CHAPTER 4
NUMERICAL RESULTS

The computer program described in the preceding chapter has been implemented. An exhaustive software development cycle became necessary along with the troubleshooting of each module individually as well as in conjunction. Particularly, variables like INDEX, FUNC and the integrand expressions that migrate between the various subroutines to obtain the final moment matrix elements required specific attention. Convergence of the integrals and moment matrix elements was found to be of importance. Upon completing these efforts, as described in the following, this program is utilized to analyze a prototype waveguide shunt-slot fed microstrip antenna element based on the proposed geometry.

The operating frequency for this prototype is selected as 6 GHz i.e. C-Band as discussed in the following section. A baseline design for the prototype is obtained using various available design formulae and empirical approaches for similar configurations. This design is used for further detailed analysis of this prototype using the developed computer program. Parametric studies are made upon the various design parameters of the proposed configuration with a view to establish their impact on the performance indices of the antenna element. By examining the behaviour of antenna input characteristics, it is possible to determine the resonant frequency and the range of frequencies over which acceptable performance is achievable. Also, these studies are aimed to evolve design guidelines for the proposed geometry with regard to the variables. The far-field characteristics are also calculated for the odd- and even-order basis functions. These are used to determine the far-field patterns for the prototype antenna using the expressions derived in previous chapters.
4.1 Prototype Antenna Element Design

The geometry of the proposed antenna element is as already presented in Figs. 2.3 & 2.4 earlier. The operating frequency of the antenna is the fundamental parameter from two considerations – as the resonant frequency of the patch radiator and as a swept variable to determine the parametric behavior of the proposed antenna. The key design parameters are the patch dimensions, the coupling shunt-slot dimensions, its position with respect to the patch centre, the waveguide internal dimensions and the offset of the slot from the longitudinal axis of the rectangular waveguide (over the broadwall surface.) The procedure adopted for selecting the values for these parameters *ab initio* is described in the following.

4.1.1 Frequency of Operation (Resonant Frequency of Patch)

The computer program developed in this thesis is completely general in nature. The expressions derived herein may be used at all microwave frequencies since the formulation is full wave in nature without approximations as are implicit in other analysis methods like transmission line or cavity models. For a prototype, which must be realized and characterized, a suitable frequency of operation must be chosen. Since the present association of the author is with the satellite communication antennas, a convenient choice of frequency would lie in one of the satcom bands. This was felt to be helpful in terms of hardware / accessories necessary for the measurements.

The S-band satcom band of 2.5 to 2.7 GHz would result in very large hardware elements. On the higher side, the Kᵤ-band (11.45 to 11.70 GHz downlink; 14.0 to 14.5 GHz uplink) would result in unduly tight fabrication tolerances on patch and slot.
Hence, the Extended C-Band (5.850 to 6.425 GHz) was chosen for the prototype design, analysis and realization that relaxes the constraints from both aspects.

Hence, the resonant frequency for the antenna element is chosen as 6 GHz.

### 4.1.2 Input Rectangular Waveguide Dimensions

The standard waveguide employed for this frequency band is known by its EIA designation as the WR159 waveguide [47]. The waveguide has an aspect ratio of 2:1 and its internal dimensions are 1.590” X 0.795” hence the prototype parameters are:

\[
\begin{align*}
a &= 40.39\text{mm}; \\
b &= 20.19\text{mm}.
\end{align*}
\]

The cut-off frequency of this waveguide is 3.714 GHz and its usable range is 4.64 to 7.05 GHz which ensures that sufficient margin exists for the selected resonant frequency.

### 4.1.3 Substrate Selection

For a microstrip radiator, a substrate with a lower dielectric constant is preferable in order to maximize the fringing fields responsible for radiation. Conversely, a higher-$\varepsilon_r$ substrate will result in smaller patch dimensions along with a reduced bandwidth and greater surface-wave excitation [35]. A thick substrate will accrue higher bandwidth but also reduce coupling to the waveguide while increasing the surface wave effects.

For this prototype model, it was desirable to have a mechanically stiff substrate with a low enough dielectric constant. Low thickness would be preferable to avoid surface wave modes. Each of these modes results in a TE- or TM-pole in the integrand...
expressions encountered during the moment matrix element determination. Considering the substrates available and the above aspects, the Rogers RO-3003 substrate was selected. The relevant electrical parameters for this are:

\[ \varepsilon_r = 3.0; \quad \tan \delta = 0.0013; \quad \text{and} \quad d = 1.524\text{mm (60 mil)}. \]

### 4.1.4 Rectangular Microstrip Patch Dimensions

The dimensions of the rectangular microstrip patch element for 6 GHz are obtained in accordance with the procedure outlined in [48]. The resonant length of the patch is half of the effective wavelength in the microstrip medium. Further the length must be reduced to the extent of fringing field extension on either side that depends on the substrate height and the patch width (non-resonant dimension.) Also for the case of the waveguide-excited patch, a resonant element is preferable to maximize the coupling between the waveguide and free space [35].

This set of design equations has been implemented in the form of a Microsoft Office Excel Sheet. This allows a user to enter the resonant frequency of the patch and the substrate parameters while a nominal patch (non-resonant) width needs to be initialized. The non-resonant dimension is not critical but may be controlled by specifying an aspect ratio. The Excel Sheet allows the user to fix this value and then the width needs to be iterated a few times to get its exact combination with the resonant length. Using this design sheet, the following dimensions were obtained for the patch:

\[ L_p = 14.12\text{ mm}; \quad W_p = 9.41\text{mm} \quad (\text{aspect ratio 1.5}). \]
The aspect ratio of 1.5 is chosen to prevent the excitation of the orthogonal $\text{TM}_{10}$ mode in the patch which would result in higher cross-polarization and introduce difficulties in impedance matching.

The patch dimensions were verified by using an open domain antenna calculator available on http://www.emtalk.com/mpacalc.php. The analyzed resonant frequency matched our design value to within 0.5%.

4.1.5 Coupling Slot Dimensions

The length of a coupling slot is recommended to be below a quarter-wavelength in the required coupling scenario [35]. As regards width, a 10:1 ratio is conventionally employed in aperture-excited patches. More generally, a slot width of $0.01\lambda$ to $0.02\lambda$ is recommended. The slot size is found to control the coupling factor and also slightly tunes the frequency of the patch resonance.

Owing to these considerations, the nominal coupling slot dimensions are chosen as:

$$L_s = 10.0 \text{ mm} \quad ; \quad W_s = 1.0 \text{ mm}.$$  

4.1.6 Patch Position

The coupling from the slot to the patch is found to be a maximum when the latter is centred over the aperture although the exact alignment accuracy need not be very stringent [35]. Although the developed computer program allows the patch to be positioned off-centre, this is the nominal location selected for the prototype. Thus:

$$x_p = 0.0 \text{ mm} \quad ; \quad y_p = 0.0 \text{ mm}.$$
4.1.7 Longitudinal Offset of the Coupling Slot

A shunt slot interrupts little wall currents if located along the median line of the waveguide broad-wall. Consequently, in order to couple any energy out through it, the longitudinal slot must be offset from the centre-line of the waveguide to interrupt a greater amount of wall currents and radiate. Also, the extent of the offset is found to control the coupled energy for the case of a radiating slot [49]. In fact, this property is one of the advantages of the proposed geometry that will be studied to a greater extent in this thesis. At present, a nominal offset is selected for the sake of a baseline analysis as:

\[ x_s = 30.195 \text{ mm (Offset = 10.0mm)} \]

This completes the definition of the prototype microstrip patch element excited with a waveguide shunt slot. We now proceed to employ the developed computer program to study this configuration in greater detail.

