound influence on the solidification process, ensuring good mixing of alloying elements and the continual fragmentation of the snow flake-like crystals which form in the melt. The result is a fine structured homogenous ingot. In other casting operations magnetic fields are used to dampen the motion of the liquid metal. The use of magnetic damping promotes a more quiescent process, thus minimizing contaminations. Another common application of MHD in metallurgy is magnetic levitation or confinement. There are many other applications of MHD in engineering and metallurgy. These include, electromagnetic (non-contact) casting of aluminium, vacuum-arc re-melting titanium and nickel-based super alloys, electromagnetic removal of non-metallic inclusions from melts, electromagnetic launchers and the so called cold-crucible induction melting process, in which the melt is protected from the crucible walls by thin solid crust of its own material. This latter technology is currently finding favor in the nuclear industry where it is used to vitrify highly active nuclear waste. It seems that MHD has now found a substantial and permanent place in the world of material processing.
1.2 Dynamics of Rotating Fluids and its Applications

Investigation of theory of rotating fluids (Greenspan, 1969) is of considerable significance because of the occurrence of various natural phenomena due to rotating fluids and for its applications in various technological situations which are governed by the action of Coriolis force. Oceanography, meteorology, atmospheric science and limnology all contain some important and essential features of rotating fluids. Large scale circulation in the atmosphere and oceans, construction of turbines and other centrifugal machines are some of the areas of application of rotating fluids. Rotation of a fluid generates two forces, namely, Coriolis and centrifugal forces on the fluid particles. The balance between Coriolis force and the pressure gradient including centrifugal force with the correction for viscous action at the boundaries emerges as the backbone of the entire theory of rotating flows. Rotating fluids have an intrinsic stability in the sense that if a fluid particle is displaced from its equilibrium position of rigid body rotation, Coriolis force acts as a restoring force. If a fluid particle is displaced from its equilibrium radius, it will oscillate with twice the angular velocity of rotation about its equilibrium position. This frequency of oscillations in rotating fluid is called inertial frequency.

The study of Magnetohydrodynamics of rotating fluids is motivated by several important problems, namely, maintenance and secular variations of the Earth’s magnetic field, internal rotation rate of the Sun, planetary and solar dynamo problems, structure of rotating magnetic stars and centrifugal machines viz. rotating hydromagnetic generator (Yantovisky and Tolmach, 1963), rotating-drum separators for liquid-metal MHD applications (Lenzo et al., 1978) etc. Hydromagnetic flow in the Earth’s liquid core is somehow responsible for the main geomagnetic field associated with the magnetohydrodynamic dynamo (Hide and Roberts, 1960a). This toroidal magnetic field is considered to be induced from the main dipole field by rotation of the fluid in the Earth’s liquid core. Geomagnetic data reveal that in the Earth’s liquid core, Coriolis force is much stronger than inertia and viscous forces and is comparable with the hydromagnetic force in strength. The coupling between Coriolis and hydromagnetic forces gives rise to the possibility of attributing irregular changes in the length of a day to the rotational
momentum transfer between core and mantle of the Earth. It is generally accepted that a number of astronomical bodies (i.e. Sun, Earth, Jupiter, magnetic stars, pulsars) possess fluid interior and magnetic fields. Changes in the rotation rate of such subjects suggest the possible importance of hydromagnetic spin-up which is one of the several problems of Magnetohydrodynamics of rotating fluids. Continuous emission of matter by the Sun, so called solar wind, has important implications for the history of solar rotation, since the wind carries away a non-negligible amount of angular momentum (Dicke, 1964).

1.3 Hall Current

It is noticed that in an ionized fluid where the density is low and/or if a very strong magnetic field is present so that the cyclotron frequency \( \omega_c = eB_0 / m_e \) (\( e \) and \( m_e \) are charge and mass of the electron respectively and \( B_0 \) is magnetic flux density) exceeds collision frequency, charged particle can gyrate round the lines of force several times before suffering collisions with other particles. This results in a drift of charged particles in a direction perpendicular to the directions of electric and magnetic fields. Thus if an electric field is applied at right angle to the magnetic field, total current will not flow along electric field. This tendency of electric current to flow across an electric field in the presence of a magnetic field is called Hall effect and resulting current is known as Hall current, which produces electrical conductivity normal to the lines of force so that electrical conductivity becomes anisotropic. Hall effect was discovered by Hall (1879). A comprehensive discussion on Hall effects is given by Cowling (1957) in his monograph. It plays a vital role in determining flow features of fluid flow problems. Hall effects is likely to be important in MHD power generation, nuclear power reactors, underground energy storage system, Hall current accelerator, magnetometers, Hall effect sensors, spacecraft propulsion etc. and in several areas of astrophysics and geophysics.

1.4 Convective Heat Transfer

Investigation of the problems of heat transfer is important due to its varied and wide applications in the problems of science and technology, namely, in designing of power
stations, chemical and food plants, aerodynamic heating, cooling of high power motors, extraction of energy from atomic piles, high speed aircraft, atmospheric re-entry of vehicles, utilization of heat stored in subterranean layer of the Earth, heat exchangers using liquid metal coolant etc.

