INTRODUCTION

This thesis is in the field of Algebra.

In abstract algebra, the concept of a module over a ring is a generalization of the notion of vector space, where instead of requiring the scalars to lie in a field, the scalars lie in arbitrary ring.

The notion of purity was first introduced in abelian groups. This notion was extended to modules over principal ideal domains (PID). In 1952, Kaplansky [18] introduced the notion of purity for modules over Dedekind domains. He has generalized most of the results in abelian groups to modules over a Dedekind domain. P.M.Cohn [5] was first to define the purity in terms of tensor product for modules over an arbitrary ring and has shown that this notion is equivalent to one introduced for modules over a PID. Later Stenström [30] has studied various notions of purity. Warfield Jr.[33] has given projective characterization for purity and has introduced the notion of RD-purity which is same as purity in abelian groups. Hiremath [15] has dualized the notion of purity as copurity and has made detailed investigation of copurity. Wisbauer[33] has introduced the notion of P-purity for any class P of R-modules. This includes Cohn’s purity since the latter is P-purity for P to be the
class of all finitely presented modules.

Choudhary and Tiwari [4], Simmons [29] and Divani-Aazar [7] have respectively studied cocyclic copurity, cyclic purity and cyclically purity.

In this thesis, we have made detailed study of cyclic purity and its dual cocyclic copurity.

The thesis consists of five chapters. The Chapter-I is introductory in nature, consists of all the preliminary notions needed in the further chapters and includes all the notations and terminology used in later chapters.

In Chapter-II, we make a detailed study of cyclic purity. This concept is introduced by Simmons [29], in case of commutative integral domains. Here we study the concept in general rings. Also the concept of cyclically purity given by Divaani-Aazar [7], is studied in detail and compared with cyclic purity.

Chapter III is the continuation of Chapter-II. In Chapter-II we concentrate mainly on cyclic pure submodules and here we study cyclic pure exact sequences. The concepts of absolutely cyclic purity, c-flatness and c-regularity are introduced by considering cyclic pure exact sequences. Also c-pure projective, c-pure injective are introduced and studied. The
concept of quasi-c-pure injectivity is studied and several non-trivial examples are given for this concept.

In Chapter-IV, we introduce a generalization of cyclic purity namely $n$-$c$-purity. Then we study the concepts $n$-absolutely $n$-$c$-purity, $n$-$c$-flat and $n$-$c$-regularity, by considering the $n$-$c$-pure exact sequences.

The concept of dualization plays an important role in module theory. Generally with respect to a property of the concept the dualization is studied. In Chapter-V, we study a dualization of $c$-purity, namely cocyclic co-purity. We have compared this concept with cyclic purity and purity. Cocyclic copure exact sequences are also studied. The concept of solvability is introduced and studied. We have given several examples for solvability using purity and copurity studied in earlier chapters.