4.2 Convergence Criteria for the Moment Matrix Terms

The convergence criteria for the various moment matrix terms need to be established before the developed formulation can be effectively used for the analysis of the proposed antenna geometry. To this end, the first of the relevant parameters is the number of modes to be retained in the double summation of the waveguide admittance matrix term. Next, the required discretization interval in \( \alpha \) and \( \beta \) for a converged numerical integration needs to be established. Finally, the upper limit of \( \beta \)-integration needs to be established that yields converged integrals for all four of the matrix terms based on the Green’s function integration i.e. the slot admittance term.
Numerical Results

the slot-to-patch and patch-to-slot coupling terms and the patch impedance terms. These various aspects are discussed in the following subsections.

4.2.1 Number of Modes for Computing Waveguide Admittance Matrix Terms

The waveguide self-admittance matrix term \[ Y'_{pt} \] represented by eqn. (2.16) and given by the double series expansion in eqn. (2.34) will have its convergence governed by the values of \( m \) and \( n \). These numbers are the modal indices for the various TE- and TM-modes supported by the input rectangular waveguide in the proposed geometry. As the shunt slot represents a discontinuity, a large number of modes are excited by its presence. Only a few of these modes will be sustained by the waveguide section while the others, having their cut-off frequencies higher than the input frequency, will be evanescent in nature. Although these latter may be attenuated quickly as the wave propagates down the guide, the stored energy represented by their presence in the proximity of the slot will result in an impedance change. Hence the number of modes retained in the expansion of the expression in (2.34) will determine the accuracy of the prediction of the input characteristics of the proposed antenna.

The developed computer program was executed for a progressive number of modal indices retained in the series summations of eqn. (2.34) with \( m = n = 10, 20 \ldots 100 \) (written as LM & LN in the program.) The behavior of the first term of the waveguide admittance matrix term \[ Y''_{11} \] was observed, since this term will be the most significant. The real part of this term was found to be constant at \( 3.018 \times 10^{-11} \Omega^{-1} \cdot m^2 \). However, the imaginary part shows a change as the value of \( (m, n) \) is progressively increased as illustrated by Fig. 4.1. The imaginary part is two orders of magnitude larger than the real part, hence will determine the overall contribution of the admittance term.
Fig. 4.1: Convergence Behaviour of $\text{Im} [Y_{11}^a]$ vs. Number of Waveguide Modes ( $\text{Re} [Y_{11}^a] = 3.018 \times 10^{-11} \Omega^{-1} \cdot m^2$ )
Numerical Results

It is seen that the case \( m = n = (\text{or } LM = LN = ) 50 \) shows \( \sim 1.3\% \) change w.r.t. the case when 40-terms each are retained while with 60 terms each, we obtain only \( \sim 0.27\% \) change. Beyond this the trend is found to be nearly asymptotic.

As a result, it is found to be sufficient to retain \( m = n = 60 \) in the subsequent analyses using the developed computer program.

4.2.2 Discretization Interval in \( \alpha \) and \( \beta \)

As mentioned earlier, numerical integration based on a 10-point Gauss Quadrature scheme is carried out for evaluating the moment matrix terms both \( \alpha \) and \( \beta \). The total integration range for either variable is subdivided into smaller intervals between which the integration is applied. The optimum discretization interval will depend upon the nature of the integrands. As a broad guideline, the integrand should not undergo sharp fluctuations within the chosen interval otherwise the discretization must be finer. These aspects are briefly discussed for the present case.

For \( \alpha \), the required integration range is 0 to \( \pi/2 \) i.e. \( 0^\circ \) to \( 90^\circ \) (see, for instance eqn. 2.65.) As a starting point, numerical integration in \( \alpha \) was carried out in a single run between these end values i.e. ALPHAS = \( 0^\circ \) and ALPHAE = \( 90^\circ \) or in other words across a single interval of \( 90^\circ \). The convergence behavior of the matrix terms turned out to be poor with large fluctuations in integral value. The reason for this found to be fast azimuthal variations in the integrand values especially away from origin i.e. for larger values of \( \beta \). To estimate the optimum spacing, \( \alpha \)-integration was carried out by finer discretization in the following order: 18 intervals \( \times 5^\circ \); 30 intervals \( \times 3^\circ \); 45 intervals \( \times 2^\circ \); 90 intervals \( \times 1^\circ \); 180 intervals \( \times 0.5^\circ \); and 360 intervals \( \times 0.25^\circ \) each.

From this study, it could be concluded that for the functions involved in the present
formulation, a discretization using 30 intervals of $3^\circ$ each is sufficient to assure the convergence of the $\alpha$-integration.

**For $\beta$,** a starting discretization of $10k_0$ was used from previous experience based on similar studies made earlier e.g. Harish [29]. It was verified with the developed program that this spacing is adequate for the convergence of the integrals involved in the present formulation.

The maximum value of $\beta$ upto which the integration should be performed is also a concern that is addressed next.

### 4.2.3 Truncation of Infinite Integral in $\beta$

Mathematically, the integration in $\beta$ is required to be performed from 0 to $\infty$ e.g. eqns. (2.65, 2.73 & 2.86.) In practice, the integral will converge at a particular (large) value of $\beta$ where the numerical calculations will have to be terminated. The optimum value needed will depend on the nature of the integrand.

As mentioned in Section 3.1, a convergence criterion of $< 0.5\%$ change of the integral value in a successive integration run (with a step of $10k_0$) has been implemented in the developed program. As evident from previous studies, convergence is likely to be obtained in the range $100 \, k_0 \leq \beta \leq 200 \, k_0$.

To establish this behavior for the current design parameters chosen, two sample moment matrix elements are selected: the first term of the patch-to-slot coupling matrix element, $[T_{11}^b]$ given by eqns. (2.18) and (2.73) and the first patch self-impedance term, $[Z_{11}]$ given by eqns. (2.20) and (2.86.) Numerical integration was carried out for different values of BETAUP, the maximum value of $\beta$. 
Numerical Results

Fig. 4.2: Convergence Behaviour of $\text{Re} [T_{11}^b]$ vs. $\beta$

($\text{Im} [T_{11}^b] = 0.166 \times 10^{-6} \text{ m}^2$)

Fig. 4.3: Convergence Behaviour of $\text{Im} [Z_{11}]$ vs. $\beta$

($\text{Re} [Z_{11}] = -0.126 \times 10^{-3} \Omega \text{ m}^2$)
Figs. 4.2 and 4.3 illustrate the convergence characteristics for the two chosen matrix elements. Convergence is found to occur at about $\beta = 8,000$ which corresponds to about $63k_0$ at 6 GHz. As seen from the curves in these figures, if a sufficiently large value of BETAUP is specified, the convergence criterion of $< 0.5\%$ magnitude variation ensures that the program exits integration loop before BETAUP is actually reached. Hence a suitable value of BETAUP to ensure convergence at the highest frequency of analysis may be chosen.