Convection is a mode of heat transfer in fluids, wherein the moving fluid particles carry heat in the form of internal energy. Convection occurs in a large scale in the oceans, atmospheres and planetary mantles and it provides the mechanism of heat transfer for a large fraction of the outermost interiors of the Sun and all stars. Fluid movement during convection may be invariably slow, or it may be rapid, as in a hurricane. On astronomical scales, convection of gas and dust is thought to occur in the accumulation disks of black holes; at speeds which may closely approach that of light. Convection arises due to body force acting within the fluid, e.g. gravity (buoyancy) or surface forces acting at the boundary of the fluid. Convective heat transfer is of two type viz. (i) forced convection and (ii) free convection. A convection process, when the motion is created by external influences such as pressure drop or an agitator, is known as forced convection. In incompressible fluids such flows are characterized by the fact that the distribution of velocity is not affected by temperature field but the converse is not true. In such flows, heat diffuses and at the same time is swept away by the fluid motion without in any way affecting the local density of the fluid. Hence in forced convection flow velocities are exactly as they would be if there are no temperature variations so that the parts of motion arising from the differences caused by thermal expansion can be ignored. Forced convection is typically used to increase the rate of heat exchange. Many types of mixing also utilize forced convection to distribute one substance within another. Forced convection also occurs as a byproduct to other processes, such as the action of a propeller in a fluid or aerodynamic heating. Fluid radiator systems and also heating and cooling of parts of the body by blood circulation are other familiar examples of forced convection.

On the other hand the essential feature of a free convection flow is that the distributions of velocity and temperature field are coupled. The motion here is caused entirely by the buoyancy forces arising from density variations in a field of gravity and the distribution of
density changes takes palace as soon as motion starts. Thus we find that in such flows the
distribution of velocity and temperature are interconnected and must be considered
together. If the fluid is incompressible then the density variation due to changes in pressure
are negligible. However, density changes due to non-uniform heating of the fluid cannot be
neglected since such changes are responsible for imitating free convection. It is widely
accepted that the free convection takes place in the field of gravity. In a rotating fluid it can
be also set up by the action of centrifugal force which is proportional to the density of fluid.
Flow and heat transfer in gas turbines is an example of such situation.

In case of mixed convection (free and forced occurring together) one would often like to
know how much of the convection is due to external constraints, such as the fluid velocity,
and how much due to free convection occurring in the system. The relative magnitudes of
thermal buoyancy force and acceleration force determine which form of convection
dominate. If thermal buoyancy force is much stronger than acceleration force then forced
convection may be neglected whereas if acceleration force is much stronger than thermal
buoyancy force then free convection may be neglected. If their relative magnitude is of
order one then both the free and forced convection need to be taken into account.

The subject designated as magnetohydrodynamic heat transfer can be roughly divided
into two parts viz. one in which heating is an incidental by-product of the electromagnetic
fields. This part includes devices like MHD generators or accelerators and to a lesser
degree pumps and flow-meters. These are broadly specified as channel and duct flows,
although most of the operating designs consist of variable area duct. The other one in which
the primary use of electromagnetic fields is to control heat transfer. This part includes free
or natural convection flow where geometric configurations are varied. A comprehensive
review on these basic areas is well documented by Romig (1962, 1964) and Moffatt (1964).
An interesting feature of magnetohydrodynamic heat transfer problems is that the usual
Reynolds analogy between skin friction and heat transfer, as in non-conducting fluids, does
not, in general, hold good. This is because, in addition to viscous dissipation, there is a
Joule dissipation of heat due to flow of electric current in the fluid.
1.5 Basic Equations of Magnetohydrodynamics in a Rotating Frame

The combination of Navier-Stokes equation of fluid dynamics and Maxwell’s relations of electromagnetism describes magnetohydrodynamic flow in a rotating frame. As Maxwell’s relations define the property of electric and magnetic fields, the fundamental laws of electrodynamics are governed by these relations. The electromagnetic equations remain as it is but the Ohm’s law (which relates the electric current to induced voltage) is to be modified by including induced current. The effects of rotation, electric and magnetic fields have to be taken into account to modify the basic laws of fluid dynamics comprising of conservation of mass, momentum and energy along with the thermal and caloric equations of state. Thermodynamic property of an electrically conducting fluid remains same as that of non-conducting fluid if we consider it as non-magnetic and neglect the phenomena like electrostriction. In Magnetohydrodynamics, when fluid velocity is small compared to the velocity of light, the displacement current is negligible. Also fluids are almost neutral, charge density of fluid and convection current being very small compared to conduction current, can be neglected. We mention below the basic equations of Magnetohydrodynamics for the flow of a homogenous, isotropic, electrically conducting, viscous and incompressible fluid of constant density \( \rho \), constant electrical conductivity \( \sigma \) and constant kinematic coefficient of viscosity \( \nu \) in a rotating frame which rotates with uniform angular velocity \( \Omega \) relative to an inertial frame.

The equation of continuity

$$\nabla \cdot \vec{q} = 0, \quad (1.1)$$

The momentum equation (equation of motion)

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} + 2\Omega \hat{k} \times \vec{q} = -\frac{1}{\rho} \nabla p^* + \phi + \nu \nabla^2 \vec{q} + \frac{1}{\rho} \left( \vec{J} \times \vec{B} \right), \quad (1.2)$$

The energy equation

$$\rho C_p \left[ \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T \right] = k \nabla^2 T + \mu \Phi + \frac{1}{\sigma} J^2, \quad (1.3)$$
Maxwell’s relations

$$\nabla \times \vec{B} = \mu_0 \vec{J},$$  \hspace{1cm} (1.4)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t},$$  \hspace{1cm} (1.5)

$$\nabla \cdot \vec{B} = 0,$$  \hspace{1cm} (1.6)

The constitutive field relation

$$\vec{B} = \mu_0 \vec{H},$$  \hspace{1cm} (1.7)

Ohm’s law for a moving conductor

$$\vec{J} = \sigma \left[ \vec{E} + \vec{q} \times \vec{B} \right],$$  \hspace{1cm} (1.8)

and Ohm’s law for a moving conductor taking Hall current into account

$$\vec{J} + \frac{\alpha_e}{B_0} (\vec{J} \times \vec{B}) = \sigma \left[ \vec{E} + \vec{q} \times \vec{B} \right],$$  \hspace{1cm} (1.9)

where $\vec{q}$, $\vec{B}$, $\vec{J}$, $\vec{E}$, $\vec{H}$, $\phi$, $\Phi$, $T$, $\hat{k}$, $k$, $\mu$, $C_p$, $\mu_e$, $\tau_e$, $\omega_e$, $t$ and $p^*$ are, respectively, fluid velocity, magnetic induction vector, current density, electric field, magnetic field, body force per unit mass including gravitational effects, viscous dissipation term, fluid temperature, unit vector along the axis of rotation, thermal conductivity, coefficient of viscosity, specific heat at constant pressure, magnetic permeability, electron collision time, cyclotron frequency, time and modified pressure including centrifugal force $-(1/2) \left| \vec{\Omega} \times \vec{r} \right|^2$, $\vec{r}$ being position vector from the origin to a general point at which the Eulerian fluid velocity $\vec{q}$ is measured relative to the rotating frame (Kibble, 1966).