With this, the convergence aspects of the moment matrix elements are established. The developed program may be used for further investigation of the proposed geometry with an assured convergence of the matrix elements. The convergence of the M-o-M solution, however, still needs to be studied. This is taken up in the ensuing section.

### 4.3 Convergence of the Method-of-Moments Solution

With the above choice of parameters, the various terms of the moment matrix would acquire their numerically-precise values. However, the number of basis functions necessary to assure a converged solution still needs to be established. Strictly speaking, this would depend on the problem being analyzed i.e. its geometrical parameters. Nevertheless, some general considerations would apply.

In this section, the behavior of the developed M-o-M solution is studied for convergence with the number of basis functions on the slot and patch respectively. As already mentioned in the previous chapters, the solution of the Moment Method matrix equation allows us to estimate the unknown current distributions over these two geometries. These currents are then utilized to compute the electrical parameters
of the problem geometry, both input and radiation characteristics. Hence the convergence behavior of the computed current distributions is first studied with the number of basis functions in the following.

4.3.1 Convergence of Computed Current Distributions with Basis Functions

The prototype waveguide shunt-slot fed microstrip patch antenna element designed for 6 GHz and detailed in Section 4.1 is used to study the effect of number of basis functions on the evaluated unknown current distributions across the coupling slot and the radiating patch. The geometrical parameters of the structure are the same as in that section. The number of basis functions are increased from \{NA, NB\} = \{1, 1\} to \{15, 15\} in steps of 2. An even number of functions is not selected as many of the even-order matrix terms are zero.

The developed program is executed for progressively increasing number of basis functions and the resulting slot magnetic current and patch electric current distributions are studied (see Figs. 4.4 & 4.5.) It is seen that the slot magnetic current shows large variations for a smaller number of basis functions across it. The current distribution begins to settle down after about nine basis functions but \( NA = 13 \) to 15 functions are seen to be desirable for a relatively stable current distribution.

The patch electric current distribution is relatively quick to settle down. After about seven expansion functions, the current distribution is found to move only in a narrow range. A value of \( NB \geq 11 \) is seen to yield a relatively stable electric current distribution across the patch.

A study of convergence behaviour of some input (impedance) characteristics of the proposed configuration would also be in order: this is discussed next.
Numerical Results

**Fig. 4.4:** Convergence Behaviour of Slot Magnetic Current with number of basis functions

**Fig. 4.5:** Convergence Behaviour of Patch Electric Current with number of basis functions
4.3.2 *Convergence of Input Parameters with Basis Functions*

Two input parameters, as predicted by the developed computer program, are chosen to study the number of basis functions appropriate to obtain convergence. These are the magnitude of $s_{11}$, the reflection coefficient and $P_{\text{out}}$, the fraction of power coupled out given by Eq. (2.104.) The convergence of the magnitude of $s_{11}$ represents the accuracy of the input impedance prediction for the antenna element and also allows the adequacy of the impedance matching to be assessed. With an analysis tool, the antenna designer needs a valid prediction for this parameter for carrying out the optimization of the designed microstrip radiator. $P_{\text{out}}$ is crucial for this particular geometry proposed. The other end of the waveguide (see Fig. 2.3) is matched-terminated; hence it receives that amount of e.m. energy not radiated by the patch. A good impedance match does not assure that the energy received by the proposed antenna element is actually radiated; it might be coupled to this second port. Hence the convergence behavior of $P_{\text{out}}$ is also studied in addition to $\text{Mag}[s_{11}]$ before the required number of basis functions can be frozen.

Figs. 4.6 and 4.7 show the convergence of these two parameters with the number of basis functions. A large variation is found in the value of these two parameters up to about five basis functions. After about 11 functions, $P_{\text{out}}$ seems to have converged while reflection coefficient is also converged after about 13 functions.

A value of $NA = 15$ and $NB = 11$ is selected for the further analysis of the prototype antenna element. The first investigation to be performed was the frequency variation of the input parameters of the antenna which is described in the following section.
**Fig. 4.6:** Convergence Behaviour of Magnitude of Input Reflection Coefficient with number of basis functions

**Fig. 4.7:** Convergence of Fraction of Power coupled out with number of basis functions
4.4 Swept Frequency Response of the Prototype Antenna Element

The prototype antenna element at 6 GHz (as detailed in Section 4.1) whose design is based on the proposed geometry is analyzed for the variation of its input parameters with frequency using the developed computer program. The purpose of this is to assess the efficacy of the impedance match and also to detect a resonance from the behavior of the complex input impedance.

Fig. 4.8 shows the frequency response of the 6 GHz prototype patch element fed by a shunt slot. The upper plot shows the real and imaginary parts of the input impedance of the radiator. The predicted VSWR is shown in the lower plot along with the fraction of power coupled out, $P_{\text{out}}$.

It is observed that both $\text{Re}[Z_{\text{in}}]$ and $\text{Im}[Z_{\text{in}}]$ are relatively stable over the frequency range from 4.5 to 6.5 GHz – an adequately wide range to assess resonant phenomena. The reactive part shows that the antenna element is predominantly inductive over this frequency range. The impedance match should be good since the resistive part, indicative of power transfer, is close to the normalized impedance of the input waveguide. This is borne out by the VSWR plot that indicates a very good match across the entire swept frequency range. As mentioned earlier, the input power may simply be transmitted to the second (terminated) port of the structure rather than getting coupled out through the slot. Only $P_{\text{out}}$ is found to show a distinct peak in the neighbourhood of 5.3 GHz.

We will now use the developed formulation and computer program to perform parametric studies of the proposed geometry. The objective is to understand the impact of the various design parameters on the input characteristics and to attempt to develop design guidelines.
Fig. 4.8: Swept Frequency Response of the 6 GHz Prototype Shunt-slot Fed Patch Element
4.5 Parametric Study of Prototype Antenna with Width of the Coupling Slot ($W_s$)

The coupling slot width for the prototype antenna element has been selected as $W_s = 1.0\text{mm}$ based on the considerations in Section 4.1.5. In general, the longitudinal slot should chiefly interrupt longitudinal currents across the waveguide top-wall. Thus, a smaller slot width is preferable. In this section, a frequency sweep is performed using the developed formulation for two other cases of slot width, $W_s = 0.5\text{mm}$ and $1.5\text{mm}$. These values correspond to $0.01\lambda$ and $0.03\lambda$ respectively while the baseline is $0.02\lambda$.

Fig. 4.9 shows the swept response of the prototype antenna element for $W_s = 0.5\text{mm}$ ($0.01\lambda$.) The variation of both impedance components as well as VSWR is relatively smoother. The coupled power exhibits a clear peak at $5.0\text{GHz}$.

Fig. 4.10 shows the frequency sweep of the element for $W_s = 1.5\text{mm}$ ($0.03\lambda$.) The plots are seen to more irregular in this case. The overall level of power coupled out is also higher compared to the other two cases. $P_{out}$ shows two distinct peaks at $4.8$ and $6.2\text{GHz}$ respectively.