It is appropriate to mention here that in a rotating frame, magnetic fields are same as in the inertial frame of reference i.e. magnetic fields are free of the coordinate systems while electric fields depend on the coordinate system from which they are measured. Alfvén and Fälthammar (1963) have shown that if fluid velocity is smaller in comparison to the velocity of light, magnetic field is independent of the choice of coordinate system i.e.
magnetic field is same in moving system and the system at rest while if electric field is \( \vec{E}_{\text{inert}} \) in a system at rest then in a system which moves relative to the first with velocity \( \vec{q} \), electric field becomes \( \vec{E} = \vec{E}_{\text{inert}} + \vec{q} \times \vec{B} \).

Eliminating \( \vec{E} \) and \( \vec{J} \) from the equations (1.4), (1.5) and (1.8), we obtain equation of magnetic induction which is given by

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{q} \times \vec{B}) + v_m \nabla^2 \vec{B},
\]

(1.10)

where \( v_m = 1/\sigma \mu_e \) is magnetic viscosity or diffusivity.

Induction equation for magnetic field taking Hall current into account is obtained by eliminating \( \vec{E} \) and \( \vec{J} \) from the equations (1.4), (1.5) and (1.9), which is given by

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{q} \times \vec{B}) + v_m \nabla^2 \vec{B} - \frac{\omega_e \tau_e}{v_m B_0} \left[ \left( \vec{B} \cdot \nabla \right) \left( \vec{B} \times \nabla \right) - \left( \nabla \times \vec{B} \right) \vec{B} \right].
\]

(1.11)

### 1.6 Boundary Conditions

Fluid velocity, fluid temperature and magnetic field are to be determined by solving basic magnetohydrodynamic equations mentioned in Section 1.5 under appropriate boundary conditions for the fluid velocity, fluid temperature and magnetic field.

Boundary conditions to be satisfied are

(i) Boundary condition for the fluid velocity at a surface is no-slip condition i.e. the tangential component of velocity is zero. Since most surfaces are not penetrated by the fluid, then the normal component of fluid velocity is also zero and for porous surfaces it is non-zero.

(ii) Thermal boundary condition is continuity of temperature field i.e. either fluid has the same temperature as the wall which is prescribed or heat flow through the surface is prescribed.

(iii) Boundary conditions for magnetic field.
Maxwell’s relations (1.4) and (1.5) are valid only for those points in whose neighborhood physical properties of the medium vary continuously. On the boundary of flow-field, the physical properties of medium may exhibit discontinuities on the boundary e.g. if the fluid is in contact with a solid boundary physical properties of the fluid will get changed abruptly to those of the solid boundary. Across such a surface of discontinuity, following magnetic boundary conditions hold (Pai, 1962a).

(A) Transition of normal component of magnetic induction vector $\vec{B}$ is continuous i.e.

$$\bar{n}.(\vec{B}_1 - \vec{B}_2) = 0,$$

where $\bar{n}$ is unit normal vector to the surface of discontinuity. Subscripts 1 and 2 refer to the values immediately on each side of the surface.

(B) Behavior of magnetic field $\vec{H}$ (tangential component) at this boundary is

$$\bar{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s,$$

where $\vec{J}_s$ is the surface current density. For finite electrical conductivity, $\vec{J}_s$ is zero, whereas for infinite electrical conductivity, $\vec{J}_s$ may be different from zero.

In most of the problems of Magnetohydrodynamics we neglect surface current density $\vec{J}_s$. Hence boundary conditions for magnetic field are that the tangential components of $\vec{H}$ and normal components of $\vec{B}$ are all continuous across a surface separating a body and a fluid or two fluids.

### 1.7 Boundary Layers

Boundary layers arise adjacent to a bounding surface when the influence of a physical quantity is restricted to small regions near confining boundaries. This phenomenon comes into existence when the non-dimensional diffusion parameters, namely, rotation parameter (reciprocal of Ekman number), magnetic parameter (square of Hartmann number), frequency parameter etc. are large.

When a vast amount of viscous and incompressible fluid bounded by a rigid surface is rotating rapidly then there appears a thin boundary layer near the bounding surface based
on the balance between Coriolis and viscous forces. This boundary layer, called Ekman boundary layer, was first noticed by Ekman (1905) in his study of wind stress on the surface of ocean. It plays a vital role in determining flow features of various problems of astrophysical and geophysical interest and fluid engineering (Greenspan and Howard, 1963).

When a viscous, incompressible and electrically conducting fluid flows past a rigid surface in the presence of an applied magnetic field and magnetic force is stronger than the viscous force, there arises a thin boundary layer, called Hartmann boundary layer, near the rigid boundary. This boundary layer was first observed by Hartmann (1937) while studying the flow of a viscous, incompressible and electrically conducting fluid within a channel in the presence of a uniform transverse magnetic field.

When combined effects of rotation and magnetic field on the flow of viscous, incompressible and electrically conducting fluid bounded by a rigid surface are considered and if Coriolis force is stronger than the magnetic and viscous forces then a thin boundary layer appears adjacent to the rigid boundary based on the balance between these forces. This boundary layer is known as modified Ekman boundary layer (Nanda and Mohanty, 1971). This layer can be viewed as classical Ekman boundary layer modified by magnetic field. In case, when the magnetic field is stronger than the other forces then there appears a Hartmann boundary layer near the rigid body unaffected by rotation (Nanda and Mohanty, 1971). When fluid is subjected to an oscillatory disturbance and if the Coriolis force is stronger than the other forces then in place of one boundary layer, as in the steady case, there arise multiple boundary layers near the rigid boundary, which are known as modified Ekman layers (Seth and Jana, 1980). These layers can be viewed as classical Ekman layers modified by oscillations and magnetic field. Also when the oscillations are predominant than the other forces there appear multiple boundary layers near the rigid boundary, known as modified hydromagnetic Stokes layers (Debnath, 1973, 1974, 1975; Seth and Jana, 1980) which can be considered as classical Stokes layers modified by rotation and magnetic field. However, when the magnetic field is stronger than the other forces then there appears a Hartmann boundary layer near the bounding surface (Seth and Jana, 1980).
In steady flow, when effects of Hall current is taken into account and magnetic force is stronger than other forces then there arises modified Hartmann boundary layer near the boundary surfaces (Seth and Ansari, 2009) which can be viewed as classical Hartmann boundary layer modified by Hall current. In case of unsteady flow, induced due to impulsive or accelerated movement of the rigid surface or free stream there arises Rayleigh boundary layer near the rigid surface (Seth et al., 2010a,b).

1.8 Literature Survey

This section is devoted to a brief literature review of the earlier investigations made on the Magnetohydrodynamic (MHD) channel flow and fluid flow past a flat plate with or without rotation which would provide background of the research study presented in the thesis. The problems of MHD channel flows are considerable importance due to its varied and wide applications in MHD pumps, MHD generators, MHD accelerators, MHD flow-meters and nuclear reactors and several other MHD devices. Such type of flows exhibit one of the basic phenomena of MHD viz. tendency of magnetic field to suppress vorticity perpendicular to itself against tendency of viscosity to promote vorticity.

Hydromagnetic channel or duct flow, in which fluid flow is induced due to movement of one of the plates of the channel, is called MHD Couette flow. MHD Couette flow is investigated by a number of researchers due its varied and wide applications in the areas of geophysics, astrophysics and fluid engineering. The theory of Couette flow can be used for the measurement of viscosity and estimating drag forces in many wall driven applications (Muzychka and Yovanovich, 2006).

Unsteady MHD channel or duct flow has been an active topic of research because of its varied and wide applications in cosmological and geophysical fluid dynamics and fluid engineering. Such flows have also significant practical applications in plasma aerodynamics, energy systems, nuclear engineering control and mechanical engineering manufacturing process. Investigations of the problems of unsteady MHD channel or duct flow is important from practical point of view because fluid transients may be expected at the start-up time of so many MHD devices, namely, MHD energy generators, MHD pumps, induction type pumps used in nuclear reactors, MHD accelerators, MHD flow-meters etc.
Recently several researchers investigated unsteady channel or duct flows of a viscous and incompressible fluid with or without magnetic field considering different aspects of the problem. Mention may be made of the interesting research studies of Bég et al. (2009), Vieru and Siddique (2010), Chinyoka and Makinde (2010, 2011) and Makinde and Chinyoka (2011). Tao (1960) and Katagiri (1962) investigated unsteady hydromagnetic Couette flow of a viscous, incompressible and electrically conducting fluid under the influence of a uniform transverse magnetic field when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel. Muhuri (1963) considered this fluid flow problem within a porous channel when fluid flow within the channel is induced due to uniformly accelerated movement of one of the plates of the channel. Soundalgekar (1967) investigated unsteady MHD Couette flow of a viscous, incompressible and electrically conducting fluid near an accelerated plate of the channel under transverse magnetic field. The effect of induced magnetic field on the problem, studied by Muhuri (1963), was analyzed by Govindrajulu (1969). Mishra and Muduli (1980) discussed this problem when one of the plates starts moving with a time dependent velocity which is proportional to $t^n e^{at}$ ($t > 0$, $n$ and $a$ are positive integers). In the above mentioned investigations, magnetic field is fixed relative to the fluid. Singh and Kumar (1983) studied MHD Couette flow of a viscous, incompressible and electrically conducting fluid in the presence of a uniform transverse magnetic field when fluid flow within the channel is induced due to time dependent movement of one of the plates of the channel and magnetic field is fixed relative to moving plate. They considered two particular cases of interest in their study viz. (i) impulsive movement of one of the plates of the channel and (ii) uniformly accelerated movement of one of the plates of the channel. It was found that the magnetic field tends to accelerate fluid velocity in both the cases. Vajravelu (1988) investigated both hydrodynamic and hydromagnetic unsteady Couette flow of incompressible and viscous fluid within a channel induced due to oscillations of one of the plates in its own plane about a constant mean velocity. Contribution on the investigations of unsteady MHD Couette flow are also due Datta (1963), Rathy (1963), Charles and Smith (1970), Mathur et al. (1972), Balram and Govindrajulu (1973) and Seth et al. (2011c).
It is well known that Hall effects become significant when the density of the fluid is low and/or applied magnetic field is strong. It plays a vital role in determining flow features of the fluid flow problems. Hall effects on fluid flow find applications in MHD power generation, MHD accelerators, nuclear power reactors and underground energy systems and in several areas of astrophysics and geophysics. Hall effects on steady Hartmann flow is studied by Sato (1961), Kusukawa (1962), Yamanishi (1962) and Sherman and Sutton (1962) whereas Gubanov and Lunkin (1961) investigated effects of Hall current on MHD Couette flow problem. Unsteady MHD Couette flow with Hall effects was considered by Jana and Datta (1977) when velocity of the moving plate varies with time as \( t^n \) (\( t \) and \( n \) being time and positive integer respectively). They have considered two particular cases of interest viz. (i) impulsive movement of the plate i.e. \( n = 0 \) and (ii) uniformly accelerated movement of the plate i.e. \( n = 1 \). They obtained velocity distribution and shear stress at the moving plate neglecting induced magnetic field. Attia (2007) investigated Hall effects on unsteady MHD generalized Couette flow in the presence of exponentially decaying pressure gradient.