Two important observations may be made here: 1) a relatively wider slot results in the interruption of transverse top-wall currents in addition to the desired longitudinal currents; and 2) since $\hat{y}$-directed components of the slot magnetic current are neglected (see Section 2.3), convergence may not occur with the assumed current distribution as per Eqn. (2.25) if appreciable transverse current components are interrupted by the slot. This is felt to be the reason for the irregular behaviour of the plots in Fig. 4.10. As a future extension of this work, this effect may be included in the formulation but is not within the scope of the present work.
Numerical Results

Fig. 4.9: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $W_s = 0.5\text{mm} \ (0.01\lambda)$
Fig. 4.10: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $W_s = 1.5\text{mm (0.03}\lambda)$
4.6 Parametric Study of Prototype Antenna with Length of the Coupling Slot ($L_s$)

The length of the coupling slot, $L_s$, is a primary means for controlling the coupling factor of the slot from the waveguide. The developed formulation is actually focused on investigating this particular property of the shunt slot. As the slot length is increased, it interrupts progressively greater amount of currents on the waveguide broad wall. Consequently, it is expected to allow higher coupling when $L_s$ is increased. However, the resonance of the slot is also expected to play a part. When not overlaid with the microstrip patch, it behaves as a radiative aperture, exhibiting strong radiation at its resonance. At present, the slot behaves as a coupling element. The actual radiation behavior of the element will also be strongly governed by the patch which itself is resonant in nature. In this section, the impedance characteristics of the proposed microstrip antenna configuration are investigated using the developed program by varying the length of the coupling aperture.

4.6.1 Coupling Slot Length, $L_s = 2.0$mm ($0.04\lambda$)

Fig. 4.11 shows the input characteristics of the C-Band prototype microstrip antenna element for this length of the coupling slot. This is chosen as a minimum starting value of the slot length – all the other parameters are as in Section 4.5.

A key difference observed now as compared to the earlier case when $L_s = 10.0$mm ($0.20\lambda$) – (see Fig. 4.9) is the predominantly capacitive behavior of the element input impedance. Power coupled out is very low and the perturbation offered by the presence of the slot is negligible as indicated by both VSWR and real part of the input impedance. The slot interrupts very little waveguide wall currents, hence the negligible radiation from the antenna element.
Fig. 4.11: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $L_s = 2.0\text{mm} (0.04\lambda)$
4.6.2 Coupling Slot Length, $L_s = 4.0 \text{mm (0.08})$

We increase the length of the slot by a small amount. Fig. 4.12 shows the input characteristics of the C-Band prototype microstrip antenna element with the length of the coupling slot, $L_s = 4.0 \text{mm (0.08} \lambda)$ and the other parameters unchanged.

A significant difference is observed in the response of the element compared to both the earlier cases. A distinct “shunt” resonance is observed just above 5.8 GHz as shown by the capacitive-to-inductive transition of the input reactance. At this frequency, also the coupled power is seen to be very high, $> 0.1$ i.e. -10dB. Simultaneously, the VSWR is found to peak at 1.1126 (but still equivalent to -25.47 dB return loss.) This is a good figure and the antenna element may be said to be optimally tuned at this point.

The interpretation of the observed input characteristics is that the slot & patch combination behaves as a short at frequencies away from this resonance. As a result, the waveguide appears to be a continuous geometry resulting in a nearly complete transmission into the terminated through port. Around resonance, the slot apparently couples a good amount of energy into the patch, which, being near its own resonance, presumably radiates it. Since the slot interrupts wall currents significantly, some of the scattered energy from the slot reflects back towards source, resulting in the local, relatively “poorer” return loss.

A second resonance is also observed near 5.2 GHz. This phenomenon is not fully understood at present. It may be a second resonance either within the radiating structure or induced by the presence of parasitics. Numerical instability also cannot be ruled out since the reactance does not show a resonant crossing.
Fig. 4.12: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $L_s = 4.0\text{mm}$ (0.08$\lambda$)
The impact of varying slot length shall be investigated further.

### 4.6.3 Coupling Slot Length, $L_s = 6.0\text{mm} \ (0.12\lambda)$

The length of the slot is further increased to 6.0mm. Fig. 4.13 shows the simulated input characteristics of the antenna for this case.

It is seen that the resonant behavior observed earlier is not seen for this case. The antenna exhibits inductive impedance behavior as opposed to the chiefly capacitive characteristics when the slot length was 2.0mm. The coupled power is reduced indicating that the slot-patch combination is apparently not accepting as much energy from the waveguide, even though the slot is interrupting more of wall current. The VSWR is also seen to be very good indicating that most of the energy is coupled to the through port.

### 4.6.4 Coupling Slot Length, $L_s = 8.0\text{mm} \ (0.16\lambda)$

We further increase the length of the coupling slot to 8.0mm. Fig. 4.14 illustrates the computed input characteristics of the antenna element for this case.

We again observe that the antenna input impedance is mainly inductive. Further the variation of both the real and imaginary parts of the input impedance over the frequency range is much lesser than for 6.0mm length. The coupled power is of a lower order and VSWR is uniformly good.

Since the original case analyzed in Section 4.5 (Fig. 4.9) corresponds to $L_s = 10.0\text{mm} \ (0.20\lambda)$, we see the trend seen in the preceding two cases is continued when the slot length is increased further also. The parametric behavior of the proposed element with coupling slot length may now be summarized.
Fig. 4.13: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $L_s = 6.0\text{mm (0.12}\lambda)$. 

(a) Real part of $[Z_{in}]$ in ohms

(b) Imaginary part of $[Z_{in}]$ in ohms

VSWR

Fraction of Power coupled out

Frequency, GHz

Fraction of Power coupled out

Frequency, GHz
Fig. 4.14: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $L_s = 8.0\text{mm}$ ($0.16\lambda$)
4.6.5 Observations on Parametric Study of Coupling Slot Length, $L_s$

The behaviour of the proposed waveguide shunt-slot excited microstrip patch antenna element has been investigated by varying the length of the rectangular coupling aperture. All the other parameters were retained constant during these simulation exercises. The findings of these studies may be summarized as follows.

1) The magnitude of the antenna element input impedance is close to the waveguide normalized impedance (~ 1.0.)

2) For $L_s = 2.0\text{mm (}0.04\lambda\text{)}$, the nature of the input impedance is capacitive. There appears to be a large mismatch offered by the antenna element since very little power is coupled out (see Figs. 4.11 and 4.15.)

3) At $L_s = 4.0\text{mm (}0.08\lambda\text{)}$, we observe a clear shunt resonance (Fig. 4.12) with the coupled power also exhibiting a peak around 5.8 GHz. A second resonance is also seen but the reason for this is not clear at present. Since the input reactance does not show a zero crossing, it may be a spurious resonance also.

4) As $L_s$ is increased further to 6.0, 8.0 and 10.0mm (0.12\lambda, 0.16\lambda and 0.20\lambda), the antenna element is found to exhibit inductive behaviour. No resonance is observed within the frequency range examined. The coupled power progressively reduces (see Figs. 4.13, 4.14 and 4.9) while the other input parameters exhibit progressively smaller variations with frequency.

5) It may be inferred that a slot length of $L_s = 4.0\text{mm (}0.08\lambda\text{)}$ may be optimum for achieving a clear resonance that maximizes the coupling from the waveguide to the radiator. At this size, the slot closely approximates a point source under the patch thereby achieving a good impedance match. However, departure from this length introduces a residual reactance, deteriorating the
Numerical Results

impedance characteristics. As a result, the quality of the matching reduces even though the slot, per se, interrupts a greater amount of the top-wall current.