Heat transfer characteristics of MHD channel or duct flows are investigated by several researchers during past decades due to its applications in MHD generators, accelerators, pumps, flow meters, nuclear reactors etc. Siegal (1958), Alpher (1961), Sherman and Sutton (1961), Pai (1962b), Viskanta (1963), Yen (1963), Erickson et al. (1964), Jagdeesan (1964), Snyder (1964) and Shrestha (1968) studied hydromagnetic channel or duct flow with heat transfer due to forced convection under different conditions while Poots (1961), Osterle and Young (1961) and Singer (1966) investigated free convection MHD flow within a parallel plate vertical channel considering deferent aspects of the problem. Recently Jha and Apere (2011a) considered unsteady MHD free convection Couette flow of a viscous, incompressible and electrically conducting fluid between two parallel vertical porous plates. They considered two cases of interest viz. (i) when applied magnetic field is fixed relative to the fluid and (ii) when applied magnetic field is fixed relative to the moving porous plate. Gill and Casal (1962), Gupta (1969), Das (1969), Yu (1970), Jana (1975), Ghosh et al. (2002) and Guria et al. (2007) studied combined free and forced
convection MHD channel or duct flows under different conditions. Hall effects on MHD free and forced convection channel flow is discussed by Mazumder et al. (1976), Datta and Jana (1977a), Kant (1980) and Seth and Ghosh (1987) considering different aspects of the fluid flow problem. Singha (2008) studied heat transfer aspects of Couette flow of a viscous, incompressible and electrically conducting fluid under the influence of a transverse magnetic field when (i) the two plates are kept at different but constant temperature and (ii) the upper plate is given to move with constant velocity while the lower plate is adiabatic. Contributions on MHD channel flow are also due to Terril and Shrestha (1965), Khozhainov (1970), Mitra and Bhattacharya (1981), Bhaskara Reddy and Bathaiah (1982), Attia (2009), Chinyoka and Makinde (2012) and Adesanya and Makinde (2012).

Problems of hydromagnetic flow of an electrically conducting fluid in a rotating channel are investigated by several researchers during past decades because such fluid flow problems may find applications in several areas of astrophysics, geophysics and fluid engineering, namely, maintenance and secular variations of terrestrial magnetic field due to motion of Earth’s liquid core, internal rotation rate of Sun, planetary and solar dynamo problems, structure of rotating magnetic stars, MHD Ekman pumping, rotating MHD generators, rotating drum type separators in closed cycle two phase MHD generator flow etc. Vidyanidhi (1969), Nanda and Mohanty (1971) and Mazumder (1977) studied steady Hartmann flow of a viscous, incompressible and electrically conducting fluid within a parallel plate rotating channel with non-conducting, perfectly conducting and arbitrary conducting walls respectively. Datta and Jana (1977b) analyzed Hall effects on the problem studied by Vidyanidhi (1969). Prasad and Raman Rao (1978) considered hydromagnetic flow between two parallel porous plates in a rotating system. Raman Rao and Linga Raju (1990) extended this problem by considering Hall effects. For the purpose of simplicity they assumed that Prandtl number is negligible. Seth and Ghosh (1999) considered effects of Hall current on the problem studied by Nanda and Mohanty (1971) whereas Seth and Ahmad (1999) discussed the effects of Hall current on the problem studied by Mazumder (1977). Linga Raju and Murthy (2005) obtained quasi-steady state solution of unsteady MHD ionized flow within a porous rotating channel under the influence of a uniform transverse magnetic field taking Hall current into account. Linga Raju and Murthy (2006)
also considered hydromagnetic two phase flow and heat transfer within a parallel plate rotating channel neglecting induced magnetic field.