Fig. 4.15 illustrates the power coupled out to the microstrip radiator (in dB) as the coupling slot length is varied.

![Graph illustrating the variation of coupled power to the prototype microstrip antenna with the length of the coupling slot.](image)

**Fig. 4.15: Variation of Coupled Power to Prototype Microstrip Antenna with Length of Coupling Slot, $L_s$**
4.7 Parametric Study of Prototype Antenna with Longitudinal Offset ($x_s$) of the Coupling Slot

An important parametric study for the proposed configuration is the variation of the offset of the shunt slot from the median line (parallel to waveguide axis) across the top wall of the rectangular waveguide. This is represented through the parameter $x_s$ shown in Fig. 2.3 except that it is denoted here as slot displacement from one edge of the waveguide top wall instead of a median offset. As a result, $x_s$ varies from 0 to $a$ (the waveguide section length) which is equivalent to a median offset, $\delta = \pm a/2$.

As known from waveguide-fed slot theory, a shunt slot couples little or no energy when placed exactly along the centre-line of the waveguide top-wall [49]. This is a consequence of the slot interrupting little of the current lines flowing along the broadwall when placed at that position (and the waveguide is assumed carrying only the dominant $TE_{10}$ mode.) This property is also the basis of the slotted waveguide travelling probe in the microwave bench used for VSWR measurements in laboratory work. This condition is the starting value of the parameter, $x_s$ (i.e. $a/2$.) Subsequently, the developed computer program is used to analyze the proposed antenna geometry as $x_s$ is varied in steps till the slot nearly reaches the end wall while the input characteristics of the prototype antenna are monitored. The offset of the longitudinal slot is also a convenient method of controlling the power coupled from the waveguide by the shunt slot [49].

4.7.1 Coupling Slot Axial Offset = 0.0mm ($x_s = 20.195$mm)

This case will serve as a sanity check for the developed formulation. In this condition, the antenna is expected to yield nearly zero coupling to the slot. The input impedance
Fig. 4.16: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $x_s = 20.195$mm (Axial Offset = 0.0mm)
should be close to the waveguide characteristic impedance and the VSWR should be similar to the case of a length of rectangular waveguide.

Fig. 4.16 illustrates the simulated input characteristics of the prototype antenna with the longitudinal slot located along the median line of the top-wall. The input resistance is constant (to six significant digits) at the value of 1.0. In a similar fashion, the coupled power equals 0.0 and VSWR equals 1.0 both to six significant digits. Only, the shunt resonance behaviour is still clearly exhibited by the input reactance, also in the neighbourhood of 5.8 GHz. Compared to the case of 10mm axial offset (see Fig. 4.12), we observe that the overall value of the reactance is very small (by about 12 orders of magnitude.) This implies that the slot offers virtually no residual reactance. In spite of this, the resonant characteristics are still evident at this low level of reactance also. This is an interesting result from this test case for the proposed geometry.

4.7.2 Coupling Slot Axial Offset = 2.0mm ($\delta \approx 0.1\delta_{\text{max}}; x_s = 22.195\text{mm}$)

We observe the shunt resonance seen earlier for the 10mm offset almost exactly at 5.8GHz. Corresponding peaks are shown by VSWR and coupled power also. At resonance, the peak coupling level is 0.005 (i.e. -23.01dB.) The second resonance is also observed around 5.2 GHz. As discussed earlier, this may be a spurious resonance.

4.7.3 Coupling Slot Axial Offset = 4.0mm ($\delta \approx 0.2\delta_{\text{max}}; x_s = 24.195\text{mm}$)

The shunt resonance is seen again but the power coupling has increased to 0.0188 ( -17.26 dB.) The second resonance around 5.2 GHz shows a different behaviour now (Fig. 4.18.) The imaginary part has shown two zero-crossings which were not observed earlier. However, a definite conclusion cannot be drawn at this stage.
Numerical Results

Fig. 4.17: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $x_s = 22.195\text{mm}$ (Axial Offset, $\delta = 2.0\text{mm} \approx 0.1\delta_{\text{max}}$)
Fig. 4.18: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $x_s = 24.195$mm (Axial Offset, $\delta = 4.0 \text{mm} \approx 0.2\delta_{\text{max}}$)
4.7.4  **Coupling Slot Axial Offset = 6.0mm (δ ≈ 0.3δ_{max}; x_s = 26.195mm)**

We observe a reactance zero-crossing again at 5.8 GHz, representing a shunt resonance (see Fig. 4.19.) The maximum coupled power is 0.04199 (-13.77 dB.) The behaviour of the reactance curve around 5.2 GHz again is reversed from that seen in the previous case in Fig. 4.18. The lowest VSWR at resonance is 1.044 which corresponds to -33.3 dB return loss. This indicates that the slot does not cause an appreciable degradation in the input return loss due to the interruption of the current lines.

4.7.5  **Coupling Slot Axial Offset = 8.0mm (δ ≈ 0.4δ_{max}; x_s = 28.195mm)**

The shunt resonance is seen to slightly shift towards lower frequency but is still essentially in the vicinity of 5.8GHz. The maximum power coupling in this sweep is found to be 0.0739 (-11.31 dB.) The lowest VSWR at the same point is 1.0849; this corresponds to -27.80 dB. The trend of reactance variation in the second resonance appears to have reversed again.

4.7.6  **Coupling Slot Axial Offset = 10.0mm (δ ≈ 0.5δ_{max}; x_s = 30.195mm)**

This is the baseline case already investigated in Section 4.6 (see input characteristics in Fig. 4.12.) Resonance, as observed earlier, is seen around 5.8 GHz. The coupling factor is 0.1007 (-9.97 dB) and the minimum VSWR is 1.1126 (R. L. = -25.47 dB.) The trend of the input parameters is similar to the case in Fig. 4.19.

4.7.7  **Coupling Slot Axial Offset = 12.0mm (δ ≈ 0.6δ_{max}; x_s = 32.195mm)**

The shunt resonance is seen around 5.8 GHz again. The maximum coupled power is 0.1525 which corresponds to -8.17 dB. The maximum VSWR at resonance is 1.1824.
Fig. 4.19: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $x_s = 26.195\text{mm}$ (Axial Offset, $\delta = 6.0\text{mm} \approx 0.3\delta_{\text{max}}$)
Fig. 4.20: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $x_s = 28.195$mm (Axial Offset, $\delta = 8.0$mm $\approx 0.4\delta_{max}$)
Fig. 4.21: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $x_s = 32.195\text{mm}$ (Axial Offset, $\delta = 12.0\text{mm} \approx 0.6\delta_{\text{max}}$)
Numerical Results

This is equivalent to a return loss of -21.56 dB. The second resonance is again seen at 5.2 GHz (see Fig. 4.21.) However, the coupled power is seen to be of a lower order, as is the perturbation caused to the input resistance at the antenna input port.

4.7.8 Coupling Slot Axial Offset = 14.0mm ($\delta \approx 0.7\delta_{\text{max}}$; $x_s = 34.195\text{mm}$)

Fig. 4.22 illustrates the simulated input characteristics of the antenna element for this case. The shunt resonance seems to drift slightly to the higher side but is still near 5.8 GHz. The peak power coupled out is 0.1466 (-8.34 dB.) This indicates that the trend of increasing power coupling from the waveguide by the slot is now nearing saturation. We observe that the VSWR has also acquired a stationary response with a value of 1.1825 at resonance (return loss -21.55 dB.) The behaviour of the input curves is not changed significantly at the second resonance.