Bhat (1982, 1983) considered heat transfer characteristics due to forced convection of the problem studied by Nanda and Mohanty (1971) and Vidynidhi (1969). Mazumder (1977) also considered heat transfer characteristics of flow due to forced convection. Soundalgekar and Bhat (1992) considered MHD flow and heat transfer of a rarefied gas in a rotating channel with finitely conducting walls. Jana et al. (1977) investigated steady hydromagnetic Couette flow and heat transfer in a rotating system considering stationary plate perfectly conducting and moving plate non-conducting. They found that the rate of heat transfer at the stationary plate is unaffected by magnetic field and rotation. Seth and Maitti (1982) discussed the same problem considering both plates of the channel as non-conducting. They found that induced electric field has significant effect on the flow-field and heat transfer characteristics. Jana and Datta (1980), Mandal et al. (1982), Mandal and Mandal (1983) and Seth and Ahmad (1985) studied effects of Hall current on steady MHD Couette flow in a rotating system under different conditions. Seth and Singh (2011) and Seth et al. (2011a) discussed MHD Couette flow of class-II with heat transfer due to forced convection in a rotating system under different conditions. Nagy and Demeny (1995) investigated Hall effects on Couette-Hartmann flow in a rotating channel under general wall conditions. In their problem, flow within the channel is induced due to movement of the upper plate as well as applied pressure gradient along the channel walls. They also considered heat transfer characteristics of the flow due to forced convection. Ghosh et al. (2009) studied effects of Hall current on hydromagnetic flow in a rotating channel with perfectly conducting walls. Heat transfer characteristics of this problem due to forced convection is also studied. Seth and Banerjee (1996) investigated combined effects of free and forced convection flow of a viscous, incompressible and electrically conducting fluid in a rotating channel with non-conducting walls taking induced magnetic field into account. Ghosh and Bhattachrjee (2000) and Seth and Singh (2008) studied the problem considered by Seth and Banerjee (1996) when the walls of the channel are perfectly conducting. Seth et al. (2010c) investigated combined free and forced convection flow in a rotating channel with arbitrary conducting walls. Seth and Ansari (2009) studied effects of Hall current on
the problem considered by Seth and Singh (2008). Prasada Rao and Krishna (1982) analyzed the effects of Hall current on steady free and forced convection flow in a rotating channel neglecting induced magnetic field. Seth and Jana (1980) investigated oscillatory Hartmann flow in a rotating system neglecting induced magnetic field. Flow within the channel is induced due to applied pressure gradient which oscillates harmonically in time. Subsequently Ghosh (1993), Seth et al. (2011d) and Ansari et al. (2011) extended this problem taking induced magnetic field into account by considering walls of the channel as non-conducting, perfectly conducting and finitely conducting respectively. Seth et al. (1988) investigated oscillatory MHD Couette flow in a rotating system neglecting induced magnetic field when the fluid motion is induced due the harmonic oscillations of the upper plate. Singh (2000) considered oscillatory hydromagnetic Couette flow of class-II of an electrically conducting fluid between two parallel plates in a rotating system, in which one of the plates is at rest and the other is oscillating in its own plane. He found that even the claim of Ganapathy (1994) that the solution of Mazumder (1991), which explains the important phenomenon of resonance in a rotating system, is incorrect. Seth and Mahto (1986) considered the oscillatory Couette-Hartmann flow in a rotating channel neglecting induced magnetic field. Hayat et al. (2004a) investigated MHD Couette flow of an incompressible and electrically conducting Oldroyd-B fluid in a rotating system. Hayat et al. (2004b) also considered this problem for the flow of second grade (non-Newtonian) fluid. Mukherjee and Debnath (1977) studied unsteady hydrodynamic and hydromagnetic boundary layer flow within a parallel plate channel with porous walls in a rotating system when the fluid flow is induced due to small amplitude non-torsional oscillations of the walls with the given frequency at time $t = 0$. Seth et al. (1982) investigated unsteady MHD Couette flow in a rotating system when the lower plate is set into motion with the time dependent velocity in its own plane and magnetic field is fixed relative to the fluid. Two particular cases of interest of the general solution, namely, (i) impulsive start of the plate and (ii) accelerated start of the plate, are considered. Chandran et al. (1993) studied unsteady MHD Couette flow in a rotating system when magnetic field is fixed relative to moving plate. Flow within the channel is induced due to impulsive movement of one of the plates of the channel. Subsequently Singh et al. (1994) considered this problem when the
fluid flow within the channel is induced due to uniformly accelerated movement of one of the plates of the channel and magnetic field is fixed relative to moving plate. Das et al. (2009a) considered the effects of rotation on unsteady Couette flow in a rotating channel in the presence of a uniform transverse magnetic field when the fluid flow is induced due to impulsive movement of one of the plates of the channel. They found general solution (Batchelor, 1967) for velocity field which converges slowly for small values of time \( t \). Due to this reason they also obtained solution for small values of time \( t \) by Laplace transform technique. Subsequently Seth et al. (2011b) considered the problem studied by Das et al. (2009a) within porous walls. Seth et al. (2010a) investigated unsteady hydromagnetic Couette flow within porous plates in a rotating system when magnetic field is fixed relative to the moving plate. Fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel. Seth et al. (2010b) also considered unsteady hydromagnetic Couette flow induced due to accelerated movement of one of the porous plates of the channel in rotating system when magnetic field is fixed relative to the moving plate. Contribution on the topic is also due to Sacheti and Singh (1992), Nagy and Demendy (1993), Singh and Mathew (2008), Seth et al. (2009), Sibanda and Makinde (2010), Jha and Apere (2010, 2011b), Bég et al. (2010), Pop et al. (2001), Ghosh and Pop (2006), Prasad et al. (2008), Singh and Kumar (2009), Singh and Pathak (2012) and Chauhan and Agrawal (2012).

Free/natural convection flow is frequently encountered in science and engineering problems viz. chemical catalytic reactors, petroleum reservoirs, geothermal systems, nuclear waste repositories, fiber and granular insulation etc. Unsteady free convection flow past an infinite or semi-infinite vertical oscillating plate is investigated by a number of researchers considering different aspects of the problem. Mention may be made of research studies of Soundalgekar (1973, 1979), Turbatu et al. (1998), Revankar (2000), Gupta et al. (2003), Toki and Tokis (2007), Toki (2008, 2009) and Makinde and Olanrewaju (2010). Free convection flow is extensively studied as it is evident from the several review articles and books published (Ede, 1967; Gebhart, 1973; Gebhart et al., 1989; Jaluria, 1980; Raithby and Hollands, 1985). Free convection flow in the presence of magnetic field is of much significance in liquid metals, electrolytes and ionized gases. Free convection flow
over a semi-infinite vertical plate in the presence of magnetic field is investigated by several researchers considering different aspects of the problem. Mention may be made of research studies of Gupta (1960, 1962), Osterle and Young (1961), Sparrow and Cess (1961), Cramer (1963), Singh and Cowling (1963), Pop (1969), Takhar (1971) and Nanousis and Tokis (1984).

The investigation of convective heat transfer from a solid body with different geometries in a porous and non-porous media is important from practical point of view due to its application in many areas of science and engineering, namely, drying of porous solids, thermal insulation, geothermal reservoirs, enhanced oil recovery, cooling of nuclear reactors, underground energy transport etc. Cheng and Minkowycz (1977) obtained similarity solution for free convection flow from a vertical plate embedded in a fluid saturated medium. Nakayama and Koyama (1987) investigated combined free and forced convection flows in Darcian and non-Darcian porous media. Hsieh et al. (1993) obtained non-similar solution for mixed convection from vertical surfaces in porous medium. Lai and Kulacki (1991) considered non-Darcy mixed convection flow along a vertical plate in a fluid saturated porous medium. Contributions on this topic are also due to Singh et al. (1989), Singh and Kumar (1993) and Hossain et al. (2000). Comprehensive reviews of porous media thermal/species convection are well presented by Pop and Ingham (2002), Vafai (2005) and Nield and Bejan (2006).