4.7.9 Coupling Slot Axial Offset = 16.0mm ($\delta \approx 0.8\delta_{\text{max}}$; $x_s = 36.195\text{mm}$)

The shunt resonance is seen slightly below 5.7 GHz in this case (see Fig. 4.23.) The coupling factor is seen to be very high – 0.2334; this corresponds to a coupling level of -6.32 dB. As expected, such a large coupling indicates a significant effect on the input match due to the energy scattered by the slot in the input direction. The VSWR at resonance is found to be 1.3227 which implies a return loss of -17.14 dB. At this VSWR, it is probably not advisable to use the slot as a coupling structure. However, it is possible to obtain a better match by adjusting the patch dimensions and thus obtain an acceptable antenna element. This is not being attempted at present, but the developed formulation allows this to be carried out. Of more interest is that the second resonance is not observed at this slot offset. Although the disturbance to the input reactance is still seen, both $P_{\text{out}}$ and VSWR register no significant perturbation.
Fig. 4.22: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $x_s = 34.195\text{mm}$ (Axial Offset, $\delta = 14.0\text{mm} \approx 0.7\delta_{\text{max}}$)
Numerical Results

Fig. 4.23: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $x_s = 36.195\text{mm}$ (Axial Offset, $\delta = 16.0\text{mm} \approx 0.8\delta_{\text{max}}$)
4.7.10 Coupling Slot Axial Offset = 18.0mm (δ ≈ 0.9δ_max; x_s = 38.195mm)

The shunt resonance is again seen in the vicinity of 5.8 GHz (see Fig. 4.24.) The perturbation to the input parameters appears to be lesser, as indicated by the resistance and coupled power curves. The maximum coupling is seen to be 0.1576 i.e. -8.02 dB. The coupling factor is close to the numbers obtained for the cases of 12 and 14mm offsets (Figs. 4.21 & 22 respectively.) The maximum VSWR at resonance is 1.2050 i.e. a return loss of -20.63 dB. Finally, the second resonance at 5.2 GHz is observed once again. However, the perturbation to input resistance as well as to coupled power are very small.

4.7.11 Observations on Parametric Study of Coupling Slot Axial Offset, x_s (or δ)

A complete parametric analysis of the prototype microstrip patch antenna element has been carried out by varying the axial offset of the coupling slot from the centre line of the feeding waveguide. The starting point is the case where the slot is positioned along the centre-line of the waveguide (zero offset.) From this, the slot is offset in steps of 2mm up to 18mm – this corresponds to a case study with the offset, δ = 0.0 to about 0.9δ_max. Analysis was not carried out beyond this limit as the basis functions across the slot may not be valid. Observations during this parametric study are summarized.

1) The coupling factor is found to progressively increase from zero for the starting no-offset case to about -6 dB. However, after about 12mm offset (δ = 0.6δ_max), the coupling appears to have levelled off (see Fig. 4.25.) It is interesting to note that computations for a direct-radiating longitudinal slot in a text have also been limited to about 0.55δ_max [49].
Fig. 4.24: Swept Frequency Response of the Prototype Shunt-slot Fed Patch Element for $x_s = 38.195\text{mm}$ (Axial Offset, $\delta = 18.0\text{mm} \approx 0.9\delta_{\text{max}}$)
2) In an application where a series of slot-patch radiators is required to be implemented in the broadwall of a rectangular waveguide, these analyses will serve as a valuable design aid. The arrangement is in the form of a series-fed linear array of longitudinal slots – also referred to as a “stick” array [35, 49]. As mentioned there, this basic linear array can be used to build a planar array by stacking several such “sticks” together in the transverse direction.

3) In this case, it may be preferable to choose an offset in the range of 0.1 to $0.5\delta_{\text{max}}$ to avoid high back-scatter from the slot. This is seen to degrade the VSWR behaviour with higher slot offsets. Since a large number of slots would be used, a large fraction of the input power would be eventually radiated.

4) The behaviour of the VSWR or return loss characteristics needs mention. For the zero offset case, the waveguide is unperturbed by the presence of the slot and we obtain near-ideal matching (see Fig. 4.26.) Thereafter, the VSWR at resonance is seen to be progressively higher as the offset is increased. However, it stays well below -20 dB even for a relatively large offset of 14.0mm ($\delta = 0.7\delta_{\text{max}}$.) As a result, the antenna element will deliver acceptable VSWR performance even up to large slot offsets without needing special efforts for impedance matching.

5) Finally, we may draw a conclusion regarding the second resonance observed in the vicinity of 5.2 GHz for all the cases analyzed. The behaviour of the reactance curves is not consistent with the condition for resonance. Also, the behaviour reverses as the slot offset is varied. As a result, with the available results, it may be concluded that the behaviour is a spurious resonance attributed to numerical limitations.
Fig. 4.25: Parametric Variation of Coupled Power to Prototype Microstrip Antenna with Axial Offset of Coupling Slot, $\delta$

$\delta = x_s - 0.5a$
Fig. 4.26: Parametric Variation of peak VSWR (Return Loss) at resonance with Slot Axial Offset from Waveguide Centre Line, $\delta$ in mm
4.8 Impact of Patch Position relative to Coupling Slot

A final parametric investigation carried out with regard to the input characteristics of the proposed antenna is the behaviour if the patch centre is displaced away from the coupling slot centre. This is denoted in the present antenna geometry through the parameters \((x_p, y_p)\) shown in Fig. 2.4(b). With the centre of the coupling slot as a reference, these numbers represent the position of the patch centre away from it.

It is possible to specify a value for both of these i.e. the patch may be displaced away from the slot centre along both \(x\) and \(y\). However, the basis function representing the current distribution across the patch (Eqn. 2.26) assumes no variation along the \(x\)-direction (due to the dominant mode operation of the patch.) Displacing the patch in that direction would excite some amount of higher-order modes that have current variation along \(x\) also. As a result, change of patch position along \(x\) would require an extended formulation with current variation in that direction also to simulate correctly. This is presently outside the scope of this thesis but is a valid direction for further work on the proposed geometry. For the present case, the basis function of Eqn. 2.26 is only able to handle the change of patch position along \(y\)-direction.

The developed formulation is used for a parametric study of input characteristics of the prototype antenna element when the patch position is displaced from the slot centre \((y_p = 1.0, 2.0 & 3.0\, \text{mm})\)

4.8.1 Patch Position Offset from Slot Centre, \(y_p = 1.0\, \text{mm}\)

The initial input characteristics of the proposed prototype antenna element are for the case when the patch is positioned directly over the slot centre – this is as per Fig. 4.12. When offset by 1.0mm, we observe a drop in coupling level from 0.1007 (-9.97dB) to
Fig. 4.27: Swept Frequency Response of the Prototype Shunt-slot Fed Patch with Transverse Offset of Patch from Slot Centre, $y_p = 1.0\, \text{mm}$
0.0865 (-10.63dB) – see Fig. 4.27. The overall response of the impedance curves is similar except the resonance has shifted towards the higher side of the spectrum (to about 5.9GHz).