Investigation of hydromagnetic free convection flow in porous and non-porous media under different conditions is carried out by several researchers due to significant effect of magnetic field on the boundary layer control and on the performance of so many engineering devices using electrically conducting fluids viz. MHD energy generators, MHD pumps, MHD accelerators, MHD flow-meters, nuclear reactors using liquid metal coolants etc. This type of fluid flow problem also find application in plasma studies, geothermal energy extraction, metallurgy, petroleum and chemical engineering etc. Raptis and Kafousias (1982) investigated steady free convection flow past an infinite vertical porous plate through a porous medium in the presence of magnetic field. Raptis (1986) considered unsteady two-dimensional natural convection flow past an infinite vertical
porous plate embedded in a porous medium in the presence of magnetic field. Chamkha (1997b) studied transient MHD free convection flow through a porous medium supported by a surface. Chamkha (1997a) also considered hydromagnetic natural convection flow from an isothermal inclined surface adjacent to a thermally stratified porous medium. Jha (1991) investigated unsteady hydromagnetic free convection and mass transfer flow past a uniformly accelerated moving vertical plate through a porous medium. Aldoss et al. (1995) considered combined free and forced convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Takhar and Ram (1994) discussed MHD free convection flow of water at 4°C through porous medium. Kim (2000) studied MHD natural convection flow past a moving vertical plate embedded in a porous medium. Ibrahim et al. (2004) investigated unsteady hydromagnetic free convection flow of micropolar fluid and heat transfer past a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. Makinde and Sibanda (2008) studied steady laminar hydromagnetic heat transfer by mixed convective flow over a vertical plate embedded in a uniform porous medium in the presence of uniform normal magnetic field. Das et al. (2009b) considered free convective flow and heat transfer of a viscous incompressible electrically conducting fluid past a vertical porous plate through a porous medium with time dependent permeability and oscillatory suction in the presence of transverse magnetic field and heat source. Makinde (2009) investigated MHD mixed convection flow and mass transfer past a vertical porous plate with constant heat flux embedded in a porous medium. Contributions on this topic are also due to Makinde (2012a), Singh and Makinde (2012) and Makinde et al. (2012).

Investigation of the effects of heat generation or absorption in fluid flow is of much significance in several physical problems of practical interest viz. fluids undergoing exothermic and/or endothermic chemical reaction (Vajravelu and Nayfeh, 1992), applications in the field of nuclear energy (Crepeau and Clarksean, 1997), convection in Earth’s mantle (McKenzie et al., 1974), post accident heat removal (Baker et al., 1976), fire and combustion modeling (Delichatsios, 1988), development of metal waste from spent nuclear fuel (Westphal et al., 1994) etc. Keeping in view of this fact, Sparrow and Cess (1961) considered temperature-dependent heat absorption in their research study on steady.
stagnation point flow and heat transfer. Moalem (1976) studied steady heat transfer in a porous medium with temperature-dependent heat generation. Vajravelu and Nayfeh (1992) investigated the effects of free convection and heat transfer in the presence of heat generation or absorption on the flow of a viscous heat generating fluid near a cone and a wedge. It was found that heat source parameter had the dominating effect on the flow and heat transfer. Chamkha (1996) studied non-Darcy hydromagnetic free convection flow of an electrically conducting and heat generating fluid over a vertical cone and a wedge adjacent to a porous medium. The governing equations were solved numerically by an implicit finite-difference scheme. Chamkha (2000a) considered steady hydromagnetic boundary layer flow over an accelerating permeable surface in the presence of thermal radiation, thermal buoyancy force and heat generation or absorption. Sahoo et al. (2003) investigated unsteady MHD free convection flow of a viscous, incompressible and electrically conducting fluid past an infinite vertical porous plate in the presence of constant suction and heat absorption. It was found that magnetic field tends to retard fluid velocity and also it has tendency to reduce mean skin friction and mean rate of heat transfer of the conducting fluid. Chamkha (2004) studied unsteady hydromagnetic boundary layer flow of a viscous, incompressible, electrically conducting and heat absorbing fluid along a semi-infinite vertical permeable moving plate embedded in a uniform porous medium. It was found that heat absorption coefficient reduced fluid temperature which resulted in a decrease in the fluid velocity. Also heat absorption coefficient has tendency to reduce rate of heat transfer. Contributions on the topic are also due to Acharya and Goldstein (1985), Rahman and Sattar (2006), Ibrahim et al. (2008), Khedr et al. (2009), Chamkha et al. (2009) and Sharma and Singh (2009).

Investigation of convective flow with radiative heat transfer is of considerable importance due to its application in various areas of astrophysics, geophysics and fluid engineering (Hotzel and Sarofim, 1967) viz. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology, space vehicle reentry, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. Cess (1966) presented a seminal study of radiative convective boundary layer flow with
conducting and optically thin fluid with radiative heat transfer near a moving infinite flat plate in a rotating medium by imposing a time dependent perturbation on a constant plate temperature. The temperatures involved are assumed to be very large so that radiative heat transfer becomes significant. Mbeledogu and Ogulu (2007) considered heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer. Mahmoud Mostafa (2009) studied thermal radiation effect on unsteady MHD free convection flow of an electrically conducting fluid past an infinite vertical porous plate taking into account the effects of viscous dissipation. Palani and Abbas (2009) considered effects of radiation on hydromagnetic free convection flow past an impulsively started isothermal vertical plate. Ogulu and Makinde (2009) investigated unsteady hydromagnetic free convection flow of a dissipative and radiating fluid past a vertical plate with constant heat flux. Contributions on this topic are also due to Makinde (2012 b, c).

Investigation of heat transfer characteristics of fluid flow due to free/natural convection from a vertical plate with a number of finite size heat sources has applications to design of microelectronic circuit boards and other energy dissipating equipments under limiting situations. Taking into account this fact Schetz (1963), Schetz and Eichhorn (1964), Hayday et al. (1967), Smith (1970), Kelleher (1971), Kao (1975), Sokovishin and Erman (1982) and Lee and Yovanovich (1991) investigated free convection flow from a vertical plate with step discontinuities in the surface temperature considering different aspects of the problem. Schetz (1963) developed an approximate analytical model and Schetz and Eichhorn (1964) studied experimentally natural convection problems in the case of a vertical flat plate with a step change in wall temperature while Hayday et al. (1967) applied a numerical approach. Kelleher (1971) and Kao (1975) obtained series solution. Later Lee and Yovanovich (1991) developed a new analytical model for discontinuous wall temperature variations. They obtained a set of approximate solutions for the temperature and velocity distributions. They demonstrated the validity and accuracy of the model by comparing their results with the results of Hayday et al. (1967), Kelleher (1971) and Kao (1975). Chandran et al. (2005) investigated unsteady natural convection flow of a viscous and incompressible fluid near a vertical plate with ramped wall temperature. They
presented two different solutions for the fluid velocity viz. (i) valid for the fluids with Prandtl numbers different from unity and (ii) valid for the fluids with Prandtl number unity. Natural convection flow near a ramped temperature plate was also compared with the flow near a plate with uniform temperature. Seth and Ansari (2010) studied hydromagnetic natural convection flow past an impulsively moving vertical plate embedded in a porous medium with ramped wall temperature in the presence of thermal diffusion and heat absorption. It is found that magnetic field, heat absorption and thermal diffusion had a retarding influence on fluid velocity whereas thermal buoyancy force, time and porosity of medium have an accelerating influence on the fluid velocity for both ramped temperature and isothermal plates. Heat absorption and thermal diffusion tend to reduce fluid temperature. As time progresses, fluid temperature is increased for both ramped temperature and isothermal plates. Recently, Seth et al. (2011e) studied MHD natural convection flow with radiative heat transfer past an impulsively moving vertical plate with ramped temperature in a porous medium taking into account the effects of thermal diffusion. It is found that magnetic field tends to retard fluid flow whereas thermal buoyancy force, radiation and permeability of medium have tendency to accelerate fluid flow in the case of both ramped temperature and isothermal plates. Also fluid velocity is getting accelerated, as time progresses, in both the cases. Radiation tends to enhance fluid temperature. As time progresses, fluid temperature is getting enhanced for both ramped temperature and isothermal plates.

Unsteady hydromagnetic flow past a flat plate in a rotating medium is extensively investigated since the publication of research article by Hide and Roberts (1960b) in which they considered the effects of Coriolis force on the Stokes problem in the presence of a magnetic field. Kulshreshtha and Puri (1969) studied unsteady hydromagnetic flow an arbitrary time dependent moving plate in the presence of a transverse magnetic field. This problem was later considered by Soudalgekar and Pop (1970) and Debnath (1972, 1974) considering different aspects of the problem. Puri and Kulshreshtha (1976) investigated the effects of transverse magnetic field on initial value Stokes-Rayleigh problem when fluid flows past a porous plate in a rotating medium. They considered the impulsive, accelerated and oscillatory flows including the transient effects. Their work on oscillatory flows,
neglecting the transient effects was studied by Tokis and Geroyannis (1981) who included induced magnetic field in their analysis. Seth et al. (1981) considered unsteady hydromagnetic flow past a porous plate in a rotating medium with time dependent free-stream in the presence of a transverse magnetic field. They considered two cases of particular interest viz. (i) impulsive movement of free stream and (ii) accelerated movement of free stream. It is found that the fluid motion attained steady state quicker in the case of impulsive movement of the free-stream velocity than in the case of accelerated movement of the free steam velocity. Hayat et al. (2008a) obtained analytical solution for MHD transient rotating flow of an incompressible second grade fluid in a porous half space. Singh (1983, 1984) considered hydromagnetic free convection flow from a vertical porous plate in a rotating medium for the impulsive and accelerated motion of the plate considering Prandtl number \( P_r = 1 \). Tokis (1986) extended Singh’s work when \( P_r \neq 1 \). An extensive development of MHD free convection flows from a flat plate in a rotating medium is presented by Kyth and Puri (1987). The case of impulsive and ramped type thermal forcing effects on unsteady MHD free convection flow near a flat porous plate was studied by Kyth and Puri (1988a). Kyth and Puri (1988b) also studied unsteady MHD free convection oscillatory flow near an infinite porous vertical plate in a rotating medium in the presence of a uniform transverse magnetic field. Raptis and Singh (1985) investigated the effects of rotation on MHD free convection flow past a uniformly accelerated vertical plate in the presence of a transverse magnetic field when the quadratic convection term in the Navier-Stokes equations vanished in a natural way. Tokis (1988) obtained exact solution for the free convection flow near an infinite vertical moving plate in a rotating medium in the presence of a foreign mass and transverse magnetic field. Nanousis (1992) studied the effects of thermal diffusion on MHD free convective and mass transfer flow past a moving infinite vertical plate in a rotating medium. Contributions on steady or unsteady hydromagnetic flow past a plate in a rotating medium are also due to Gupta (1972), Soundalgekar and Pop (1973), Gupta and Soundalgekar (1975), Debnath et al. (1979), Hayat et al. (2001a,b), Takhar et al. (2002), Hayat and Mumtaz (2005), Ghosh and Ghosh (2008) and Singh et al. (2009).

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