### 4.8.2 Patch Position Offset from Slot Centre, \( y_p = 2.0\text{mm} \)

With a further offset to the patch, the coupling level is seen to reduce further to 0.0726 (i.e. -11.39 dB.) The reactance zero-crossing occurs even higher around 6.0GHz – interestingly, the original design frequency of the prototype antenna!

### 4.8.3 Patch Position Offset from Slot Centre, \( y_p = 3.0\text{mm} \)

Finally, a frequency sweep is carried out for an offset of 3.0mm. The coupling factor is found to 0.0675 i.e. -11.70 dB. The resonance is shifted almost to the top of the frequency sweep range at 6.3 GHz.

With this patch position, the slot lies almost at the lower edge of the patch (width, \( W_p = 9.41\text{mm} \). Hence, further offsetting of the slot is not attempted. We summarize below the findings of the parametric exercise.

### 4.8.4 Observations on Parametric Study of Patch Position Offset from Slot Centre

The two key parameters affected by the offsetting of the patch position are the coupling factor and the resonant frequency (see Fig. 4.30.) We note the following.

1) It is observed that the coupling factor assumes a maximum value when the patch is centred exactly over the slot. As the patch is offset, the coupling factor is seen to reduce but the decrease is gradual and is significant even when the slot is placed nearly at the patch margin (\( y_p = 3.0\text{mm} \).) The reduction occurs as the dominant mode current peak is located at the patch centre. However, the
Fig. 4.28: Swept Frequency Response of the Prototype Shunt-slot Fed Patch with Transverse Offset of Patch from Slot Centre, \( y_p = 2.0 \text{mm} \)
Fig. 4.29: Swept Frequency Response of the Prototype Shunt-slot Fed Patch with Transverse Offset of Patch from Slot Centre, $y_p = 3.0\text{mm}$
Fig. 4.30: Parametric Variation of Coupling & Resonant Frequency with Transverse Offset of Patch from Slot Centre
alignment accuracy requirement of the slot and patch may not be critical as the reduction in coupling is seen to be gradual. This is along the lines of the observations made by Sullivan et. al. [20] and by Chang et. al. [35].

2) As the coupling reduces, the resonant frequency, signified by the peaking of the coupled power to the antenna element, is seen to increase. This is expected as the standing-wave current distribution along the patch width will offer a different impedance condition to the exciting slot. As a result, the impedance locus will shift position in the Smith Chart [20] resulting in a different frequency where feed reactance vanishes. The upward shift in the resonant frequency is also consistent with the findings in earlier work. For instance, it is known that the slot dimension needs to be increased to compensate the change in frequency due to the position offset.
4.9 Far-Field Radiation Pattern Plots for Prototype Patch Element

The preceding sections have presented a detailed study of the input characteristics of the proposed C-Band prototype waveguide shunt-slot fed microstrip patch antenna. Parametric studies have also been reported with regard to the chief design parameters of the antenna configuration. In this section, we present the far-field radiation characteristics.

The expressions for the far-field quantities have already been derived in Section 2.11. The basis of the computation, as explained in Chapter 2, is the reciprocity theorem invoked by placing an elementary dipole at the far-field position and computing its field at the position of the present antenna geometry. The computed far-field quantities represent the individual contributions of the current distributions across the coupling aperture as well as the patch surface, for each of the basis functions respectively into the two principal planes. We discuss the features of these individual far-field contributions in the following.

4.9.1 Radiation Pattern Plots of Far-Field Contribution by Patch Currents

**E-Plane:** The function \((P_l^{m1})^y_{x}\) in eqn. 2.126 represents the contribution to the E-plane radiation pattern due to the y-directed patch currents. The corresponding pattern plot for the first-order basis function is illustrated in Fig. 4.31 and is representative of odd expansion functions. The pattern shows a peak along boresight with a gradually tapered pattern with nulls in the plane of the antenna. As the order, \(l\), of the basis function is increased; the pattern will be tapered faster in inverse proportion as seen from the expression. No contribution is seen in the rear half-plane on account of the Green’s functions being originally for an infinite grounded dielectric slab.
Fig. 4.31: E-Plane Radiation Pattern for First-Order (Odd) Basis Function of Electric Current along Patch in Proposed Geometry
Fig. 4.32: E-Plane Radiation Pattern for Second-Order (Even) Basis Function of Electric Current along Patch in Proposed Geometry
For the even order basis functions along the patch surface corresponding to $l$ even in eqn. 2.126, we observe a null along boresight in addition to those in the plane of the patch (see Fig. 4.32.) This is a typical difference pattern arising from boresight cancellation due to the anti-symmetrical current distribution across the patch by virtue of the basis function. In the co-pol, this function may increase sidelobes or fluctuations in the “sum” pattern of Fig. 4.31. The function may also be responsible for cross-polar components when both orthogonal current components are assumed to exist on the patch surface.

**H-Plane:** The function for the H-plane radiation pattern contribution of the patch currents derived in this thesis is $(P_l^{m1})_{yy}$ as given by eqn. 2.128. For the case of $l$ odd, the function also exhibits a peak along boresight. However, the roll-off is sharper than for the case of E-plane described above (see Fig. 4.33.) Also, as expected, there are nulls along the plane of the antenna.

For the case of $l$ even, eqn. 2.128 indicates that there is identically no contribution of the currents associated with the even-order patch basis function in the H-plane. This is due to anti-symmetrical arrangement of the currents to either side of this plane.
Fig. 4.33: H-Plane Radiation Pattern for First-Order (Odd) Basis Function of Electric Current along Patch in Proposed Geometry
4.9.2 Radiation Pattern Plots of Far-Field Contribution by Slot Currents

**E-Plane:** The slot contribution to the radiation pattern in the E-Plane for the dominant mode of the antenna element is derived as \( (P_m^{m^2})_{\phi_x} \) in eqn. (2.135.) It is seen that the even-order expansion functions have a null response over all angular space and only the slot basis functions of odd order contribute to the far-field behaviour of the overall element. The pattern is seen to be the characteristic omnidirectional pattern of the slot with a very slight roll-off (~ 0.001dB) along the ground plane (see Fig. 4.34.)

**H-Plane:** The expression for the slot contribution into the H-plane, \( (P_m^{m^2})_{\phi_y} \), is derived in eqn. (2.133.) For the case of \( m \) odd, this function exhibits a peak on boresight (see fig. 4.35) with nulls in the plane of the radiator. The behaviour bears resemblance to the H-plane contribution of the patch currents as from the similarity of the expressions.

For the case of \( m \) even, there is a null seen in the boresight direction (see Fig. 4.36.) This is due to the anti-symmetrical distribution of the currents represented by the even-order basis function. There is similarity also to the corresponding expression for even-order basis function on the patch also.

With these individual contributions of the currents across the patch and slot respectively being computed, the net radiation pattern of the prototype antenna element can be determined by the superposition of these contributions. This is addressed in the following sub-section.
Fig. 4.34: E-Plane Radiation Pattern for First-Order (Odd) Basis Function of Magnetic Current along Aperture in Proposed Geometry
Fig. 4.35: H-Plane Radiation Pattern for First-Order (Odd) Basis Function of Magnetic Current along Aperture in Proposed Geometry
Fig. 4.36: H-Plane Radiation Pattern for Second-Order (Even) Basis Function of Magnetic Current along Aperture in Proposed Geometry
4.9.3 Analysis Results of Prototype Waveguide-fed Microstrip Patch Antenna

A complete solution for the proposed waveguide shunt slot-fed microstrip patch antenna may now be written. The developed formulation based on the method of moments (M-o-M) expands the unknown current distributions in terms of a set of entire-domain basis functions. Solution of the resulting matrix equation yields the unknown complex coefficients for the slot magnetic currents and the patch electric currents thus determining the unknown currents. These can now be used to compute the input characteristics of the antenna element as well as the far-field radiation patterns. This last step is carried out as per eqn. (2.119.)

We shall consider the case of the C-band prototype element assumed in Section 4.1. Since the originally selected parameters may not be optimum as determined from parametric analyses carried out in the subsequent sections, the following parameter set is used at present.

Waveguide dimensions: \( a = 40.39 \text{ mm} \); \( b = 20.19 \text{ mm} \);

Substrate Parameters: \( \varepsilon_r = 3.0 \); \( \tan \delta = 0.0013 \);

and \( d = 1.524 \text{ mm (60 mil)} \);

Patch Dimensions: \( L_p = 14.12 \text{ mm} \); \( W_p = 9.41 \text{ mm} \);

Coupling Slot Dimensions: \( L_s = 4.0 \text{ mm} \); \( W_s = 0.5 \text{ mm} \);

Axial Offset of Slot: \( x_s = 30.195 \text{ mm} \) \( \text{(Offset = 10.0 mm.)} \)

The other parameters pertaining to the numerical algorithms that are part of the formulation and solution process are also fixed as per the results obtained in the
Numerical Results

previous sections. These relate to the number of waveguide modes retained; the maximum value of $\beta$ where the numerical integration is truncated, etc. and are enumerated below:

Number of waveguide modes: $m = n = 50$;

Number of basis functions: $NA = 15$; $NB = 11$;

Truncation Limit in $\beta$, $\beta_{\text{max}} = 2.0 \times 10^5$.

The frequency for the solution was chosen as 5.8 GHz which is the resonance as seen in Fig. 4.12 for these values of the design parameters. The developed program was executed to obtain the complete solution for this case.

The obtained current distributions for both the slot and patch show a single prominent maximum at the centre of those structures (see Figs. 4.37 & 38.) It is well known that this is the characteristic current distribution for the case of a radiator at or near resonance. The predicted radiation pattern based on the field components described in the last sub-section was calculated using the coefficient vector determined as part of the M-o-M solution process (shown in Fig. 4.39.) We observe that there is a boresight peak in both principal planes; the H-plane pattern shows nulls along the ground plane while the E-plane pattern shows about 7-dB roll-off but no nulls. This is on account of the direct radiation by the slot in those directions. The current distributions as well as patterns are seen to be in close agreement to those reported by Harish [29] although he uses an end-wall slot to excite the microstrip patch and sub-domain basis functions for analysis. By this observation, the present analysis appears to be validated.
Numerical Results

Fig. 4.37: Computed Slot Magnetic Current for Prototype C-Band Antenna Element

Fig. 4.38: Computed Patch Electric Current for Prototype C-Band Antenna Element
Fig. 4.39: Computed Principal Plane Radiation Patterns for C-Band Prototype Microstrip Patch Antenna Element at 5.8 GHz
4.9.4 *Analysis Results of Prototype Antenna with Patch Position, y_p = 3.0mm*

Another analysis is carried out for the case when the patch is offset from the slot centre by setting \(y_p = 3.0\) mm. As seen from Fig. 4.29, resonance condition for this case is shifted towards higher side of the spectrum and peak power coupling occurs at 6.3 GHz. The developed program was executed for the prototype element with all other parameters as in the previous section except \(y_p\).

The resulting current distributions across the slot and patch respectively are seen to be different from the previous case (see Figs. 4.40 & 41.) We observe a reduction in the overall magnitude of both slot magnetic current as well as patch electric current compared to the centred-patch case of previous section. This is corroborated by the relatively lower coupling level seen in Fig. 4.29 also. A slight asymmetry is seen in the slot current in Fig. 4.40 but is more pronounced in the patch current distribution (Fig. 4.41.) This is due to the asymmetric excitation by the slot and the peak current occurs close to where the slot is located under the patch.

Very little impact is seen on the far-field pattern characteristics (see Fig. 4.42) due to the offset slot feed. Both H- and E-Plane patterns are seen to taper off relatively faster as compared to the previous case but symmetry is retained. This is because the slot itself is non-resonant and does not radiate appreciable levels of energy compared to the patch. The latter is still operating in the fundamental \(TM_{01}\) mode even though the excitation is off-centre. The relatively faster roll-off is primarily on account of the higher frequency of operation.
Numerical Results

Fig. 4.40: Computed Slot Magnetic Current for Prototype Element with $y_p = 3.0$ mm

Fig. 4.41: Computed Patch Electric Current for Prototype Element with $y_p = 3.0$ mm
Fig. 4.42: Computed Principal Plane Radiation Patterns for Prototype Antenna Element (with $y_p = 3.0$ mm) at 6.3 GHz
4.10 Summary

This chapter has presented the analytical results obtained from the computer program based on the developed formulation developed in this thesis for a prototype C-Band microstrip antenna element excited by a longitudinal waveguide slot. A set of parameters are first obtained for a C-Band antenna element resonating nominally at 6 GHz. Considerations for the choice of design resonant frequency, waveguide section, substrate parameters, patch / slot dimensions and transverse offset of coupling slot are detailed. Next, the convergence criteria for the moment matrix terms have been established viz. the number of waveguide modes to be retained, the discretization interval in $\alpha$ and $\beta$; and the truncation limit for the infinite integral in $\beta$. The number of basis functions necessary to obtain a converged solution is determined by observing the current distributions and input parameters obtained therefrom. This is followed by swept frequency analyses of the prototype antenna. Parametric studies are presented for the key design dimensions of the antenna viz. coupling slot length & width; axial offset of coupling slot; and patch position w.r.t. slot centre. Computed input parameter plots are presented in detail for all settings of the design parameters. Later, the key performance parameters are plotted showing the variation with the parameter. Useful design guidelines are obtained from these studies in this manner. A narrow slot width yields the smoothest response since the assumed current distribution neglects variation across slot width. Choice of the slot length is critical in obtaining resonance. A monotonic control range for the power coupling to the radiator is obtained by using the transverse offset of the slot from the waveguide centre-line. Patch centering above slot is not critical in terms of tolerance but detunes the radiator slightly. Finally, the far-field contributions to the principal planes by the various basis functions on the slot and patch are presented. The odd-order basis functions produce a
peak on boresight. The even-order functions radiate a typical difference pattern with a null on-axis. The functions corresponding to the individual pattern contributions are then used to compute the aggregate far-field pattern including the effect of actual current distributions determined in the M-o-M solution along with weighting by the computed coefficient vector. Radiation patterns show a single lobe in the forward half-plane in both principal cuts. The H-plane pattern shows nulls along the plane of the substrate while the E-plane shows considerable radiation in that direction. The latter effect is the direct contribution of the slot radiation. The current distributions and pattern plots are given for two cases – that of the prototype element and with the patch displaced from the slot centre slot. For the latter case, asymmetry in the induced patch current distribution is also observed.